ASSESSMENT OF THE REPUBLIC OF SERBIA'S SYSTEMIC RISK AND THE LIKELIHOOD OF A SYSTEMIC CRISIS

Darko Kovačević

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Financial Stability Department

THE NATIONAL BANK OF SERBIA

Belgrade, 12 Kralja Petra Street,

Telephone: (+381 11) 3027 100

Belgrade, 17 Nemanjina Street,

Telephone: (+381 11) 333 8000

www.nbs.rs

Assessment of the Republic of Serbia's Systemic Risk and the Likelihood of a Systemic Crisis

Darko Kovačević

Abstract: The purpose of this paper is to set up the Systemic Stress Indicator (SSI) of the Republic of Serbia’s financial system based on the proposed modification of the systemic stress testing approach that allows for a more appropriate aggregation of the observed indicators within the financial system segment. It also weighs up the advantages of the proposed approach compared to the aggregation methods most frequently used in literature. It proposes a mathematical formulation of the systemic risk level of the financial system and an analytical framework of the early warning system based on an assessment of the likelihood of a systemic crisis occurrence in case of an arbitrary number of regimes. The SSI has demonstrated the ability to correctly identify crisis periods and the systemic risk level of the Republic of Serbia’s financial system. It is suggested that probabilities of a systemic crisis occurrence in a given period are in perfect sync with the dynamics of undetected periods. An optimal period in the case of used indicators and relatively short time series is six months, which may provide timely signals to policy makers to mitigate negative effects on financial and macroeconomic stability.

Key words: Financial stability, Systemic risk, Financial crisis, Macro-financial linkages, Markov-switching model, early warning model
JEL Code: C32, G01, E44

Non-Technical Summary

An important lesson learned from the global financial crisis of 2008 was that national regulators and other financial system supervisory authorities lacked adequate tools to facilitate the analysis and measurement of systemic risk in real time. The preconditions for the development of analytical tools for systemic risk identification implied, on the one hand, the understanding of economic processes leading to the build-up of these risks, and, on the other hand, the analysis of events that may generate material costs in the real economy. Financial instability may become systemically significant through an exogenous and/or endogenous shock, if a larger part of the financial system is hit at the same time. The start of a crisis at a systemic level is usually accompanied with a trigger event, while recovery is a long-standing process, and the moment of exiting the crisis is often unknown. Moreover, although each financial crisis is unique in terms of causes and the channels of transmission to different market segments, it is important to compare different systemic events by using the indicators to measure the level of systemic stress.

The aim of this paper is to construct the Systemic Stress Indicator (SSI) of the Republic of Serbia's financial system, based on the modification of approach to systemic stress evaluation proposed by Hollo and a group of authors, which enables the aggregation of the observed indicators within financial system segments. As the composite indicator is based on movements in different financial market segments, it enables a solid evaluation of linkages between these segments, i.e. it allows for the possibility to evaluate the systemic component and individual risk factors. We have presented a detailed analytical framework for the calculation of the composite indicator and the systemic risk component, based on Markov switching models with dynamic transitive probabilities. We have identified the advantages and shortcomings of different approaches to the aggregation of sub-indices of financial system segments, based on which we have proposed an approach that is consistent with the composite indicator methodology and enables exact mathematical formulation of the systemic risk level. We have also validated the number of systemic crisis regimes by applying the Gaussian component method. Both approaches assessed in the same way the number of regimes and moments of SSI distributions, which suggests the uniqueness of the obtained results.

The paper also contains an analytical framework for early warning signals, which is based on the methodology used for SSI calculation and which is unique not only for the assessment of the current stress level in the system, but also for the assessment of probability of a systemic crisis in future in case of an arbitrary number of regimes. The quality of classification of the proposed model was validated by assessing the measures of the quality of classification in case of an arbitrary number of regimes, and based on public statements of economic policy makers and experts.

It has been demonstrated that the SSI can accurately identify crisis periods, the level of systemic risk of the financial system, and that it can assess the probability of a systemic crisis occurrence, providing significant information about the degree of risk accumulation in financial markets and potential implications for financial and macroeconomic stability.

Contents

1 Introduction

The global financial and economic crisis of 2008 revealed weaknesses in the regulation of financial systems internationally. One of the important lessons learnt during the crisis is that national regulators and other authorities responsible for supervising the financial system did not have appropriate tools to facilitate real-time risk perception and assessment. Another problem was that even when regulators were aware of the risk and when unfavourable trends that could affect the overall financial system were assessed, they did not have appropriate regulatory mechanisms to take emergency intervention. The financial crisis showed that so-called macroprudential regulation and supervision should be added to microprudential supervision of financial markets and intermediaries (Borio 2003) in order to identify imbalances and vulnerabilities of the overall financial system. Therefore, a large number of initiatives were introduced with the aim of reviewing existing regulatory frameworks. At the same time, intensive research work was initiated in order to develop new analytical tools, which the authorities responsible for macroprudential supervision would use for timely decision-making.

The prerequisites for the development of analytical tools for identifying systemic risks include the understanding of processes leading to the accumulation of these risks, on the one hand, and the analyses of events that may result in material costs on the other (an occurrence of a financial crisis with considerable costs in the real economy (Crotty 2009)). The three main sources of systemic risk are mentioned in scientific literature: the accumulation of financial imbalances (Obstfeld and Rogoff 2009), e.g. robust credit growth is linked to a sharp rise in the price of assets, exogenous external and/or internal shocks affecting financial market actors (Houben et al. 2004), and the financial contagion effect (Schwarcz 2008).

International experience shows that financial crises with high costs for the real economy are often preceded by persistent and excessive asset growth. During these periods, growth in consumption and investments, as well as expansion of lending for financing further growth, may become a self-perpetuating process, accompanied by risk accumulation. These periods are usually characterised by the weakening of banks' standards during the loan approval process, which results in riskier categories of borrowers being able to access banks' funds. This seemingly sustainable credit growth, consumption and investments may be interrupted by even minor financial shocks if they affect several segments of the financial system at the same time.

Financial instability may become systemically insignificant, through an exogenous external and/or internal shock, if a larger segment of the financial system is concurrently affected. As a result, depending on the risk accumulation and balance sheet imbalances, many financial institutions may face a lower capital quality and business problems, which are often called the primary effect. A financial contagion can occur endogenously regardless of the above processes. If one or more banks adversely affect the business sustainability of other financial institutions or even financial markets as a whole, as a result of negative external and/or internal shocks, a contagion effect called the secondary effect is present.

It most often occurs in case of unexpected problems with the operation of systemically important financial institutions, which may reflect on other financial institutions and financial markets. It is important to emphasise that in this case the effects do not necessarily result from the macroeconomic environment, but from the inadequate business policy, operating model and risk management function of a single, systemically important, financial institution. The onset of any crisis is usually accompanied by a triggering event, whereas the recovery is a long-lasting process with an often unknown ending. In addition, although each financial crisis is unique in terms of its cause and the channel of transmission to various market segments, it is important to compare various systemic events with certain indicators used to measure the systemic stress level.

The financial stress level largely depends on the size of the shock that hit the system, the risk accumulation level and financial system imbalances and responses of decision makers responsible for preserving macrofinancial stability, as well as market expectations in terms of these responses. The first step in establishing a uniform methodology for calculating indicators used to assess financial sector soundness, which are implemented by a large number of national regulators, are financial soundness indicators proposed by the International Monetary Fund for the purpose of a cross-country comparison and establishment of a macroprudential policy framework at national levels (IMF 2008). Unlike numerous indicators that are related to the economic activity and can be expressed in a money equivalent, the financial stress level does not have its unique form in the real economy. Since it cannot be directly measured, the idea behind the creation of the SSI, presented in this paper, is to measure different stress manifestations. The SSI aims to aggregate different financial segments into a single composite indicator. Since the composite indicator focuses on financial market movements, it provides a good estimate of linkages between these segments, i.e. the possibility to assess a systemic component and movement of individual risk factors.

The main objective of this paper is the construction of the SSI of the Republic of Serbia. The main or general objective of the SSI and other financial soundness indicators is an assessment of the current financial system stress level or its lack. The paper provides a detailed analytical framework for the creation of indicators, even in case of the lack of high-frequency data, simultaneously considering the characteristics of the local financial system that are related to a relatively high level of euroisation and significant foreign ownership in the banking sector, where one of the primary monetary policy transmission mechanisms is the exchange rate channel. The aggregation of financial system segment indicators is discussed in detail, and the strengths and weaknesses of each approach are elaborated upon. The paper also provides a framework for assessing the likelihood of a systemic crisis occurrence based on the same framework, which is a natural upgrade of the said methodology.

The paper is divided into four parts. After the introduction, we present an analytical framework for the aggregation of different financial system segments and creation of a composite indicator in the second part by using dynamic Markov-switching regimes. In addition, the indicators of different segments of the Republic of Serbia's financial system are discussed in detail and historical episodes of increased stress are identified.

The third part deals with the development of a methodology and mathematical model for calculating the likelihood of a systemic crisis occurrence in an arbitrary time horizon based on the presented SSI methodology. We select the best early warning model based on classification quality measures in case of an arbitrary number of regimes. We also validate the obtained probabilities of a systemic crisis occurrence based on public statements of the Republic of

Serbia's economic policy makers about the condition of the financial system from 2008 until early 2021.

Finally, the fourth part contains a conclusion and the main results of the conducted analysis and contributes to scientific literature in the form of the overall methodology not only for assessing individual risks in the financial system and systemic risk as a whole, but also for providing systemic crisis warning signals.

2 Creation of a composite indicator of systemic stress

In order to answer some of the above questions, the present analysis will introduce a financial stress index called the Systemic Stress Indicator (SSI), based on the Composite Indicator of Systemic Stress (CISS), proposed in the paper by Hollo et al. (2012). The main or general objective of the CISS and other financial soundness indicators is an assessment of the current financial system instability/stress level. Therefore, a financial system can be defined as a set of financial markets, intermediaries and infrastructure. A separate sub-index of each financial system segment is calculated after a proper transformation of individual indicators. The main methodological innovation during the development of this type of indicator, presented in the paper by Hollo et al. (2012), is the use of the portfolio theory in the sub-index aggregation into a composite indicator. The portfolio aggregation takes into account concordance measures between different sub-indices. As a result, an indicator assigns a larger weight when stress prevails in several market segments simultaneously, which indicates that systemic risk/stress is higher if financial instability is widespread in most of the financial system.

Another aggregation element that characterises a systemic risk is the fact that portfolio weighting functions are calibrated for each sub-index based on their impact on the industrial production index as the economic activity measure of a country.

The first step in calculating the SSI is the creation of sub-indices for each of the selected financial system segments. The most frequently used aggregation approach is the equal variance method, which assigns the same weights to all observed indicators. It is desirable for the methodology to include a common variability factor that occurs in the data. For the effects of individual data series to be comparable, input data need to be standardised. We use the empirical cumulative distribution function to reduce each indicator to an interval between zero and one, using the following formula:

$$z_t = F_T(x_t) = \begin{cases} r/T & for\ x_{[r]} \le x_t < x_{[r+1]}, r = 1,2,\ldots,T-1 \\ 1 & for\ x_t \ge x_{[T]} \end{cases} \tag{1}$$

Where $x_{[r]}$, $r = 1,2,\ldots,T-1$, $t = 1,2,\ldots,T$ and T is the sample length. By doing so, each data point obtains its rank that corresponds to the quantile of the cumulative probability distribution function ranging from 0 to 1.

According to the portfolio aggregation of the composite indicator (Hollo et al. 2012), a high correlation between individual risks results in an increase in the overall portfolio risk. On the other hand, if the correlation between individual portfolio segments is low, the overall portfolio risk is reduced, i.e. individual risks are diversified.

Unlike the original approach, in which the authors obtain the sub-index value using the equal variance method, i.e. a weighted sum of indicators within a segment, we suggest a different creation of these indicators that would comply with the composite indicator creation approach and enable an exact mathematical formulation of the systemic risk level Sys . An analysis of the existing and proposed aggregation approach is discussed in detail in Appendix 1. If the overall risk of an individual segment is observed as one sub-index, the value of this indicator, based on the portfolio risk aggregation theory, can be obtained as follows:

$$SI_i^t = \sqrt{(\boldsymbol{W}_i \circ \boldsymbol{Z}_i^t)\mathbb{C}_t(\boldsymbol{W}_i \circ \boldsymbol{Z}_i^t)^T}, \tag{2}$$

Where $z_{i,j}^t$ represents transformed indicators within the segment i, w_i represents a time-invariant weighting function of equal variances, and $\mathbb{C}_t$ represents a correlation matrix, whereas $\circ$ is the Hadamard product:

$$\begin{aligned} \boldsymbol{Z}_i^t &= (z_{i,1}^t, z_{i,2}^t, \dots, z_{i,j_i}^t) \\ \boldsymbol{W}_i &= (w_{i,1}, w_{i,2}, \dots, w_{i,j_i}), \end{aligned} \tag{3}$$

where j_i is the number of financial system segment indicators i. The following are weighting functions: $w_{i,j_m} = w_{i,j_n}, \forall m, n \in J_i$.

Out of individual SI_i^t sub-indices, the SSI is obtained based on the following equation:

$$SSI^t = \sum_{j=1}^{q} (\boldsymbol{W_k} \circ \boldsymbol{SI}^t\ )\boldsymbol{I}\,(\boldsymbol{W_k} \circ \boldsymbol{SI}^t\ )^T + (\boldsymbol{W} \circ \boldsymbol{SI}^t\ )(\mathbb{C}_t - \boldsymbol{I})(\boldsymbol{W} \circ \boldsymbol{SI}^t\ )^T, \tag{4}$$

Where $W_k = [w_k]$, $w_k = \begin{cases} 1 & \mathrm{k = j} \\ 0 & \mathrm{k \neq j} \end{cases}$, q represents the number of the observed segments, I is a unit matrix, $SI^t\ = (SI_1^t, SI_2^t, \dots, SI_q^t)$ is an appropriate vector of the sub-index value and $\mathrm{W} = (w_1, w_2, \dots, w_q)$ represents an appropriate vector of sub-index weights.

The first part of the equation (4) concerns the impact of individual segments, i.e. sub-indices, while the second part of the equation (4) exactly determines the systemic risk level Sys at the moment t:

$$Sys^t = (\boldsymbol{W} \circ \boldsymbol{SI}^t\ )(\mathbb{C}_t - \boldsymbol{I})(\boldsymbol{W} \circ \boldsymbol{SI}^t\ )^T. \tag{5}$$

Typically, in case of negative shocks in a financial system, many financial system segments are concurrently affected, which results in a high correlation between segments (Lo Duca and Peltonen 2011). In addition, an adequate sum of members can be shown in a more compact form that is used in the paper by Hollo et al. (2012):

$$SSI^t = (\boldsymbol{W} \circ \boldsymbol{SI}^t\ )\mathbb{C}_t(\boldsymbol{W} \circ \boldsymbol{SI}^t\ )^T \tag{6}$$

Scientific literature provides numerous definitions of systemic risk. It is defined mainly based on the final effect on the real economy (De Bandt et al. 2009). Here we see a correlation between a financial system and the real economy that we discussed at the beginning of this paper. The sub-index weighting vector $\boldsymbol{W}$ is consistent over time and is estimated using the maximum reliability method by means of an analysis of sub-index effects on the y-o-y industrial production growth rate – IP:

$$\hat{l}(\boldsymbol{W}|\boldsymbol{SI}) = \frac{1}{\mathrm{T}}\sum_{t=1}^{\mathrm{T}} ln\big(f(\boldsymbol{X}|\boldsymbol{SI})\big), \text{ where}$$
$$f(\boldsymbol{SI}|\boldsymbol{W}) = \frac{1}{2\pi^{T/2}\sqrt{\det|\boldsymbol{\Sigma_t}|}} exp\left(-\frac{1}{2}\big((\boldsymbol{W}^T\boldsymbol{SI} - \boldsymbol{IP})\boldsymbol{\Sigma_t}(\boldsymbol{W}^T\boldsymbol{SI} - \boldsymbol{IP})^T\big)\right) \quad (7)$$

With restrictions:

$$\sum_{i=1}^{q} w_i = 1 \text{ и } w_i > 0$$

In case of a sub-index calculation and the SSI, $\mathbb{C}_t$ represents a matrix of time-varying correlation coefficients, which is defined as:

$$\mathbb{C}_t = \begin{bmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1q,t} \\ \rho_{21,t} & 1 & \cdots & \rho_{2q,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{q1,t} & \rho_{q2,t} & \dots & 1 \end{bmatrix} \quad (8)$$

The general member of this matrix is the correlation coefficient between the variables i and j at the moment t:

$$\rho_{ij,t} = \frac{\sigma_{ij,t}}{\sigma_{i,t}\sigma_{j,t}} \quad (9)$$

where $\mathrm{i} = 1, \dots, \mathrm{q}$ and $j = 1, \dots, q$, $\sigma_{ij,t}$ is a covariance between the variables *i* and *j* at the moment t, and $\sigma_{\mathrm{i,t}}$ and $\sigma_{j,t}\sigma_{\mathrm{ij,t}}$ are standard deviations of the variables *i* and *j* at the moment *t*, which are simultaneously estimated using the exponentially weighted moving average (EWMA) based on the following formulas:

$$\sigma_{ij,t} = \lambda\sigma_{ij,t-1} + (1-\lambda)(s_{i,t-1} - \mu_{i,t-1})(s_{j,t-1} - \mu_{j,t-1}) \quad (10)$$
$$\mu_{i,t} = \lambda\mu_{i,t-1} + (1-\lambda)s_{i,t-1}$$

The parameter λ refers to a persistence of the covariance. High values $\lambda \to 1$ indicate high inertia and low reaction of the process to previous values and vice versa (Alexander 2009). Here $i = 1, \dots, q$ and $j = 1, \dots, q$, $\mu_{i,t}$ represents the expected variable value, whereas the parameter λ takes on a constant value of 0.93 (Louzis and Vouldis 2013).

2.1 Financial system segments and selected indicators

The main difference between the SSI and other financial soundness indicators is that the SSI is focused on the systemic dimension of financial stress. The SSI includes 25 indicators that reflect the financial stress level in six crucial segments of the Republic of Serbia's financial system: foreign exchange market, public finances, money market, capital market, banking sector and foreign environment.

One of the objectives during the selection of variables for the SSI construction is the coverage of the local financial system segments and characteristics of the foreign environment, which can be seen in Appendix 2. The segments are named as follows: foreign exchange market – FX, public finances – GOV, money market – MON, capital market – EQU, banking sector – BANK and foreign environment – FOR. The range of the local financial system indicators largely reflects the characteristics of the Republic of Serbia's financial system. The data cover the period from January 2008 to March 2021. Even though the time series includes a period of only 13 years, we deem that the systemic stress indicator efficiently defined the periods that occurred during the said time interval. The variables are taken on a monthly basis even though the methodology itself offers the option of using higher-frequency data. Due to a small number of financial institutions in the capital market and the lack of market indicators, monthly data from regulatory reports of financial institutions were used. Thus, some of the indicators used in developed economies (Hollo et al. 2012) could not be applied in the case of the Republic of Serbia.

The first analysed segment of the financial system was the foreign exchange market. The used indicators take into consideration the characteristics of the local financial system that are related to relatively high euroisation and significant foreign ownership in the banking sector, where one of the key monetary policy transmission channels, apart from the interest rate channel, is the exchange rate channel. Apart from the fact that euroisation reduces the efficiency of monetary and fiscal policies, exchange rate fluctuations are highly important in circumstances of excessive foreign-currency lending, as it is the case in Serbia, and the currency-induced credit risk. The exchange rate is a result of economic developments, the interest rate and specificities of the economy and its environment. Apart from exchange rate fluctuations, the volatility of the dinar against the euro is used to calculate the SSI. Volatility indicators are generally observed as a reflection of uncertainty, i.e. the higher the volatility the higher the values of the indicators used for measuring the stress level. The FX market may respond to new information on expected exchange rate fluctuations. However, it turns out that not all information will always have a critical impact on the exchange rate. After some time, it can be ascertained that some information was not relevant due to asymmetric information available to the participants. Therefore, exchange rate fluctuations observed over a longer period are expected to be smaller than those observed over a short period (Dornbusch 1976). Another FX market indicator used to create the SSI is the average daily difference between the buying and selling exchange rate of the dinar against the euro, based on indicative quotations in the interbank FX market, which is related to the liquidity of the currency pair. In general, the liquidity of assets is related to the speed and ease at which they can be traded. The difference between buying and selling exchange rates contains an incorporated liquidity risk premium and thus, a difference increase contributes to an increase in the SSI value. The last indicator refers to the net value of FX interventions of the National Bank of Serbia in the domestic FX market. All exchange rate features explained so far are related to the rate freely formed in the domestic foreign exchange market on the basis of the supply and demand of currencies. The National Bank of Serbia pursues a managed float exchange rate regime and intervenes in the interbank market to ease excessive short-term volatility of the dinar against the euro, with no intention to influence the exchange rate level or its trend, and to preserve price and financial system stability and maintain an adequate level of FX reserves. This rule of the National Bank of Serbia complies with its Memorandum on Inflation Targeting as a Monetary Strategy, which has been officially applied since January 2009, and with its monetary policy programmes. When intervening in the interbank market, the National Bank of Serbia absorbs the stress that would otherwise reflect on the dinar/euro exchange rate. With this in mind, the value of FX interventions in the domestic FX market reflects pressure on the local currency and thus impacts the SSI.

The second financial system segment used to assess the SSI is the money market, which is a financial market segment where short-term financial instruments are traded. In general, assets whose maturity is shorter than one year are traded. Transactions with giro money, short-term loans, discount and Lombard transactions and short-term securities transactions are mainly conducted in this market. Since the money market enables an applicant to trade certain assets for cash within the shortest possible time, liquidity is the main feature of this market. The first indicator of the money market condition is BEONIA (Belgrade OverNight Index Average), which is an interest rate formed as a weighted average of interest rates on overnight interbank loans. Pursuant to the Decision on Submitting Data on Overnight Interbank Money Market Loans to the National Bank of Serbia, an overnight loan means lending dinar funds from one bank to another with a repayment deadline being the end of the following business day from the lending day. By including BEONIA, we assume that in case of severe turbulences in the banking sector, banks might start borrowing excessively, which would result in an increase in the average interbank interest rate, leading to the increase of the SSI. However, if we look at historical fluctuations of this rate, we can see that BEONIA is usually in sync with the repo rate. Apart from the BEONIA value, we also use the standard deviation of the rates at which individual transactions are concluded in the interbank money market against the BEONIA rate. A significant deviation of the rates at which interbank market transactions are conducted from the BEONIA rate may be the signal of money market uncertainty. An increase in the deviation leads to an increase in the SSI value. Apart from the BEONIA rate, we also use the difference between the BEONIA rate and the key policy rate. The key policy rate is the main instrument of the National Bank of Serbia's monetary policy. A negative value of this indicator shows a sufficiently high liquidity level of the banking system, which reduces the SSI value as expected. On the other hand, if banks are forced to borrow at an interest rate higher than the key policy rate, it should signalise that they are facing a liquidity shortage. Apart from the above indicators, we also included the difference between an interest rate ceiling on overnight interbank market loans and the interest rate on credit facilities. According to the Decision on Interest Rates applied by the National Bank of Serbia in the Implementation of Monetary Policy, the interest rate on an overnight loan for maintaining banks' daily liquidity (credit facility) is determined at the level of the key policy rate plus 0.9 pp. As a pledge for this loan, banks need to deposit a certain amount of dinar securities. If a bank failed to take such loan from the National Bank of Serbia and was forced to borrow in the interbank market at an unfavourable interest rate, it means that it was in urgent need for additional liquidity, which can be interpreted as a sign of stress, and therefore the positive value of this indicator should increase the SSI value. Another liquidity ratio in the money market is the required reserve allocation. The average required reserve allocation for a given period on the last day of the maintenance period should be equal to or higher than 100% of calculated required reserves. Even though it is best that the allocated required reserve be exactly 100% of the calculated reserves, a higher amount is not particularly worrying as it implies that banks are liquid more than necessary. An amount lower than 100% implies a liquidity shortage, which could imply that banks are not able to allocate sufficient funds for the required reserve, i.e. they are not liquid enough. Relatively high euroisation of our economy makes us dependent on euro fluctuations in international markets. Therefore, the SSI includes the difference between three-month EURIBOR and the interest rate on three-month German Bunds (government bonds). EURIBOR is an average interbank interest rate offered among major European banks. In addition, the difference between three-month EURIBOR and an EONIA (Euro OverNight Index Average) interest rate was used. The EONIA interest rate is important as it does not only contain market expectations about the European Central Bank’s monetary policy, but it also limits fluctuations of longer-maturity interest rates. The dynamics of difference fluctuations was the focus of empirical studies on the effects of financial crises on the money market (Tamakoshi and Hamori 2015), which indicate that certain fluctuations of this indicator may be an early warning signal of potential money market turbulences.

The capital market, as the third financial segment used to calculate the SSI, is a market where cash and cash equivalents are supplied and demanded long-term. It is an institutionalised market with clear rules of conduct for its actors and product standardisation. Trading in the capital market provides its participants with investment diversification and optimum maturity asset transformation. The global financial crisis showed that its effects can be considerable in spite of opinions about its limited effects on shallow and insufficiently developed markets, such as the Serbian capital market. The first representative capital market indicator is the CMAX transformation (Illing and Liu 2006) of BELEX15 index of the most liquid stocks on the Belgrade Stock Exchange, whose purpose is to identify a sudden or an extended drop in the equity price, which represents a capital market stress symptom. BELEX15 quantifies the performance of the most liquid segment of the domestic capital market. A stock index basket is comprised of common shares in the regulated market, which had minimum 80% of trading with concluded transactions over the past two quarters. Those stocks that meet this criterion are ranked against market capitalisation in free circulation. The CMAX transformation measures the maximum cumulative loss in a given period. The maximum cumulative loss over the past 12 months is observed for the purposes of this paper. Higher values of this indicator mean a bigger cumulative loss and consequently an increase in the SSI value and vice versa. If the value of company’s shares is high, the market estimates that such company is valuable and stable. Money invested in it will not be lost as the risk of going out of business is low. Investors assess all relevant information when trading and their risk perception is recorded in the stock price. Hence, a value increase in this indicator implies a lower risk. The reason why stock exchange data are interpreted in a simple manner is because a stock exchange is one of the most liquid markets. All stock exchange transactions are conducted in real time and therefore the information is quickly incorporated in the price. Thus, a price or yield formed on the stock exchange is an anticipating indicator. In addition to the price index, the turnover index that is often linked to information asymmetry of market participants is also of particular interest (Chae 2005). Market participants cannot be ideally informed about business operations of entities and therefore they trade considerably less out of caution during financial stress periods. Lower trading values of the BELEX15 index indicate a higher SSI level. The indicators that we will use are realised volatility of the BELEX15 index stated through the closing price and realised volatility of the BELEX15 turnover. If price/trading fluctuations are significant, investors cannot assess with certainty the company they invest in. A high uncertainty level is interpreted as a stress signal and results in an increase in the SSI value.

The effects of financial crises on public finances are reflected in a decrease in public revenue due to economic downturn. At the same time, there is an increase in expenditures and deficit, which is covered with higher government borrowing in domestic and foreign markets, which becomes more expensive in case of financial imbalances. In addition, the necessity for government interventions in order to preserve stability of the economy becomes more pronounced. The global financial crisis faced countries with a challenge to maintain a consumption level that supports future economic growth. In case of longer crisis periods, there is a need to support aggregate supply as it will ease the pressure on public consumption, which is further aggravated if the deficit was high before the crisis. This can result in a sovereign debt crisis that is defined as economic and financial effects triggered by the perception of the government’s inability to repay its public debt. It usually occurs when a country reaches a critical level of external debt and low economic growth at the same time. The first indicator of public finances is the difference between the yield to maturity of the domestic 10Y government eurobond and the 10Y German government bond, which implies the risk perception of the Republic of Serbia’s debt against Germany’s low risk debt. When the yield difference increases, the value of the SSI also increases since a higher potential yield is related to a higher risk. Another indicator is the difference between the quotes of the yield to maturity of the 10Y government eurobond. The difference between yield quotes has an analogous meaning, as explained in the paper. A substantial difference implies that investment in this bond is perceived as risky and results in an increase in the SSI value. High-risk premia incorporated in the difference suggest liquidity risk. Volatility indicators point to investors’ uncertainty about the measurement of assets, i.e. the higher the volatility the higher the values of the indicators used for measuring the stress level, which indicates increased market turbulences. Therefore, the SSI calculation includes the realised volatility of the yield to maturity of the 10Y government eurobond. The Emerging Markets Bond Index Global Serbia (EMBI Global Serbia) is one of the sub-indices based on which EMBI Global is formed (Cavanagh and Long 1999) and it has an analogous meaning. Increased risk perception leads to an increased EMBI and consequently a higher SSI. High values of the consolidated fiscal deficit (IMF 2001) in relation to the gross domestic product imply unsustainable public debt of a country and its vulnerability as a debtor, which is for investors often a signal of an elevated fiscal policy sustainability risk. Attinasi et al. (2009) showed that countries facing the highest increase in yield margins on government securities are precisely the countries that had a high deficit-to-GDP ratio in the pre-crisis period. Thus, a constantly high deficit-to-GDP ratio is the indicator of the accumulation of fiscal risks and results in a higher SSI.

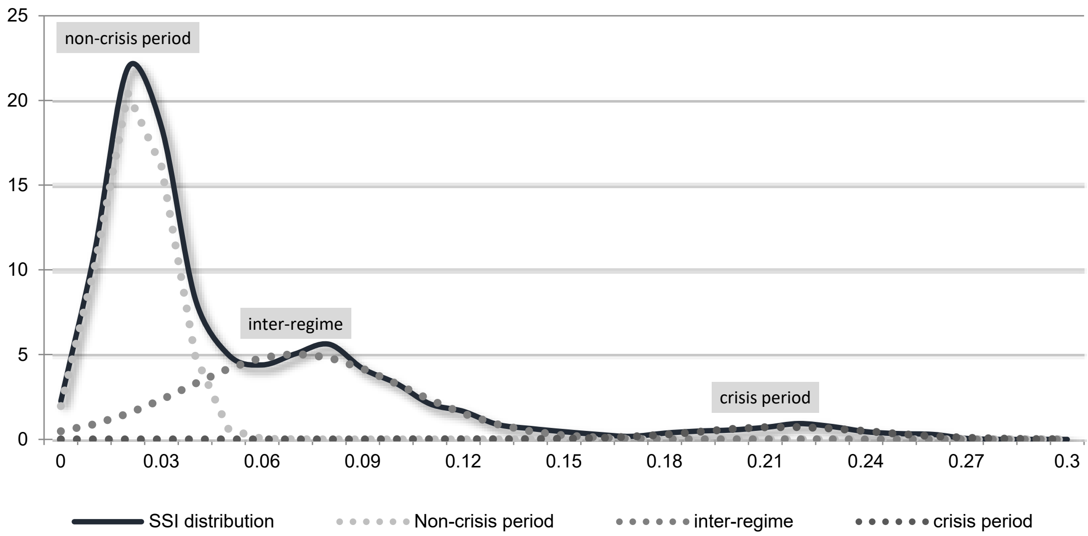
The banking sector constitutes a particularly important segment of the financial system. Taking into consideration that the Serbian financial system is bank-centric, with the banking system’s share of more than 90% of financial sector assets, a significant impact on the SSI level is related to this segment. Since only a small number of banks are listed on the Belgrade Stock Exchange, no market indicators, but only regulatory and reporting data were used. The representative indicators used are the deviation or the total deposit gap that represents a cyclical component of deposits, assessed as a one-sided Hodrick-Prescott filter where the lambda parameter takes value 14400 (Louzis and Vouldis 2013). The advantage of applying gaps concerns the fact that they contain information on the cumulative effect of imbalances. In that regard, gaps are a more accurate imbalance measure in comparison to growth rates. If deposits are below their long-term trend at a certain moment, the gap is negative, and banks might face reduced funding sources. On the other hand, a positive deposit gap has a favourable effect on funding sources and liquidity of banks and consequently on the SSI. The next indicator is a credit gap that is also determined as a cyclical component of credit fluctuations using the one-sided Hodrick-Prescott filter. A negative gap implies that banks approve less loans than usual. It may occur due to the tightening of credit standards as a result of an increased risk perception or, on the other hand, reduced demand for loans that is related to dented consumption. In both cases it is an unfavourable signal that increases the SSI value. Taking into consideration a significant foreign ownership of the banking sector, the credit default swap (CDS) of parent banks was used, in which a weighting factor is proportionate to the share in total balance sheet assets of the domestic sector (Dumičić 2014). An assessment of the CDS includes an evaluation of the country's credit rating, fiscal and macroeconomic stability and the credit rating of the observed financial institution that covers operating indicators and a development strategy. The CDS is a leading indicator that captures the dynamics of crisis periods in real time and it therefore may be used to create early warning signals (Podpiera and Ötker 2010). This is supported by the fact that the values of the credit default swap of some countries increased several times in 2007, while financial soundness indicators did not show any risk accumulation signs until 2008.

The last segment used to calculate the SSI is the external environment, which is becoming more important due to market regionalisation and the effects of global trends on small, open economies, such as Serbian. The first indicator that was used to assess the external environment risk is the EMBI Global (Cavanagh and Long 1999). It defines markets of developing countries using the per capita income data and public debt restructuring history. During the aggregation into a composite EMBI Global, instruments that are more relaxed than other indicators are introduced and developing countries are included, which makes the indicator's range wider. Weighting factors per individual countries during the aggregation comply with their market capitalisation. The openness of the Serbian economy and inclusion in international capital flows emphasises the impact of international factors on domestic markets. Thus, it is particularly important to monitor capital markets in neighbouring countries taking into consideration a high correlation with the Serbian capital market, and to include an indicator related to the average value of stock exchange indicators in neighbouring countries as it may point to stress accumulation in these markets and a potential negative impact on domestic markets.

2.2 Assessment of a systemic stress period and the number of regimes

The objective of an analytical framework is to separate high financial stress periods from moderate or low stress periods. The framework assumes that the characteristics of the SSI depend on a space-state representation. For instance, financial stress is assumed to have the tendency to cluster around different local attractor levels in different regimes, showing persistence in different intervals, and only a transition between different regimes is sudden and unpredictable (i.e. stochastic). The SSI characteristics may be identified based on the empirical probability distribution function shown in Figure 1.

Figure 1 **Systemic Stress Indicator expressed as a sum of Gaussian distributions**



The SSI probability distribution is visibly multi-modal and largely asymmetrical towards the right-skewed tail. This feature of the probability distribution function suggests that the empirical probability function may be presented as a sum of appropriate distributions, whereby each of them characterises a separate switching regime. The Gaussian mixture model (Press 2007) in Figure 1 shows moments of parametric distributions presented in Table 1. Gaussian mixture models are parametric functions of the probability density obtained by weighted sums of Gaussian distributions.

Table 1 **Main statistics of Gaussian mixture distributions depending on the number of regimes**

	2	3	4	5
AIC	-655.38	**-688.76**	-685.42	-682.60
BIC	-640.03	**-664.21**	-651.66	-639.64
Likelihood	-332.69	-352.28	-353.71	**-355.31**

The model below provides estimates of Gaussian distribution moments in case of three different switching regimes based on the values of information criteria AIC and BIC, presented in Table 1.

Table 2 **Gaussian distribution moments in case of three regimes**

	Expected value	Standard regime deviation	Weighting function
Non-crisis period	0.0223	5.04E-05	56.2%
Inter-regime	0.0696	5.82E-04	36.2%
Crisis period	0.1929	1.60E-03	7.6%

As it can be seen from a preliminary regime analysis, in addition to an increased expected value, the indicator volatility rises in riskier periods.

Markov-switching models (Hamilton 1989) are one of the most popular non-linear techniques for analysing time series in scientific literature. These models use multidimensional structural equations that explain the behaviour of a space-state representation in case of several switching regimes. These models can also analyse complex dynamic behaviour patterns in time series by allowing switchings between structural equations. The improvement of Markov-switching models is a mechanism that enables switchings between different states with support of an undetected state variable – X^S. A significant advantage of introducing the state variable is that it enables dependence of a model structure change and probabilities of switching from one regime to another.

Starting from the definition of the generalised Markov-switching model:

$$y_t = \sum_{i=1}^{N_{nS}} \beta_i \, x_{i,j}^{N_{nS}} + \sum_{i=1}^{N_S} \Phi_j \, S_t x_{i,j}^{N_S} + \varepsilon, \tag{11}$$

Where N_S denotes switching parameters and N_{nS} non-switching parameters.

A model with only switching parameters included is obtained for $x_{i,j}^{N_{nS}}=0$, i.e.

$$y_t = \sum_{i=1}^{N_S} \Phi_j \, S_t x_{i,j}^{N_S} + \varepsilon \ , \tag{12}$$

where $x_{i,j}^{N_S} = [1 \quad 1 \quad \cdots \quad 1], i = 1,2,\dots,N_S$ and $|S_t|$ is the number of switching regimes. The number of regimes is not *a priori* known. Thus, the distribution of the dependent variable y_t must be assumed $\overline{y_t}$.. Under the assumption of the local normality of a dependent function $(y_i|S_t = i; \theta_i) \sim N(\mu_i^F, \sigma_i^F)$, where $i = 1,2,3$ and θ_i^F represents a set of parameters of an unrestricted or a full regime model, i the density of the conditional probability of the dependent variable is y_i obtained as follows:

$$f(y_t|S_t = i, \theta_i^F) = \frac{1}{2\pi^{n/2}\sigma_i^F} \exp\left(- \frac{\left((y_t - \mu_i^F)\right)}{2\sigma_i^{F^2}} \right) \tag{13}$$

The conditional regime probability j can be expressed through the following equation:

$$P_r(S_t = j|\psi_{t-1}, \theta_j^F) = \sum_{i=1}^{|S_t|} P_r(S_t = j, S_{t-1} = i|\psi_{t-1}^S, \psi_{t-1}', \theta_j^F) =$$

$$= \sum_{i=1}^{|S_t|} P_r(S_t = j|S_{t-1} = i, \psi_{t-1}^S, \theta_j^F) \, P_r(S_{t-1} = i|\, \psi_{t-1}', \theta_i^F), \tag{14}$$

Where $P_r(S_t = j|S_{t-1} = i, \psi_{t-1}^S, \theta_i^F)$ is a conditional probability of a transition from one regime to another based on historical information ψ^S, and θ_i^F represents an assessed vector of transition probabilities parameters in relation to the state variable X^S. A conditional transition probability can be defined by means of probit transformation $\Phi(\cdot)$:

$$P_r(S_t = j|S_{t-1} = i, \psi_{t-1}^S, \theta_j) = \Phi(\boldsymbol{X}^S, \boldsymbol{\psi}^S) \tag{15}$$

When new information on the probabilities of being in a regime j is obtained, the equation (13) can be updated:

$$\mathrm{P}_r(\mathrm{S_t} = j|\psi_t\ ,\theta_j^F,\mathrm{y_t}) = \frac{f(\mathrm{S_t} = j,\mathrm{y_t}|\psi_{t-1},\theta_j^F)}{f(\mathrm{y_t}|\psi_{t-1},\theta_i^F)} =$$
$$= \frac{f(\mathrm{y_t}|\mathrm{S_t} = j|\psi_{t-1},\theta_i^F)P_r(\mathrm{S_t} = j|\psi_{t-1},\theta_i^F)}{\sum_{\mathrm{i=1}}^{|\mathrm{S_t}|} f(\mathrm{y_t}|\mathrm{S_t} = i|\psi_{t-1},\theta_i^F)P_r(\mathrm{S_t} = i|\psi_{t-1},\theta_i^F)} \tag{16}$$

based on which a log-likelihood function is calculated:

$$logL = \sum_{j=1}^{|S_t|}\sum_{i=1}^{|S_t|} f(\mathrm{y_t}|\mathrm{S_t} = j,\mathrm{S_{t-1}} = i,\psi_{t-1}^S,\theta_j^F)\cdot \mathrm{P}_r(\mathrm{S_t} = j,\mathrm{S_{t-1}} = i|\psi_{t-1}^S,\theta_j^F) \tag{17}$$

A set of final model parameters is obtained by maximising the log-likelihood function $\max_{\theta_j^F}(\mathrm{logL})$.

In a generalised case, a set of switching regime equations is obtained:

$$\begin{aligned} y_1 &= \mu_1 + \varepsilon_{1t}; \varepsilon_{1t} \sim \mathrm{N}(0,\sigma_1\ ) \\ y_2 &= \mu_2 + \varepsilon_{2t}; \varepsilon_{1t} \sim \mathrm{N}(0,\sigma_2\ ) \\ &\vdots \\ y_{|\mathrm{S}_t|} &= \mu_{|\mathrm{S}_t|} + \varepsilon_{|\mathrm{S}_t|t}; \varepsilon_{1t} \sim \mathrm{N}(0,\sigma_{|\mathrm{S}_t|}) \end{aligned} \tag{18}$$

whereas a transition regime matrix at the moment t is defined as:

$$P_t = \begin{bmatrix} p_{(1|1)}^t(\mathbf{X}^{\mathrm{S}},\boldsymbol{\psi}^S\ ) & p_{(1|2)}^t(\mathbf{X}^{\mathrm{S}},\boldsymbol{\psi}^S\ ) & \cdots & p_{(1|\mathrm{S}_t)}^t(\mathbf{X}^{\mathrm{S}},\boldsymbol{\psi}^S\ ) \\ p_{(2|1)}^t(\mathbf{X}^{\mathrm{S}},\boldsymbol{\psi}^S\ ) & p_{(2|2)}^t(\mathbf{X}^{\mathrm{S}},\boldsymbol{\psi}^S\ ) & \cdots & p_{(2|\mathrm{S}_t)}^t(\mathbf{X}^{\mathrm{S}},\boldsymbol{\psi}^S\ ) \\ \vdots & \vdots & & \vdots \\ p_{(\mathrm{S}_t|1)}^t(\mathbf{X}^{\mathrm{S}},\boldsymbol{\psi}^S\ ) & p_{(\mathrm{S}_t|2)}^t(\mathbf{X}^{\mathrm{S}},\boldsymbol{\psi}^S\ ) & & p_{(\mathrm{S}_t|\mathrm{S}_t)}^t(\mathbf{X}^{\mathrm{S}},\boldsymbol{\psi}^S\ ) \end{bmatrix} \tag{19}$$

Where $\mathrm{X^S}$ denotes a state variable vector and ψ^{S} denotes a system parameter vector. Detailed information on the expectation-maximisation algorithm used to assess model parameters can be found in Dempster et al. (1977)..

2.3 Analysis of SSI movements

Different specifications using a lagged dependent variable $\psi_{\mathrm{t}}^{\mathrm{S}} = \mathrm{y_{t-h}}$ as a state variable for different number of regimes i are contained in Appendix 3. An auto-regression model with a unit lag follows more adequately the dynamics of the SSI movement during a crisis period, but caution is advised when including a dependent variable in the form of auto-regression terms in the equation (11). Inclusion of an auto-regression term and enabling its coefficients to have a switching property allow the smallest indicator variations, even at extremely low values, i.e. when an indicator is in the least risky regime, to be interpreted as stress episodes. As a rule, these models have better values of the information criteria AIC and BIC and the values of maximum likelihood. Numerous transitions between opposite regimes (e.g. from a non-crisis regime to the crisis one) indicate model's instability. Moreover, transition probability matrices show the tendency of not having maximum transition probabilities diagonalwise. This problem is mainly related to the inability of an optimisation algorithm to converge to a global optimum in case of a multi-modal problem or a large number of parameters being optimised. Therefore, during an assessment of the likelihood function in equation (17) the approach (Kovačević et al. 2014) was used, enabling the finding of a global optimum, even in case of a large number of parameters. Based on equations (4) to (7), as well as equation (17), a weighting vector W, per defined financial system segments, was obtained as:

$$\begin{aligned} w_{FX} &= 0.202 \\ w_{GOV} &= 0.125 \\ w_{MON} &= 0.229 \\ w_{EQU} &= 0.125 \\ w_{BAN} &= 0.194 \end{aligned} \tag{20}$$

Based on the weighting vector shown in (20), it is evident that the money market, FX market and the banking sector segments have the strongest estimated impact on the SSI, whereas the public finance segment, capital market and external environment have slightly lower values.

The specification of the preferred model is represented by an auto-regression term with a unit-lag of state variable $\mathrm{SAR}(1)$ with three switching regimes in which a free member and the residual variance allow for switching parameters. We will name this model a full dynamic Markov model with three switching regimes, with a unit-lag of state variable F-DMS(3)-SAR(1). The preferred model has the best statistics related to information criteria values, as seen in Appendix 3. Additionally, the presented results also include a regime classification measure, proposed in the paper by Ang and Bekaert (2002) and Baele (2005), in case there are more than two regimes, which is obtained based on the following equation:

$$\mathrm{RCM}(|\mathrm{S_t}|) = 100\left(1\text{-}\frac{|\mathrm{S_t}|}{|\mathrm{S_t}|\text{-}1}\frac{1}{\mathrm{T}}\sum_{t=1}^{\mathrm{T}}\sum_{i=1}^{|\mathrm{S_t}|}\left(\mathrm{p_{i,t}}\text{-}\frac{1}{|\mathrm{S_t}|}\right)^2\right) \tag{21}$$

Lower RCM values denote a better regime separation ability. As it can be seen in Appendix 3, a full model F-DMS(3)-SAR(1) had the best regime classification results. In case of three switching regimes, a generalised Markov-switching model is obtained, based on the equations listed in Chapter 2.2.

$\mathrm{y_1} = 2.31\text{E-}02 + \varepsilon_{1t};\ \varepsilon_{1t} \sim \mathrm{N}(0, 6.30\text{E-}05)$ for non-crisis period
$\mathrm{y_2} = 7.40\text{E-}02 + \varepsilon_{2t};\ \varepsilon_{2t} \sim \mathrm{N}(0, 3.95\text{E-}04)$ for inter-regime
$\mathrm{y_3} = 1.87\text{E-}01 + \varepsilon_{3t};\ \varepsilon_{3t} \sim \mathrm{N}(0, 1.86\text{E-}03)$ for crisis period

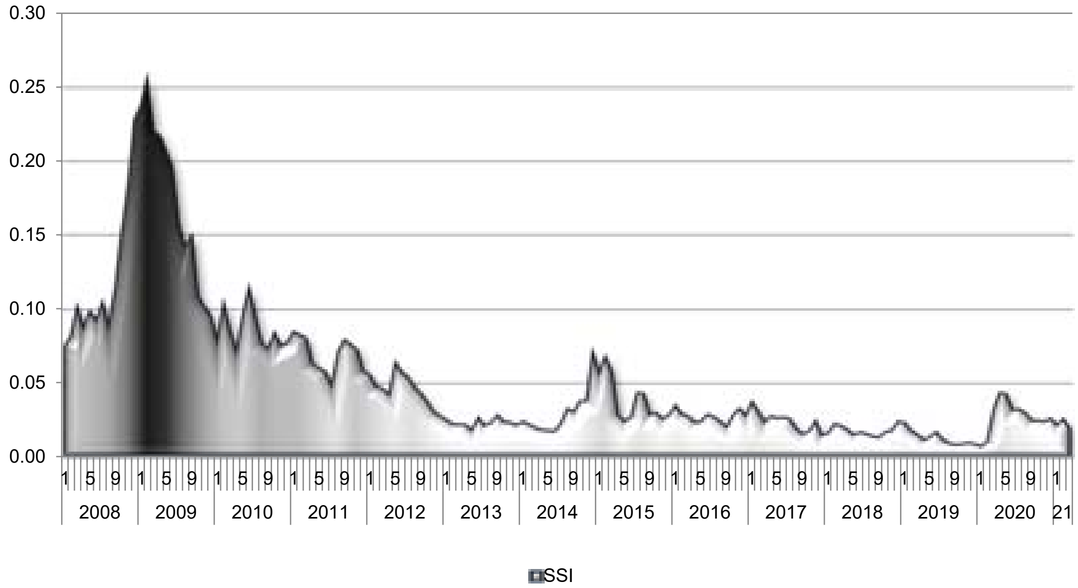
Since it is a full model F-DMS(3)-SAR(1), we will designate the model parameters as ***F***:

$$\mu_1^F = 2.31\mathrm{E}-02, \delta_1^F = 6.30E-05$$

$$\mu_2^F = 7.40E-02, \delta_2^F = 3.95E-04$$

$$\mu_3^F = 1.87E-01, \delta_3^F = 1.86E-03$$

Figure 2 **Movement of SSI**



The assessed expected value and volatility differ statistically in different regimes. The values also comply with a preliminary analysis that was obtained using the Gaussian mixture model shown in Table 2. The expected value level is the lowest in case of a non-crisis period: $\mu_1^F = 2.31E\text{-}02$. In case of a crisis period, it is $\mu_3^F = 1.87E\text{-}01$, which complies with the expected behaviour of the SSI, i.e. the higher the regime risk the higher the SSI values. Correspondingly, volatility justifies economic fundamentals, where higher values of indicator fluctuations are expected, reflecting increased uncertainty in crisis periods. Volatility in a crisis period is $\delta_3^F = 1{,}86E\text{-}03$ and it is considerably higher compared to a non-crisis or an inter-regime period. In order to look into regime dynamics more closely, we have to identify the main characteristics of each period.

Figure 3 **Conditional regime probabilities using the F-DMS(3) SAR (1) model**

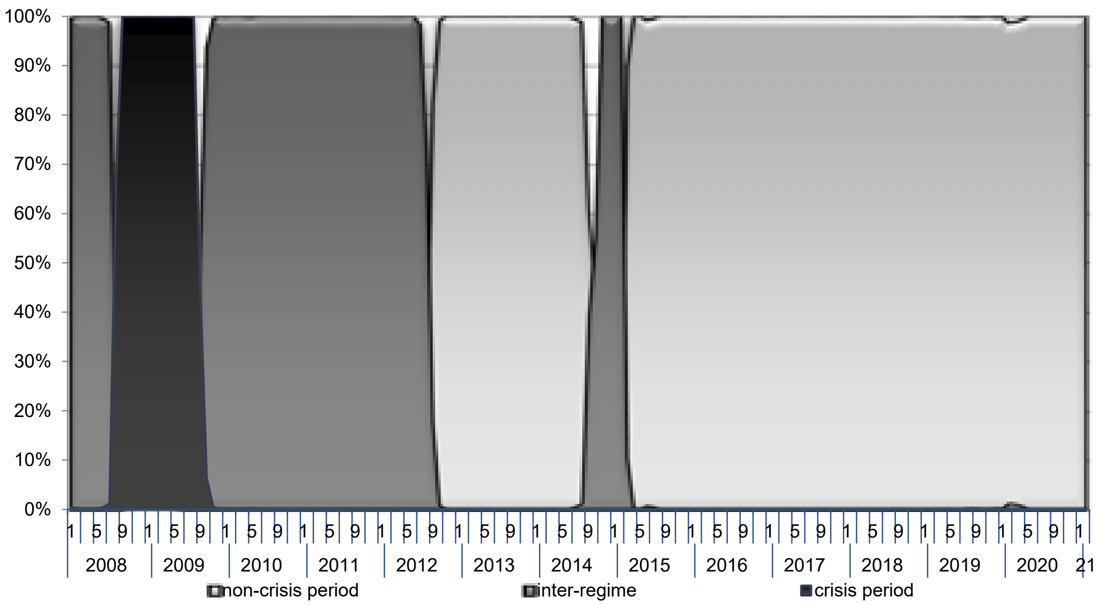
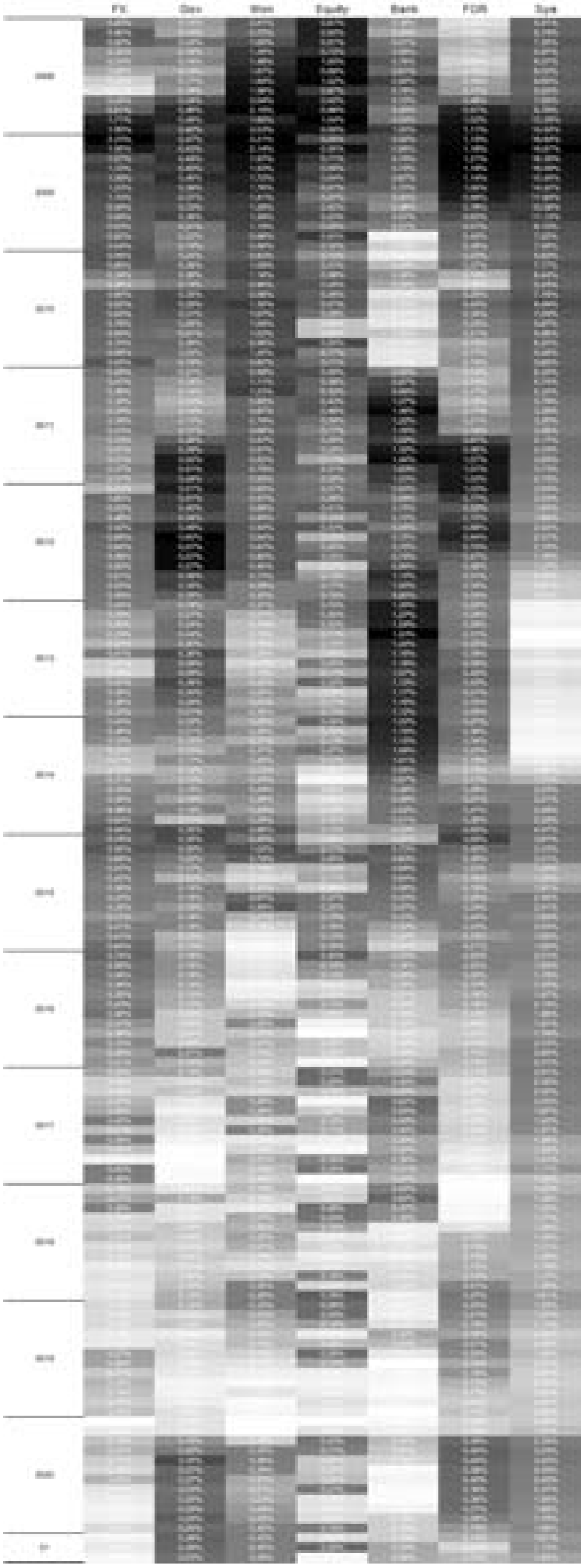


Figure 4 **Contribution of segments to the composite SSI**



2.4 Analysis of risk factors and stress level in the observed period

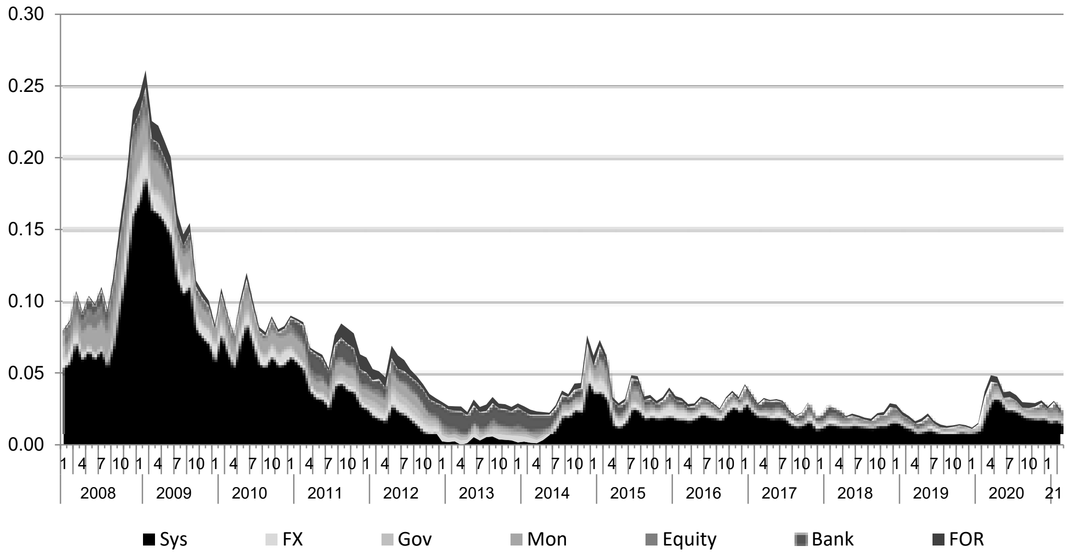
The SSI defines a pre-crisis period from January 2008 to September 2008. The first shocks in the domestic financial system first appeared in money and capital markets. The average monthly BEONIA value increased from 9.4% in January 2008 to over 17% in November and December 2008. Simultaneously, there was a reduction of excess dinar liquidity of banks, whereas the BELEX15 index lost over 70% of its value at the time.

The period between October 2008 and October 2009 was marked by the escalation of the global financial crisis. In the last months of 2008 we witnessed the worst financial crisis since the Great Depression of the 1930s. In January 2009 the prices of shares in all major global markets unexpectedly collapsed. A large number of US and European banks recorded enormous losses in their financial statements. Lehman Brothers went bankrupt in September 2008, while Goldman Sacks and Morgan Stanley sought state aid in order not to declare bankruptcy. The Federal Reserve System injected hundreds of billions of dollars as state aid into the banking system. The financial crisis spilled over into the European financial system for the first time since the introduction of the euro in 1999 as the official currency. Facing panic, regulators across Europe adopted additional measures of monetary and fiscal policies in order to ease the effects of the coming recession. At the same time, most central banks in Europe reduced their key policy rates.

The initial shocks in the domestic financial system in the first half of 2008 continued to deepen and intensify. The local currency weakened against the euro in the FX market by over 10% y-o-y, while the spread between the buying and selling rates increased, reflecting FX market uncertainty.

Public finance also suffered significant shocks. In December 2008 Serbia recorded historically the highest value of an emerging market bond risk premium of over 1,300 bp. At the same time, the consolidated fiscal deficit widened, equalling almost 4.2% of gross domestic product at the end of 2009. The average monthly BEONIA value reached its maximum of 18.2% in January 2009. Stock market indices saw a sharp drop. Over a period of eleven months, BELEX 15 dropped from over 1800 (June 2008) to 400 in April 2009. The banking sector saw a liquidity crisis as 6.2% of total deposits or RSD 1.3 billion was withdrawn from domestic banks. Thanks to the support of parent banks to their subsidiaries operating in the Republic of Serbia, the liquidity position was preserved, and major disruptions were prevented. The neighbouring countries also faced considerable shocks. Stock market indices in the neighbouring countries lost on average 55% of their value compared to January 2008, while the Emerging Market Bond Index Global for Serbia reached its maximum value in December 2008. Figures 2 and Figures 4 show that the SSI and the systemic component reached their historical maximum in February 2009 – the SSI peaked to 0.26, when the crisis culminated in the Republic of Serbia.

Figure 5 **Movement of the SSI by component and the breakdown of key risk factors**



The global economic crisis gradually subsided between October 2009 and August 2012 and assumed the local character of a sovereign debt crisis. A eurozone crisis broke out in late 2009. A few eurozone member states (Greece, Portugal, Ireland, Spain and Cyprus) were not able to repay or restructure their government debts or to inject sizeable funds into their financial systems without the support of other eurozone member states or international financial institutions. The European Central Bank responded by reducing interest rates and providing favourable loans in order to ensure the eurozone’s financial system liquidity. In September 2012 the European Central Bank calmed financial markets by announcing it would provide unrestricted support to eurozone member states. The domestic financial system was now not as affected as it had been at the beginning of 2009. The SSI level visibly increased in June 2010, September 2011 and May 2012. The volatility in the domestic FX market increased in June 2010, which resulted in robust FX interventions by the National Bank of Serbia. The banking sector saw a decline in the credit activity and an increase in Greek-owned parent banks’ risk due to an increase in Greek sovereign risk.

The next significant increase in the SSI took place in September 2011 mainly due to factors in the international environment – sovereign risk, particularly Greek and Italian, went up, spilling over to a increase in risk of banking groups in these countries. Moreover, there was a jump in the global emerging market bond index, indicating an increased investor risk perception of emerging markets. There was another mid-stress period in June 2012, when the domestic currency significantly dropped by 11.5% y-o-y. Simultaneously, for the first time since the crisis onset, the consolidated fiscal deficit exceeded 6%, while the average credit risk insurance premium of parent banks, particularly Greek and Cyprus banks, was at its peak, reflecting the sovereign credit risk.

The period between December 2012 and the beginning of 2020 was marked by a low risk with stable system risk values, except for a mid-stress period from September 2014 to April 2015. It was the period of a stable exchange rate and low and stable inflation. The BELEX15 index also stabilised. The banking sector was adequately capitalised and highly liquid, with lower credit activity during 2013 and early 2014. Individual episodes of increased stress occurred in some segments but did not significantly affect the SSI. The consolidated fiscal deficit rate reached 6.2% at the end of 2014 and started dropping afterwards, while EMBI Serbia reached somewhat higher values in comparison to the previous period.

The pandemic triggered by the coronavirus outbreak in early 2020 resulted in an unprecedented health and economic global crisis. To protect people's lives, measures were adopted, accompanied with disruptions in international financial and commodity markets, elevated uncertainty and flight to safe-haven assets, which all together resulted in a sharp drop in the global economic activity. Due to increased uncertainty in the second quarter of 2020, an increase in the SSI was mainly affected by foreign market developments. The EMBI risk premium returned to a downward path at the end of April 2020, when it peaked at 312 bp (its highest value in 2020) and amounted to 137 bp at the end of the second quarter. A relatively stable exchange rate was maintained throughout 2020 due to timely and adequate FX interventions. Moderate depreciation pressures, in place since March as a result of the pandemic outbreak, gradually weakened during the year. In November and December 2020 appreciation pressures prevailed. An extensive set of economic measures introduced to mitigate negative effects of the pandemic resulted in an increase in the fiscal deficit and public debt, which was still among the smallest in Europe. The public debt-to-GDP ratio increased from 52.0% at the end of 2019 to 57.4% at the end of 2020. The recovery of the global economy that started in mid-2020 was slowed down by the second coronavirus wave in October 2020, which required the re-instatement of restrictive measures in many countries. Solid macroeconomic fundamentals and large-scale monetary and fiscal policy measures enabled the Republic of Serbia to remedy the consequences of the coronavirus epidemic more efficiently than other European countries.

3 Assessment of the likelihood of a systemic crisis

A large number of scientific papers have recently dealt with how to improve the early warning models, mainly by means of developing new analytical tools. Apart from traditional binary models that apply Logit or probit transformations (Berg and Pattillo 1998), literature offers multidimensional tools that use a large number of indicators when developing early warning indicators (Rose and Spiegel 2012, Bussiere and Fratzscher 2006, Frankel and

Saravelos 2010), by applying machine learning (Kou et al. 2019), while the application of Markov switching models is described in detail in Abiad (2003). A paper by Kliesen et al. (2012) emphasises the fact that financial crises are almost impossible to predict. They are instigated by various triggers and occur in various forms and thus new crises are difficult to predict by relying on traditional warning models only. An additional problem is that financial stress cannot be directly measured as it does not have a money equivalent.

We will apply the so-called univariate signalling approach that is based on an assessment of a signal that appeared during previous crisis periods, based on which critical values, above which the indicator points to a strong possibility of a crisis occurrence, are identified. Such an approach enables a simpler interpretation of the achieved results. A more detailed analysis of the types of approaches to creating early warning signals can be found in Kaminsky (1999).

In this paper, we propose the early warning signal approach that is based on the likelihood of a systemic crisis on the basis of the methodology described in the previous chapter. Unlike binary models, this approach enables an analysis of a larger number of regimes that can be classified, according to their intensity, as more or less risky. Although an ex-post identification using the SSI is a good choice, in a general case of undetected crisis periods, i.e. periods that can be estimated to be crisis periods only over time and during the identification of key crisis factors in the past and in drawing conclusions about periods of higher turbulences, early warning signals, on the other hand, raise awareness among decision makers of a potential systemic crisis occurrence and offer sufficient time for a timely response.

We will define a restricted model based on the unrestricted/full model described in the previous chapter. The main characteristic of dynamic Markov-switching models are conditional state transition matrices. Unlike static Markov models, probabilities of making transitions from one regime to another are time-dependent on the endogenous state variable $\mathrm{X^S}$. Core building blocks for obtaining dynamic probabilities are presented in the paper by Diebold et al. (1994). Let us assume that the dependent variable can be defined as:

$$(\mathrm{y_i}|\mathrm{S_t} = \mathrm{i}; \theta_\mathrm{i}) \sim \mathrm{N}\left(\mu_\mathrm{i}^\mathrm{F}, \sigma_\mathrm{i}^\mathrm{F}\right), \tag{22}$$

where $\mathrm{i} = 1,2,3$ based on the results shown in Table 1 and Table 8, and θ_i represents a set of parameters of the regime i .

A restricted model is obtained using the assessed parameters of the full model that refer to distribution moments of the dependent variable $\mathrm{y_t}$. Model parameters are divided into two subsets. The first subset is related to the parameters of distribution moments of the dependent variable - $\alpha_\mathrm{i}^\mathrm{F}$ of the regime i , while the second subset refers to the state variable parameters that are used when estimating conditional transition probabilities $\beta_\mathrm{i}^\mathrm{F}$, i.e. $\theta_\mathrm{i}^\mathrm{F} = \left(\alpha_\mathrm{i}^\mathrm{F}, \beta_\mathrm{i}^\mathrm{F}\right)$.

A restricted model is obtained by maximising log-likelihood functions based on lagged state variable h :

$$\begin{aligned} logL = \sum_{j=1}^{|S_t|}\sum_{i=1}^{|S_t|} f\left(\mathrm{y_t}|\mathrm{S_t} = j, \mathrm{S_{t-1}} = i, \psi_{t-h}^S, \left(\alpha_i^F, \beta_i^R\right)\right) \cdot \mathrm{P}_r\left(\mathrm{S_t} = j, \mathrm{S_{t-1}}\right. \\ \left. = i|\psi_{t-h}^S, \left(\alpha_i^F, \beta_i^R\right)\right), \end{aligned} \tag{23}$$

where $|\cdot|$ denotes the cardinality of the set.

The maximisation of the log-likelihood function results in a set of parameters β_i^R used in the restricted model.

A conditional probability of the regime j based on historical information $\psi_{t\text{-}h}$, on the basis of full model parameters $\left(\alpha_i^F, \beta_i^R\right)$ is obtained as follows:

$$\mathrm{P}_r\left(\mathrm{S}_{t+1} = j|\psi_{t-h}, \left(\alpha_i^F, \beta_i^R\right)\right) = \sum_{i=1}^{|\mathrm{S}_t|} \mathrm{P}_r\left(\mathrm{S}_{t+1} = j, \mathrm{S}_t = i|\psi_{t-h}^S, \psi_{t-h}', \left(\alpha_i^F, \beta_i^R\right)\right) = \tag{24}$$
$$= \sum_{i=1}^{|\mathrm{S}_t|} \mathrm{P}_r\left(\mathrm{S}_{t+1} = j|\mathrm{S}_t = i, \psi_{t-h}^S, \left(\beta_i^R\right)\right) P_r\left(\mathrm{S}_t = \mathrm{i}|\ \psi_{t-h}', \left(\alpha_i^F\right)\right)$$

We use recursion to obtain a conditional probability of the regime j at the moment $t + h$ based on historical information ψ_t :

$$\mathrm{P}_r\left(\mathrm{S}_{t+h} = j|\psi_t\ , \left(\alpha_i^F, \beta_i^R\right)\right) = \sum_{i=1}^{|\mathrm{S}_t|} \mathrm{P}_r\left(\mathrm{S}_{t+h} = j, \mathrm{S}_{t+h-1} = i|\psi_t^S, \psi_t', \left(\alpha_i^F, \beta_i^R\right)\right) = \tag{25}$$
$$= \sum_{i=1}^{|\mathrm{S}_t|} \mathrm{P}_r\left(\mathrm{S}_{t+h} = j|\mathrm{S}_{t+h-1} = i, \psi_t^S, \left(\beta_i^R\right)\right) P_r\left(\mathrm{S}_{t+h-1} = \mathrm{i}|\ \psi_t', \left(\alpha_i^F\right)\right),$$

Where $\mathrm{P}_r\left(\mathrm{S}_{t+1} = \mathrm{j}|\mathrm{S}_t = \mathrm{i}, \psi_t^S, \left(\alpha_i^F, \beta_i^R\right)\right)$ represents a conditional probability of transition from one regime to another based on historical information and state variable parameters ψ^S, β_i^R is an assessed vector of transition probabilities in relation to state variables X^S.

In a general case, a conditional transition probability can be defined using a probit transformation, as in the equation (15):

$$P_r\left(S_{t+1} = j|S_t = i, \psi_{t-h}^S, \left(\beta_i^R\right)\right) = \Phi\left(\mathbf{X}^S, \boldsymbol{\beta}^R\right). \tag{26}$$

If there are more than two regimes, we can use clustering to define a set of non-crisis regimes $\{nc\}$ and a set of crisis regimes $\{c\}$. Let us observe a transition matrix as follows:

$$P_r(\cdot\mid\cdot) = \begin{bmatrix} P_r(S_{t+1} = \{nc\}|S_t = \{nc\},\cdot) & P_r(S_{t+1} = \{nc\}|S_t = \{c\},\cdot) \\ P_r(S_{t+1} = \{c\}|S_t = \{c\},\cdot) & P_r(S_{t+1} = \{c\}|S_t = \{c\},\cdot) \end{bmatrix} \tag{27}$$

A conditional transition probability of staying in non-crisis regimes $\{nc\}$ at the time $t + 1$ can be calculated as:

$$P_r(S_{t+1} = \{nc\}|S_t = \{nc\},\cdot) = \frac{\sum_{j=1}^{|S_{nc}|}\sum_{i=1}^{|S_{nc}|} P_r\ (\mathrm{S}_t = \mathrm{i}|\ \cdot)P_r(S_{t+1} = j|S_t = i,\cdot)}{\sum_{i=1}^{|S_{nc}|} P_r\ (\mathrm{S}_t = \mathrm{i}|\ \cdot)} \tag{28}$$

At the same time, a conditional transition probability of staying in crisis regimes $\{c\}$ at the time $t + 1$ can be denoted as:

$$P_r(S_{t+1} = \{c\}|S_t = \{c\},\cdot) = \frac{\sum_{j=1}^{|S_c|}\sum_{i=1}^{|S_c|} P_r\,(\mathrm{S_t} = \mathrm{i}|\,\cdot)P_r(S_{t+1} = j|S_t = i,\cdot)}{\sum_{i=1}^{|S_c|} P_r\,(\mathrm{S_t} = \mathrm{i}|\,\cdot)} \tag{29}$$

A conditional probability of transitions from non-crisis regimes $\{\mathrm{nc}\}$ to crisis ones $\{\mathrm{c}\}$ at the time $\mathrm{t}+1$ can be calculated as:

$$P_r(S_{t+1} = \{c\}|S_t = \{nc\},\cdot) = \frac{\sum_{j=1}^{|S_c|}\sum_{i=1}^{|S_{nc}|} P_r\,(\mathrm{S_t} = \mathrm{i}|\,\cdot)P_r(S_{t+1} = j|S_t = i,\cdot)}{\sum_{i=1}^{|S_{nc}|} P_r\,(\mathrm{S_t} = \mathrm{i}|\,\cdot)} \tag{30}$$

A conditional probability of a transition from crisis regimes $\{\mathrm{c}\}$ to non-crisis ones $\{\mathrm{nc}\}$ at the time $\mathrm{t}+1$ is obtained trivially based on the equation (30).

We will name the probability that at least one crisis period will occur at the time interval $t+\gamma+1$, $t+\gamma+2, \ldots, t+\gamma+hor$ a probability of the occurrence of a systemic crisis PSC which can unambiguously be calculated as:

$$\mathrm{PSC_{t+\gamma}} = 1\text{-}\Big(\mathrm{P_r}\big(\mathrm{S_{t+\gamma}} = \{\mathrm{c}\}|\mathrm{S_{t+\gamma\text{-}1}} = \{\mathrm{nc}\},\cdot\big)\mathrm{P_r}\big(\mathrm{S_{t+\gamma}} = \{\mathrm{c}\},\cdot\big)\mathrm{P_r}\big(\mathrm{S_{t+\gamma}} = \{\mathrm{nc}\}|\mathrm{S_{t+\gamma\text{-}1}} = \{\mathrm{nc}\},\cdot\big)^{(\mathrm{hor\text{-}1})} + \mathrm{P_r}\big(\mathrm{S_{t+\gamma}} = \{\mathrm{nc}\},\cdot\big)\mathrm{P_r}\big(\mathrm{S_{t+\gamma}} = \{\mathrm{nc}\}|\mathrm{S_{t+\gamma\text{-}1}} = \{\mathrm{nc}\},\cdot\big)^{\mathrm{hor}}\Big) \tag{31}$$

where $\gamma = 1,2,\ldots,\mathrm{h}$ and $\mathrm{P_r}\big(\mathrm{S_{t+\gamma}} = \{\mathrm{c}\},\cdot\big)$ is calculated recursively using the equation (25).

The equation (31) indicates that the probability of occurrence of a systemic crisis $\mathrm{PSC_{t+\gamma}}$ at the time $\mathrm{t}+\gamma$ represents an assessment of conditional probabilities that are based on the uncertainty of the regime in which we are at the previous moment $\mathrm{t}+\gamma\text{-}1$.

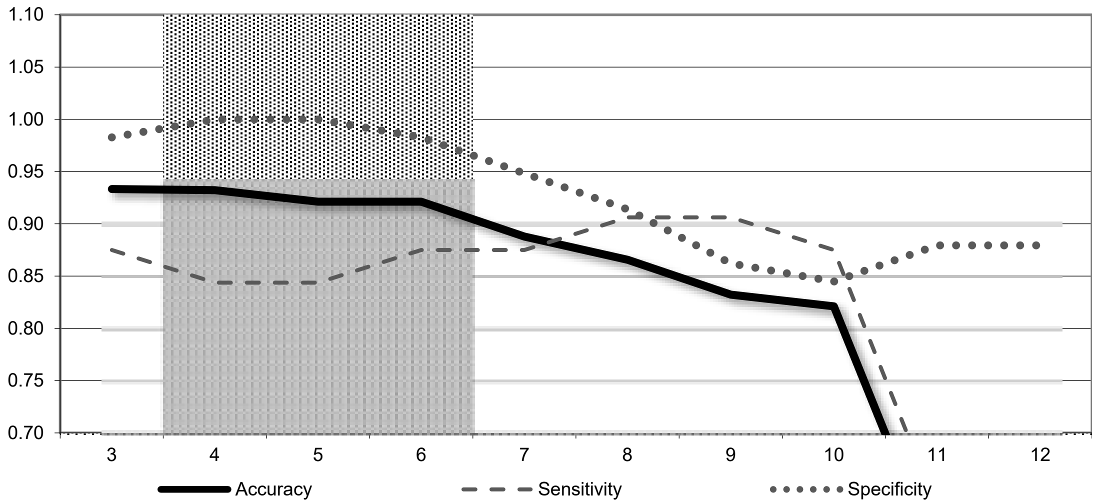
3.1 Critical levels of the likelihood of a systemic crisis occurrence and optimal forecast horizon

Appendix 4 details restricted models in which a lag h in a dependent state variable changes at an interval ranging between 3 and 12 months in case there are three regimes according to the analysis conducted in previous chapters. Prediction values below three months were not taken into consideration due to a short prediction horizon, which diminishes the usability of an early warning system. As it could be seen in the case of shorter lags, the values of the estimated likelihood are higher. Hence, in case of a three-month lag, the likelihood is 281.5, and 250.46 in case of a twelve-month lag. In addition, a measure of the regime classification quality – RCM decreases as the dependent variable lag increases, which reaches a minimum of 0.0194 in case of a ten-month lag. The values of the information criteria AIC and BIC decrease if the lag increases. In order to select the best model specification, the quality of each model classification needs to be assessed.

In three-regimes model, the equation (31) is used to calculate the probability that at least one crisis period will occur at the interval $t+\gamma+1, t+\gamma+2, \ldots, t+\gamma+$ hor for the time horizon hor = 6 months. We will name this probability of restricted models $\mathrm{PSC^R}$. Table 11 in Table11 shows the upper and lower critical values of the probability of occurrence of the systemic crisis $\mathrm{PSC^R}$, whereby we reach the maximum classification accuracy of the regimes shown in Figure 3, which were obtained based on full model probabilities.

As it can be seen in Table 11 in Table11, lower critical limits b_l in the case of all models are similar to the zero value and range at the interval of $[0{,}01-0{,}1]$. On the other hand, upper critical limits b_u, which are used to separate a non-crisis period from an inter-regime period, have a considerably wider interval $[0{,}15-0{,}8]$. Based on the accuracy of a multi-class classification (Sokolova and Lapalme 2009), we can see that the maximum classification accuracy significantly drops in case of state variable lags that exceed 10 months. In case of lags of 11 and 12 months, relevant maximum classification accuracies are around 0.57. Moreover, in case of these lags, the classification sensitivity is reduced to low values, which leads to the conclusion that an optimal prediction horizon must be strictly below 11 months. The classification specificity in the case of all models ranges from 0.84 up to the theoretical maximum value of 1.

Figure 6 **Assessment of multi-class classification parameters depending on a state variable lag**



A compromise between the classification accuracy and prediction horizon should be made. Figure 6 shows the accuracy, sensitivity and specificity of a classification depending on a prediction horizon and/or a state variable lag. It can be seen that in case of lags of up to six months, classification accuracy values of above 90% are obtained. In addition, in case of a six-month lag, classification sensitivity and specificity have high values, which justifies the use of the six-month lag and critical regime classification levels of 0.005 for the lower critical classification level and 0.760 for the upper one. Figure 16 in Appendix 4 shows families of classification accuracy curves in relation to the movement of the upper critical value in case of a six-month lag for the lower critical values set at 0.005, 0.1 and 0.2 respectively.

3.2 Assessment of the likelihood of a systemic crisis occurrence

Table 3 shows the values of transition probabilities based on the full model F-DMS(3) SAR (1) for the last moment in time t of the sample, i.e. March 2021, based on which, using the equations from (26) to (31), a transition matrix forecast was obtained at the interval of $t+1, t+2, \dots, t+6$ and shown in Table 4.

Table 3 **A transition matrix of the model F-DMS(3) SAR (1) in March 2021**

	Non-crisis	Inter-regime	Crisis
Non-crisis	98.9%	24.0%	16.8%
Inter-regime	1.1%	55.8%	34.1%
Crisis	0.0%	20.2%	49.1%

Transition probabilities behave in accordance with the expectation that the highest values are positioned diagonally, i.e. the probability of staying in the same regime is the highest. The probability of a transition from a crisis period to a non-crisis one is 16.8%, whereas the probability of a transition from a non-crisis period to a crisis one is close to 0%. Remote regimes have lower transition probabilities than the neighbouring regimes, which is in line with the expected behaviour of transition matrices. The forecasts of transition probabilities on the basis of the restricted three-regimes model with a six-month state variable lag R-DMS(3) SAR (6) show the same characteristics, which are presented in Table 4.

Figure 7 **Early warning signal of the full model F-DMS(3) SAR (1) and the restricted model R-DMS(3) SAR (6)**

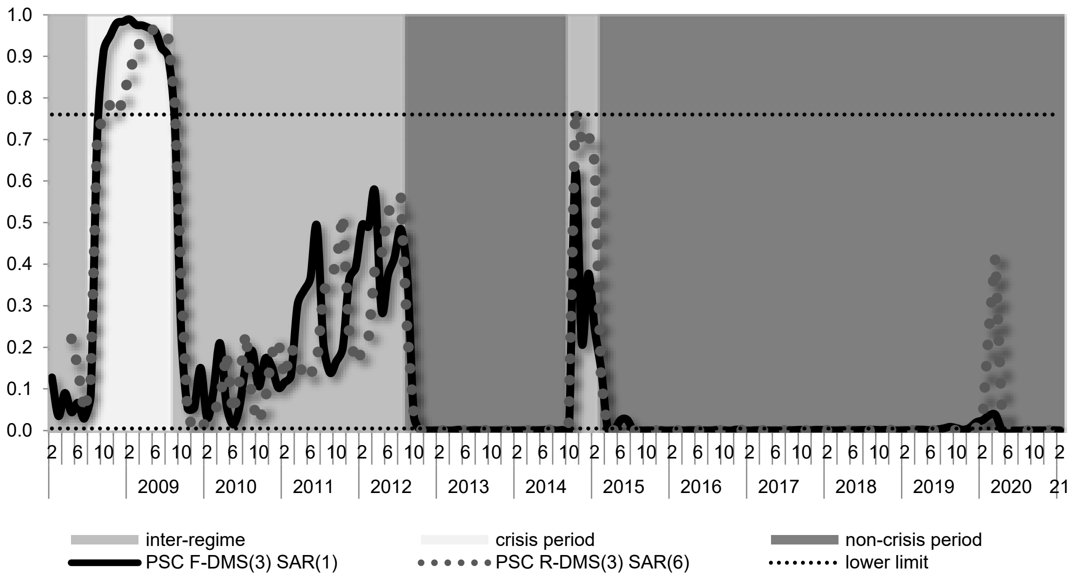


Figure 7 shows the movement of the likelihood of a systemic crisis occurrence based on the full model F-DMS(3) SAR (1) and the restricted model R-DMS(3) SAR (6). As it can be seen, the R-DMS(3) SAR (6) probabilities are in sync with the full model dynamics. However, probability inconsistencies can be detected in the inter-regime between November 2009 and August 2012, where a four-month lag in the restricted model probability is identified. Immediately after the crisis period, the restricted model probability dropped to low values in November 2009. However, this month was not incorrectly classified as a non-crisis one due to a low value of the lower critical probability level of 0.005. A lack of high probability values between October 2008 and February 2009 is also evident, but the level is above the upper critical value, which is why this period was correctly classified as a crisis period. There was also a slightly higher probability of a systemic crisis occurrence in December 2014, when the restricted model classified this month under a crisis one, while the full model probability had lower values that indicated an inter-regime. There was an apparent increase in the early warning signal of the reduced model in April and May 2020 to 0.241 and 0.416 respectively, but its levels were significantly lower than the upper critical limit values. Thereafter, the signal was on a continuous decline until April 2020. Both models classified non-crisis periods identically.

Table 4 **Forecast values of conditional transition probabilities from April 2021 to September 2021**

F-DMS(3) SAR (6)		Non-crisis	Inter-regime	Crisis
April 2021	Non-crisis	99.7%	25.1%	17.9%
	Inter-regime	0.3%	53.7%	35.4%
	Crisis	0.0%	21.2%	46.7%
May 2021	Non-crisis	99.6%	25.4%	18.2%
	Inter-regime	0.4%	53.3%	35.3%
	Crisis	0.0%	21.3%	46.5%
June 2021	Non-crisis	99.8%	23.7%	16.4%
	Inter-regime	0.2%	55.6%	35.6%
	Crisis	0.0%	20.6%	48.0%
July 2021	Non-crisis	99.2%	27.6%	20.8%
	Inter-regime	0.8%	50.3%	34.8%
	Crisis	0.0%	22.1%	44.5%
August 2021	Non-crisis	99.8%	24.0%	16.7%
	Inter-regime	0.2%	55.2%	35.6%
	Crisis	0.0%	20.8%	47.7%
September 2021	Non-crisis	98.9%	28.7%	22.1%
	Inter-regime	1.1%	48.8%	34.4%
	Crisis	0.0%	22.5%	43.4%

Figure 8 shows a regime probability forecast $\mathrm{P_r}(\mathrm{S_t} = \mathrm{j}| \cdot)$ in an observed six-month horizon, i.e. the period from April 2021 to September 2021, projected based on the equation (25). As it can be seen, an increase in the probability of being in a crisis period $\mathrm{P_r}(\mathrm{S_t} = 3| \cdot)$ was projected during the forecast period. A drop in a non-crisis regime probability $\mathrm{P_r}(\mathrm{S_t} = 1| \cdot)$ from 99.5% in the first forecast month to 98% in the sixth month was also projected in this period. At the same time, an inter-crisis regime probability $\mathrm{P_r}(\mathrm{S_t} = 2| \cdot)$ increased.

Figure 9 shows historical movement of the assessed Probability of a Systemic Crisis occurrence at any time over the next six months – PSC based on the full model PSC F-DMS(3) SAR (1) between January 2016 and March 2021. The probability forecast of a systemic crisis occurrence at any time between April and September 2021 on the basis of the restricted model PSC R-DMS(3) SAR(6) was obtained based on the equation (31). A maximum value was obtained in October 2021 with a 2.3% probability of a systemic crisis at any time between September 2021 and March 2022. These are low values that do not indicate a significant crisis occurrence probability in the observed period.

Figure 8 **Forecast regime probability movement in the observed six-month time horizon**

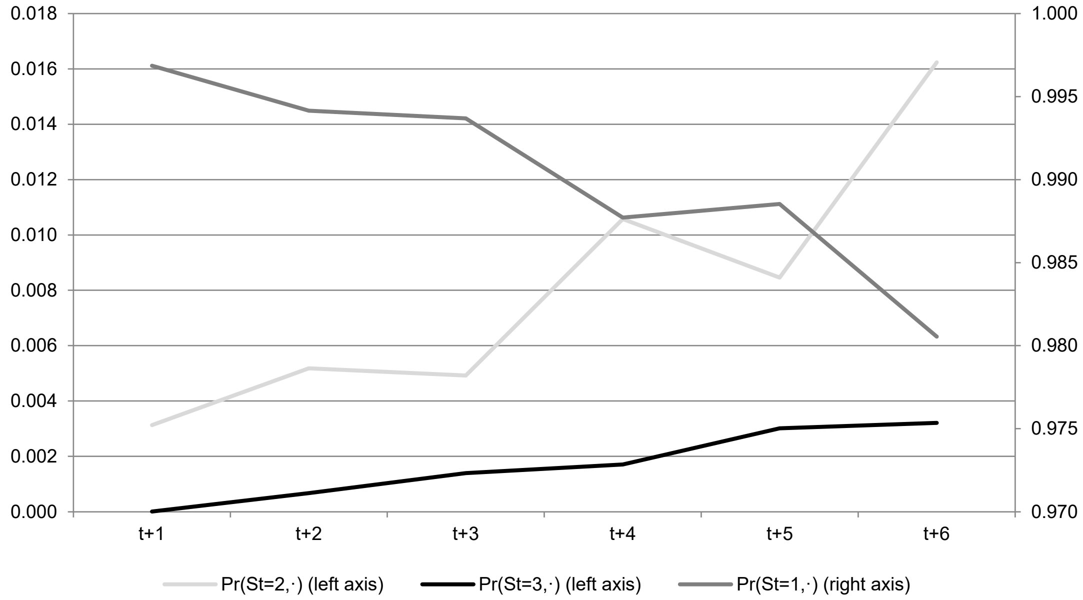
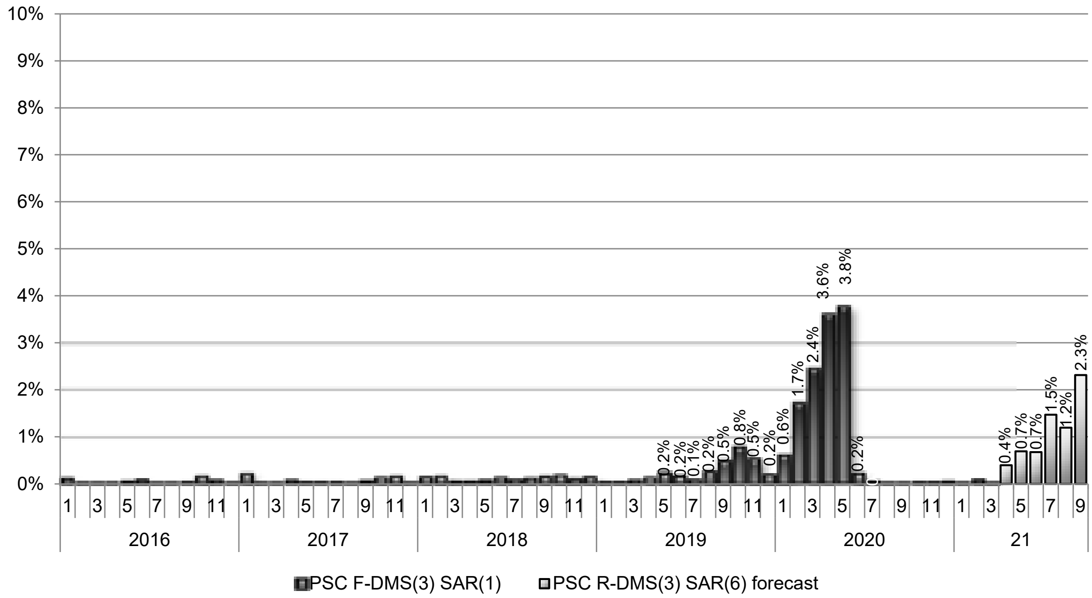


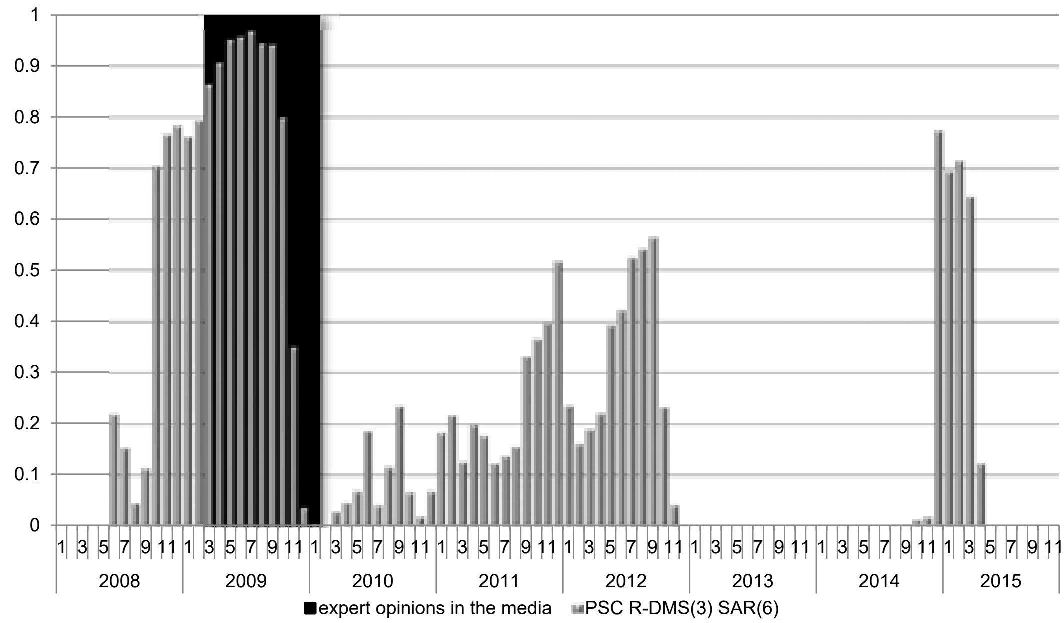
Figure 9 **Early warning signal movements of the full model PSC F-DMS(3) SAR (1) between January 2016 and February 2021 and a restricted model forecast PSC R-DMS(3) SAR(6) between March and September 2021**



3.3 Verification of an early warning signal of a systemic crisis occurrence

Figure 10 shows the movement of a systemic crisis probability, whereas the black rectangle indicates media statements on risk perception made by economic policy makers. Detailed expert statements and their dates are provided in Appendix 5. After optimistic expert announcements at the end of 2008, the first hint at a potential crisis appeared in the media in November, which suggested a possible crisis in the forthcoming period. The first public statement about the fact that the Republic of Serbia was in a crisis period appeared in March 2009. As it is evident in the figure, the systemic crisis occurrence probability approaches the upper critical level in October 2008. The systemic crisis probability appeared four months before the first official confirmation. The probability also returned to the normal levels in October 2009, i.e. three months before the first expert confirmation that the Republic of Serbia came out of the crisis, which appeared for the first time at the end of January 2010. According to the model specifications, an effective forecast period based on the probability of a systemic crisis is six months. However, in case of an empirical analysis on the basis of public expert statements, the said period is four months. There is an evident asymmetry between the crisis occurrence and end periods. It is apparent that the signalling period before the crisis occurrence is somewhat longer than the period after the crisis abates. This difference can be explained by the need for confirmation in case of bad news in order not to deepen market instabilities. On the other hand, there is apparent optimism coming from positive statements in order to ease the market pressure, which is reflected in a shorter period between early warning signals and the first official statements about the crisis end.

Figure 10 **Systemic crisis probability and media news headlines**



4 Conclusion

The Systemic Stress Indicator of the Republic of Serbia (SSI) is discussed in detail in this paper. The SSI is calculated based on the aggregation of six segments of the financial system into a composite index, which dynamically takes into account their mutual concordance measures. The paper provides a detailed analytical framework for the calculation of the composite indicator and systemic risk component, relying on the Markov-switching model with dynamic transition probabilities. Strengths and weaknesses of various approaches to the aggregation of the financial system segment sub-indices, which are included in the SSI calculation, were identified. In addition, switching regimes were validated using the Gaussian mixture model. Both approaches assessed the number of regimes and SSI distribution moments identically, which indicates the uniformity of the obtained results. The second part of the paper presents an analytical framework for early warning signals based on the methodology used for SSI calculation, providing a unique framework for assessing not only the current system stress level, but also the probability of a systemic crisis in advance. An analytical framework for assessing the probability of a systemic crisis occurrence in case of an arbitrary number of regimes is also provided. The classification quality of the proposed model is validated by means of assessments of classification quality measures in case of an arbitrary number of regimes, but it is also verified qualitatively based on public statements made by economic policy makers and experts.

It has been demonstrated that the SSI can properly identify crisis periods and the systemic risk level and assess the systemic stress component, providing significant information on the risk accumulation level in financial markets, and possible implications for financial and macroeconomic stability. The paper also contains a detailed analysis of the key instability factors in high stress periods between January 2008 and March 2021 based on SSI dynamics. The approach stemming from the composite index aggregation model was proposed to be used for aggregating indices of individual financial system segments into segment sub-indices. It also elaborates on the advantages of the proposed approach compared to the most frequently used methods in literature.

Unlike the *ex-post* identification of generally undetected crisis periods and key risk factors that triggered them in the past, early warning signals enable timely indication of a potential systemic crisis and factors that could trigger it, as well as the possibility to respond in a timely manner. The paper demonstrates that early warning signals properly monitor the dynamics of undetected periods and provide an assessment of the likelihood of a systemic crisis in case of an arbitrary time horizon. The optimal period in the case of the Republic of Serbia and relatively short time series is six months. It has been demonstrated that an effective prediction period is somewhat shorter and lasts between four and six months based on a qualitative analysis of public statements made by Republic of Serbia’s economic policy makers.

The SSI discussed in the paper can be an excellent supplement to the existing analytical tools used to assess financial stability, such as macroprudential stress tests, a financial stress index and a banking sector stability index (NBS 2021), and provides additional information on a systemic risk measure in the Republic of Serbia’s financial system.

Appendix 1 Sub-index aggregation method analysis

The Appendix contains the description of index movements using different data aggregation models and an explanation of the reasons for applying the proposed aggregation method. The first approach is the equal variance method which uses the same weights during the aggregation. The equal variance method generates an index that assigns equal importance to each variable. This is the most frequently used aggregation method in literature (Illing and Liu 2003). The main assumption of this model is the variable normality, which is at the same time its greatest weakness as it does not take into account an occurrence of a fat-tailed distribution, heteroskedasticity and volatility clustering (McNeil and Frey 2000). A strength of this approach lies in its simplicity and the fact that we do not know a priori aggregation weighting functions in a large number of cases. Therefore, the assumptions of this method are valid.

The second analysed method is a principal component analysis. This method is a statistical procedure that uses an orthogonal transformation in order to reduce data dimensionality taking into consideration a linear dependence between time series (Jolliffe 2002). Orthogonal components are called principal or main components. The number of principal components is less than or equal to the input space dimension. Principal components are ranked based on an explained variability of input space. The first principal component is defined as a component that explains the most of input data variations. The first principal component is used for each segment of the financial system and we will name this approach "the first PCA component". We will see that the key weakness of this approach is that the first principal component often explains a small amount of variation and therefore does not reflect data dynamics to a sufficient extent.

The next approach uses all principal components. Apart from the first component, each subsequent component explains the input data variability in descending order. This approach (Louzis and Vouldis 2013, Hatzius et al. 2010) is based on a weighted sum of principal component values proportional to the explained variability. To avoid including all principal components, only those that cumulatively explain more than the pre-determined amount of variation are included. The orthogonality of principal components helps to solve the multicollinearity problem in aggregating these components using a weighted sum.

Using data, defined at the beginning of this paper, we obtained standardised values for all aggregation methods shown in Figure 14. All indices have similar SSI dynamics. This is supported by high correlation between indices obtained through various aggregation methods shown in Table 5. The values of a linear correlation range from 0.79 to 0.94. The highest index inter-correlation was identified between the equal variance approach and all principal component method, whereas the lowest value was identified between the equal variance approach and the approach in which only the first principal component is used.

Table 5 **Inter-correlation values of various sub-index aggregation approaches**

	Equal variances	First PCA component	All PCA components	Portfolio aggregation
Equal variances	1.00	0.79	0.97	0.88
First PCA component	0.79	1.00	0.85	0.94
All PCA components	0.97	0.85	1.00	0.90
Portfolio aggregation	0.88	0.94	0.90	1.00

The reason is the fact that the first component is insufficient to capture data variability. This is particularly evident in segments of the FX market – FX and public finances – GOV, where the first principal components explain only 32.9%, i.e. 43.9% of the variability, respectively, as it can be seen in Table 6.

Although with similar dynamics, a significant deviation between different indices occurs in the period from May 2012 to January 2013. The indices that are based on the equal variance method and the principal components method are considerably higher compared to other two methods. When selecting a method that best describes the composite indicator of systemic stress of the Republic of Serbia’s financial system, we will analyse this period in more detail in order to assess differences in the approaches. We will focus on two methods: the equal variance method and the co-variance method.

Table 6 **Data variance percentage according to principal components**

	1st component	2nd component	3rd component	4th component	5th component	6th component	7th component
FX	32.9%	32.3%	21.2%	13.5%			
GOV	43.9%	30.3%	16.7%	7.2%	2.0%		
MON	51.6%	16.4%	14.9%	9.5%	4.2%	2.8%	0.5%
EQU	51.1%	30.0%	16.3%	2.6%			
BANK	49.8%	33.5%	16.7%				
FOR	72.5%	27.5%					

Sub-index Figure 15 of each segment of the Republic of Serbia’s financial system is shown in Figure 15. Conditional correlations are also in Figure 15 using the exponentially weighted moving average method (EWMA) of the segment’s time series. Inter-correlations are aggregated by averaging the values of inter-correlations using the formula:

$$\overline{\mathrm{R}} = \frac{1}{\mathrm{N}} \sum \mathrm{R}'_{\mathrm{i,j}} \text{ in which } \mathrm{R}'_{\mathrm{i,j}} = \begin{cases} \mathrm{R}_{\mathrm{i,j}}, \forall \mathrm{i} > \mathrm{j} \\ 0 \quad \text{else} \end{cases}: \tag{32}$$

A further analysis of asymptotic characteristics of multidimensional concordance measures is presented in the paper by Joe (1990).

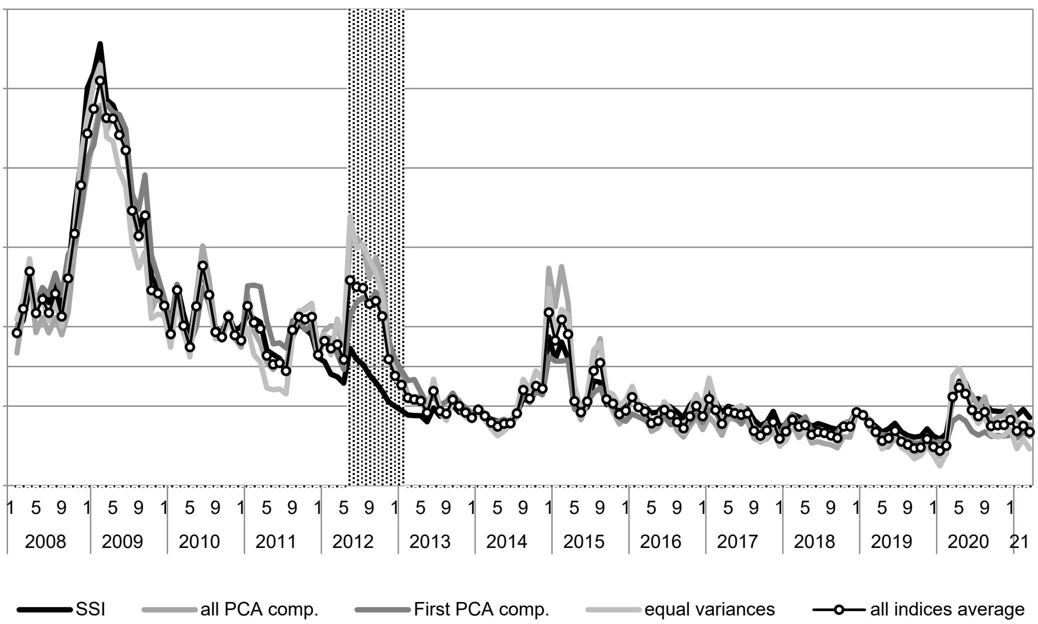
We can see a difference in the SSI level between May 2012 and January 2013, resulting from differences in individual segments, particularly in the case of public finances, money market and banking sector. Differences between sub-indices are the largest in the case of the pronounced difference between non-conditional correlation values, shown in Table 7, and conditional time-varying correlation obtained through the equations (8) to (10), and (32).

Table 7 **Values of non-conditional correlation between segment indicators**

	Non-conditional correlation
FX	0.1060
GOV	0.1426
MON	0.2593
EQU	0.3445
BANK	0.1770
FOR	0.4803

Based on an analysis of the difference of sub-indices for each individual segment and the difference between the non-conditional and conditional correlation values that are shown in Figure 15, a strong linear conditionality is evident in all cases, except in the case of the capital market, between the non-conditional correlation values and conditional correlation values in relation to a deviation of these two sub-indices. Another characteristic of the linear approximation shown in Figure 15 is that in case the non-conditional correlation and conditional correlation match, sub-indices will also match. The cross markers in Figure 15 are used to mark the period between May 2012 and January 2013, which saw the highest deviation between these two indices. As it can be seen, these are extreme values of a difference between correlations. It suggests that the principal component aggregation cannot properly follow the changing concordance measures between variables even though it offers the option of taking into account non-conditional correlations of a multi-dimensional input space. Although it is put to good use in case of input space reduction, it does not properly follow the dynamics, particularly when there is a sudden change in the input space co-variance, which occurs in case of regime changes, which is especially important when developing the SSI.

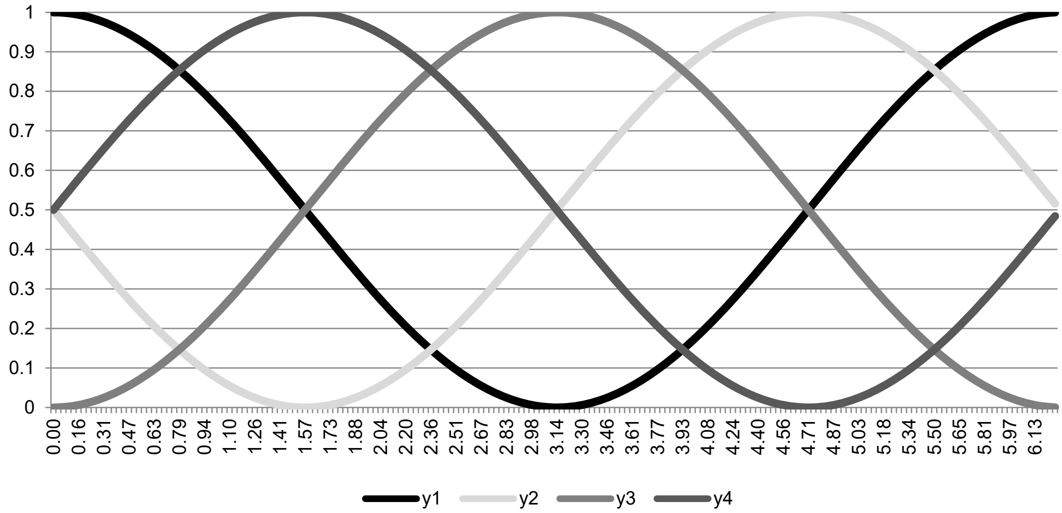
Figure 11 **Movement of the SSI using various aggregation methods**



This characteristic of the principal component methods can be seen in the following theoretical example. Assuming the input space shown in Figure 12 is in sync with the sinusoidal functions:

$$y_i = \frac{sin\left(\left(x + \left({}^{\pi}/_{2}\right)\right) i + 1\right)}{2}, i = 1,2,3,4 \tag{33}$$

Figure 12 **Example of comonotonic and anti-comonotonic signalsExample of comonotonic and anti-comonotonic signals**



It is evident that the sum of these signals in time is constant. There are also two pairs of signals that are anti-comonotonic (McNeil et al. 2015): $\{y_1, y_3\}$ and $\{y_2, y_4\}$, and they cancel each other out. After applying the all-principal method that is shown in Figure 1, it is evident that the first two components explain nearly 100% of the variability. These components are shifted in time, whereby the first component has a wider range in comparison to the second one. The third and fourth components have values that are close to zero. The index marked with a full black line is obtained by aggregating the first two components. The standardised index value ranges in the interval of $[\text{-}0{,}5 - 0{,}5]$. Normalisation that is used to reduce the standardised index to a unit interval results in both limit values in the interval, which indicates a lack of the method application. Namely, the aggregation of principal components covers the entire interval and therefore, they amplify the impact of extreme values. On the other hand, a portfolio aggregation has a lot slower dynamic. This index moves in the interval of $[0{,}1 - 0{,}25]$, which complies with the assumption about the pair index constancy of anti-comonotonic variables and the movement of a non-conditional correlation. Another advantage of this aggregation method is that it does not require additional normalisation and does not consequently generate extreme values of the unit interval and it enables a more accurate range of the index dynamics.

Figure 1 **Example of index movements based on all principal components approach and the portfolio aggregation method**

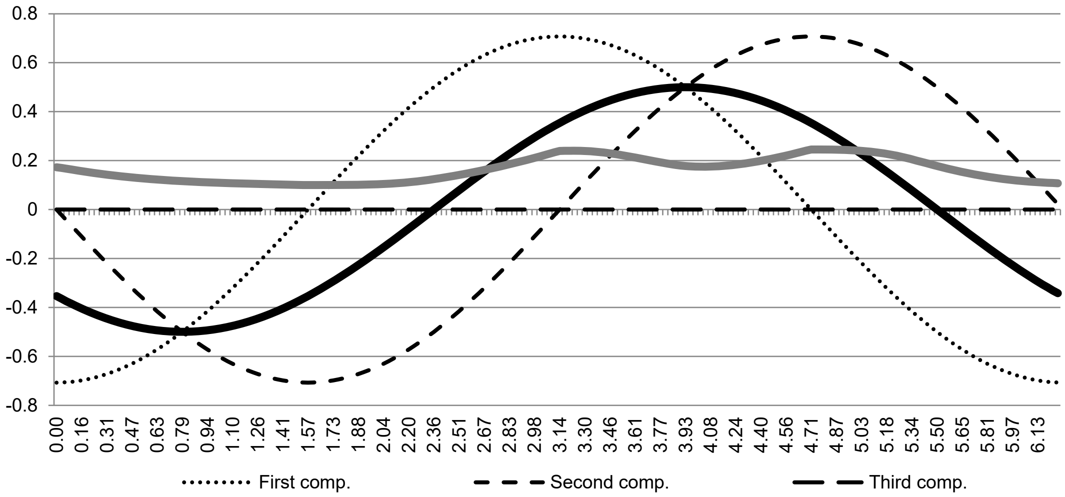


Figure 14 **Sub-index movement by applying the all principal components approach and the portfolio aggregation method by segments and conditional correlation values of sub-index series**

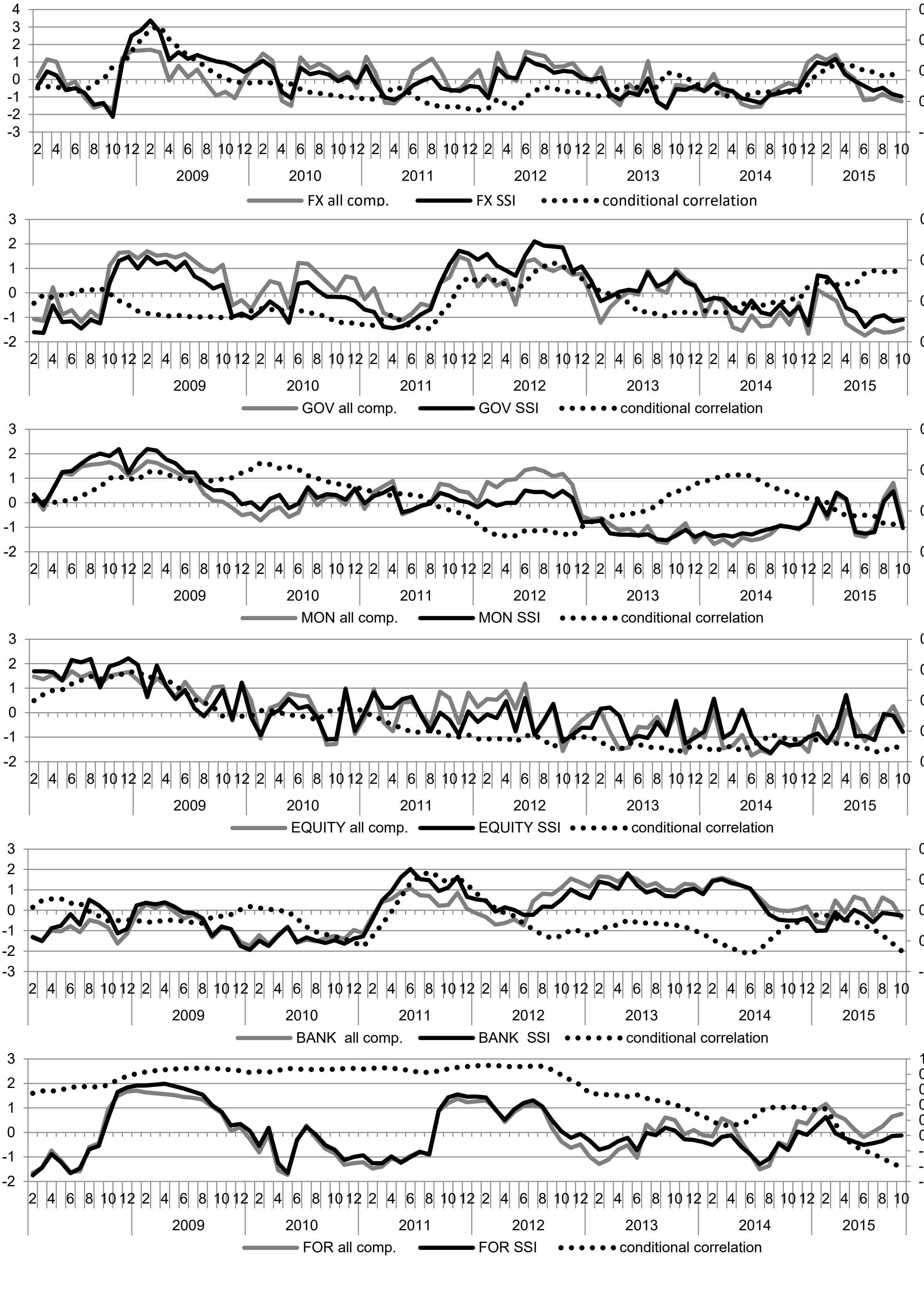
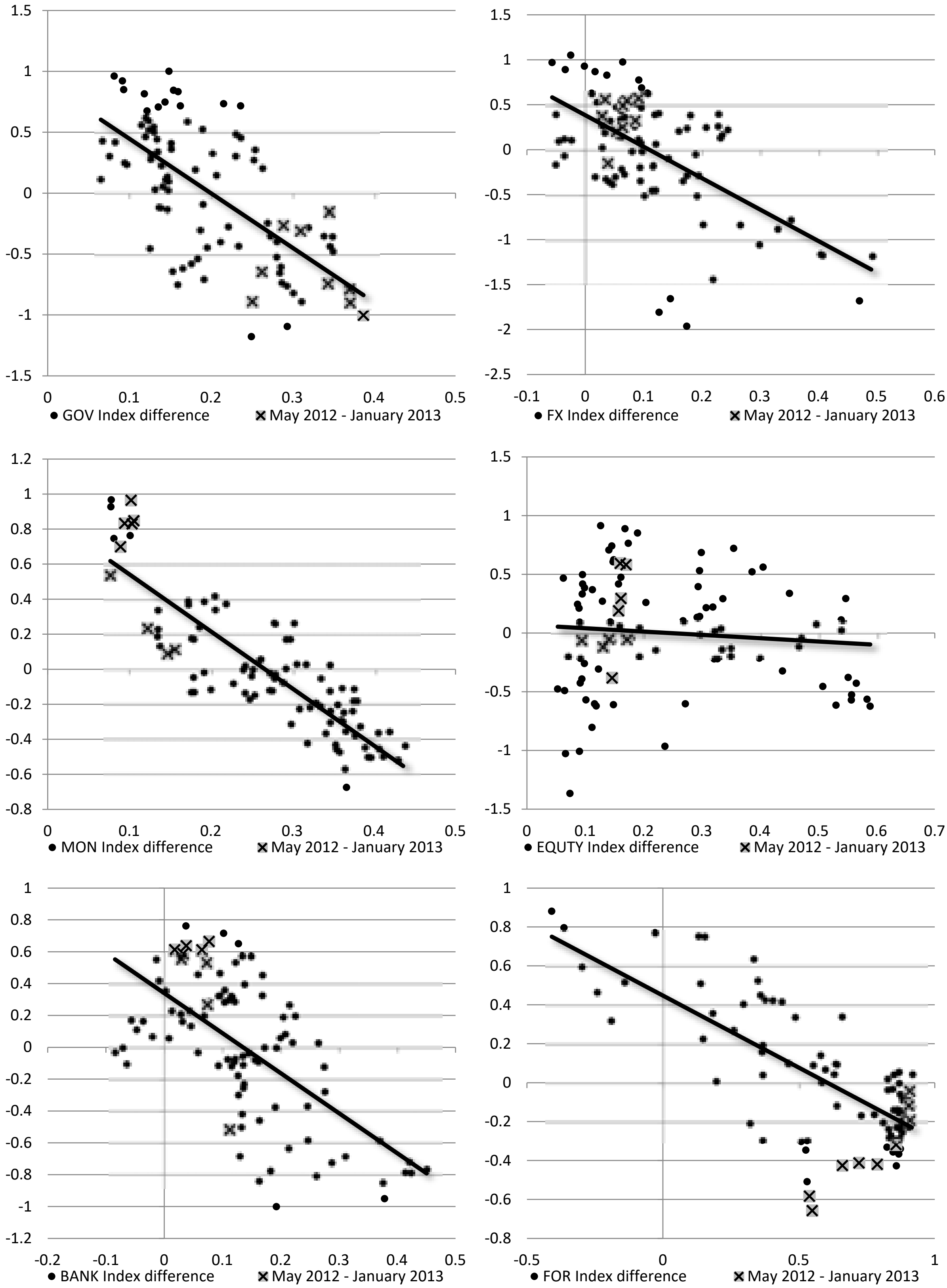


Figure 15 **Difference in sub-index values obtained by using the all principal and the portfolio aggregation method by segments depending on conditional correlation values of sub-index series**



Appendix 2 List of indicators for SSI calculation

Segment	Indicator	Designation
Foreign exchange market	Y-o-y growth of the RSD to EUR exchange rate	FX.EUR_RSD_MG
	Realised volatility of y-o-y growth of the RSD to EUR exchange rate	FX.RV
	Absolute value of the National Bank of Serbia's interventions in the FX market	FX.NBS_Interventions
	Difference between the selling and buying RSD to EUR exchange rates	FX.B_A_spread
Public finances	Difference between the yield to maturity of the 10Y government bond and the 10Y German government bond	Gov.YTM_RS_minus_BUND
	Difference between the selling and buying prices of the 10Y government bond	Gov.B_A_spread_RS
	Realised volatility of the yield to maturity of the 10Y government bond	Gov.YTM_RS_RV
	EMBI Global Serbia	Gov.EMBI_G_Serbia
	Consolidated fiscal result (% of GDP)	Gov.Deficit
Money market	Weighted average interest rate on overnight interbank market loans – BEONIA	Money.BEONIA
	Difference between BEONIA and the key policy rate	Money.BEONIA_minus_REFRATE
	Standard deviation – BEONIA	Money.BEONIA_StDev
	Difference between the maximum rate on overnight loans and interest rate on credit facilities	Money.DEPOSIT_CREDIT_Rates
	Average RSD required reserve allocation on 17th of each month	Money.Allocated_RR
	Difference between a three-month interest rate on EURIBOR and the yield to maturity of the German government bond	Money.EURIBOR_MINUS_BUND
	Difference between a three-month interest rate on EURIBOR and the overnight EONIA interest rate	Money.EURIBOR_MINUS_EONIA
Capital market	CMAX transformation of the stock market index BELEX15	Equity.CMAX_ BELEX15
	BELEX15 index turnover	Equity. BELEX15_turn
	Realised volatility of the stock market index BELEX15	Equity. BELEX15_RV
	Realised volatility of the BELEX15 index turnover	Equity.RV_ BELEX15_turn
Banking sector	Credit gap using a one-sided HP filter	Banking.Loan_Gap
	Deposit gap using a one-sided HP filter	Banking.Deposits_gap
	Weighted sum of CDS parent banks in line with a share in the balance-sheet total	Banking.weight_CDS
Foreign environment	Composite EMBI GLOBAL	FOR.EMBI_G_comp
	Average values of stock market indices in neighbouring countries	FOR.Stock_indexes

Source: NBS, MoF, Belgrade Stock Exchange.

Appendix 3 Statistics of estimated values of model parameters

Table 8 **Main statistics of different values of a dependent state variable lag and the number of regimes**

	F-DMS(2) SAR (0)	F-DMS(3) SAR (0)	F-DMS(2) SAR (1)	F-DMS(3) SAR (1)	F-DMS(2) SAR (2)	F-DMS(3) SAR (2)
Likelihood	225.93	286.29	227.84	**289.3**	226.69	285.97
Number of parameters	6	12	6	12	6	12
AIC	-439.86	-548.58	-443.68	**-554.6**	-441.37	-547.93
BIC	-424.66	-518.19	-428.48	**-524.21**	-426,24	-517.67
RCM	3.17%	2.26%	2.91%	**2.11%**	2.78%	2.64%
Normality (p value)	**3.95E-01**	1.00E-03	3.11E-01	1.00E-03	2.93E-01	1.00E-03
Autocorrelation	0.00E+00	1.13E-08	0.00E+00	2.97E-09	0.00E+00	8.47E-11
ARCH	4.47E-08	5.15E-03	2.27E-08	1.66E-02	4.23E-07	2.43E-03

Table 9 **Statistics of estimated parameter values of the F-DMS(3) SAR (1) model**

F-DMS(3) SAR (1)	Coefficient	Standard error	p-value
Variance (1)	6.30E-05	1.00E-05	0.00E+00
Variance (2)	3.95E-04	1.09E-04	4.00E-04
Variance (3)	1.86E-03	1.23E-03	1.33E-02
Expected value (1)	2.31E-02	8.61E-04	0.00E+00
Expected value (2)	7.40E-02	3.05E-03	0.00E+00
Expected value (3)	1.87E-01	1.31E-02	0.00E+00
$Pa_{(1\|1)}$	9.11E+01	8.88E+00	0.00E+00
$Pa_{(1\|2)}$	-3.81E+01	9.05E+03	9.97E-01
$Pa_{(1\|3)}$	3.61E+02	1.96E+06	1.00E+00
$Pa_{(2\|1)}$	2.47E+01	5.18E+00	0.00E+00
$Pa_{(2\|2)}$	-8.98E+00	7.55E+00	2.36E-01
$Pa_{(2\|3)}$	9.11E+01	8.88E+00	0.00E+00

Other parameter values of transition probabilities $\mathrm{Pa}_{(3|1)}$, $\mathrm{Pa}_{(3|2)}$ and $Pa_{(3|3)}$ are obtained on the basis of the presented transition probabilities.

Appendix 4 Models for assessing the probability of a systemic crisis occurrence

Table 10 **Statistics of reduced models depending on a state variable lag**

Horizon	AIC	BIC	Normality	ARCH	RCM	Likelihood
R-DMS(3) SAR(3)	-561.83	-542.53	1.00E-03	2.42E-04	0.0252	285.85
R-DMS(3) SAR(4)	-551.08	-536.01	1.00E-03	2.43E-04	0.0263	281.54
R-DMS(3) SAR(5)	-540.71	-525.71	1.00E-03	9.94E-06	0.0245	276.36
R-DMS(3) SAR(6)	-538.98	-524.05	1.00E-03	2.22E-06	0.0250	275.49
R-DMS(3) SAR(7)	-535.91	-521.05	1.00E-03	3.83E-07	0.0233	273.96
R-DMS(3) SAR(8)	-526.81	-512.01	1.00E-03	9.80E-08	0.0221	269.40
R-DMS(3) SAR(9)	-518.62	-503.89	1.00E-03	1.07E-07	0.0215	265.31
R-DMS(3) SAR(10)	-520.79	-506.14	1.00E-03	3.24E-09	0.0194	266.40
R-DMS(3) SAR(11)	-515.76	-501.18	1.00E-03	5.95E-10	0.0209	263.88
R-DMS(3) SAR(12)	-511.05	-496.54	1.00E-03	5.26E-11	0.0211	261.53

Table11 **Assessment of the reduced models classification quality depending on state variable lag**

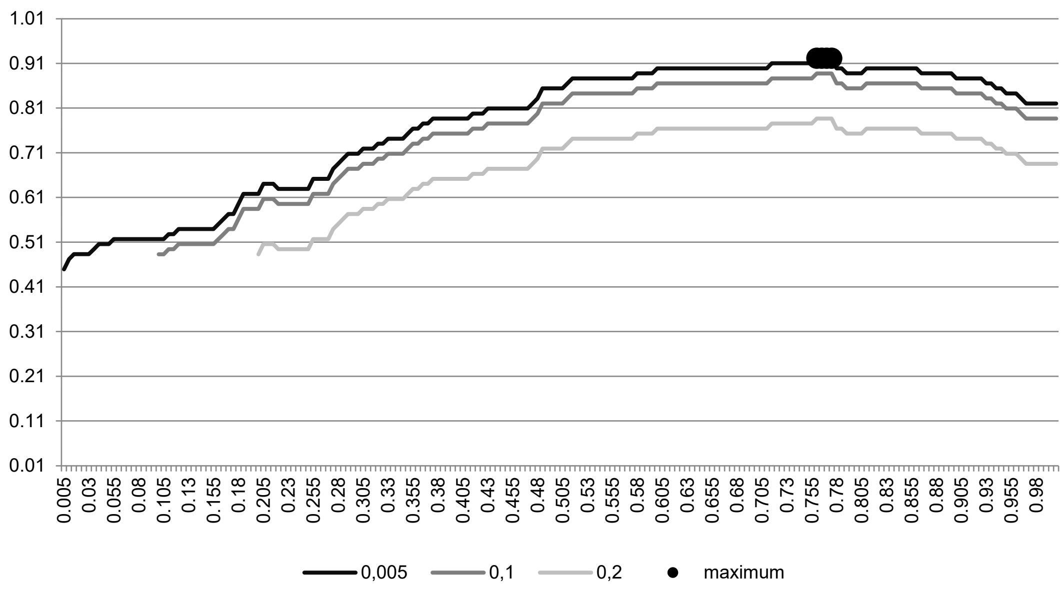
	b_l	b_u	Classification accuracy	Classification sensitivity	Classification specificity
R-DMS(3) SAR(3)	0.010	0.805	0.9333	0.875	0.983
R-DMS(3) SAR(4)	0.005	0.720	0.9322	0.844	1.000
R-DMS(3) SAR(5)	0.005	0.715	0.9211	0.844	1.000
R-DMS(3) SAR(6)	**0.005**	**0.760**	**0.9211**	**0.875**	**0.983**
R-DMS(3) SAR(7)	0.005	0.740	0.8878	0.875	0.948
R-DMS(3) SAR(8)	0.005	0.620	0.8656	0.906	0.914
R-DMS(3) SAR(9)	0.005	0.460	0.8322	0.906	0.862
R-DMS(3) SAR(10)	0.005	0.460	0.8211	0.875	0.845
R-DMS(3) SAR(11)	0.001	0.260	0.5767	0.532	0.879
R-DMS(3) SAR(12)	0.001	0.015	0.5767	0.551	0.879

Table12 **Statistics of estimated parameter values of the R-DMS(3) SAR (6) model**

R-DMS(3) SAR (6)	Coefficient	Standard error	p value
Variance (1)	6.30E-05	-	-
Variance (2)	3.95E-04	-	-
Variance (3)	1.86E-03	-	-
Expected value (1)	2.31E-02	-	-
Expected value (2)	7.40E-02	-	-
Expected value (3)	1.87E-01	-	-
$Pa_{(1\|1)}$	1.14E+02	1.94E+01	0.00E+00
$Pa_{(1\|2)}$	-2.79E+01	1.15E+01	1.65E-02
$Pa_{(1\|3)}$	-3.82E+01	5.19E+03	9.94E-01
$Pa_{(2\|1)}$	3.61E+02	1.30E+05	9.98E-01
$Pa_{(2\|2)}$	2.38E+01	4.54E+00	0.00E+00
$Pa_{(2\|3)}$	-7.25E+00	3.99E+00	7.11E-02

Other parameter values of transition probabilities $Pa_{(3|1)}$, $Pa_{(3|2)}$ and $Pa_{(3|3)}$ are obtained based on the presented transition probabilities.

Figure 16 **Family of curves of classification accuracy in case of three regimes depending on the lower and upper critical classification values**



Appendix 5 News history of the 2008–2010 crisis

“I expect that the crisis will be mitigated before its negative effects are reflected on our country.” **(Dnevnik, 3 October 2008)**

“Serbia will come out of this great global financial crisis intact…. It turned out that our restrictive monetary policy represents steady protection against crises such as the current one.” **(Dnevnik, 16 October 2008)**

“Global crisis threatens Serbia in 2009“ **(Stratfor, 4 November 2008)**

“Crisis came to Serbia earlier than expected and the responsible ones are ‘domestic economic actors’.” **(B92, 3 March 2009)**

”Crisis has not ended but it is slowing down.“ **(RTV Pink, 12 May 2009)**

“Serbia has emerged from the crisis according to economic indicators.” **(BETA, 20 January 2010)**

“Serbia has both formally and statistically come out of the crisis.“ **(Danas, 17 May 2010)**

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