Double Contagion: The Impact of Globalization and Exchange Rate Regime on Financial Fragility *

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Abstract

We study the impact on financial fragility of globalization and a switch from managed to freely floating exchange rate regime in the context of a two-country multi-region model a la Allen and Gale (Journal of Political Economy, 2000) with open-economy monetary features of Chang and Velasco (Journal of Economic Theory, 2000). In this economy, both banking and exchange rate crises may develop. In contrast to Allen and Gale, we find that increased globalization, i.e. a more complete structure of links among regions, can increase financial fragility, if the smaller country maintains a floating exchange rate regime. Furthermore, in a globalized world a higher level of financial fragility may result from a switch to the floating exchange rate regime in the smaller country. The intuition behind these results is that the smaller economy with floating exchange rate regime 're-exports' negative shocks to the neighboring region(s) of the larger economy via the exchange rate depreciation rather than absorbs them. These regions cannot follow suit, and so are more likely to suffer from the financial meltdown. These findings rationalize the phenomenon of 'fear of floating' in many emerging market economies in 1990s and 2000s.

Keywords: financial crisis, contagion, fear of floating

JEL classification: F34

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1 Introduction

Over the past 30 years the incidence of financial crises all over the world has increased dramatically. Often a crisis in one country turns into a 'virus' that spreads contagiously to other countries causing a banking crisis and/or a currency crisis. In this paper we pose two interrelated questions: 1) Does globalization contribute to financial fragility? 2) Does a switch to a flexible exchange rate regime ameliorate or amplify financial fragility? Surely the past 3 decades saw globalization *and* rising fragility. But was it a causal relationship? At the same time most developed and emerging market economies liberalized their domestic financial system, which could also contribute to the financial fragility. Allen and Gale (2000) analyze the relationship between interconnectedness and financial contagion using a multi-region closed-economy Diamond-Dybvig-style model.[1] They find that the higher degree of market completeness (interconnectedness) reduces the fragility of the banking system. This suggests that in a global world with a single currency (or exchange rates permanently fixed) globalization *reduces* financial fragility. But how would the Allen- and-Gale result change if different parts of the world economy had different exchange rate regimes? As for the second question, the conventional wisdom since mid-1990s is that a switch to free floating rates can reduce the financial vulnerability. Most crisis episodes over the last 30 years occurred in countries with managed exchange rate regimes. In Mexico in 1994, and then again in Asian countries in 1997 and in Russia in 1998, attempts to maintain the exchange rate peg invited speculative attacks that made the resulting devaluation deeper. So, it became a commonly shared view that emerging market economies should adopt 'corner solutions,' i.e. either a fully flexible exchange rate regime, or a complete dollarization (euroization). The collapse of the currency board in Argentina in 2001 was considered further evidence in favor of the floating exchange rates. Chang and Velasco (2000) develop a monetary open-economy model to support this new paradigm. They show that the flexible exchange rate regime completely eliminates both currency and banking crises in a small open economy while under fixed exchange rate regime each of them is possible. However, this finding brings up an issue of the impact of switching from the managed exchange rate regime to the flexible one by some particular country on the fragility of the *world* financial system when other parts of the world still have a fixed exchange rate regime. Another reason to doubt the universality of the 'flexible exchange rate paradigm' is the wide spread of the 'fear of floating' among emerging

[1] see Diamond and Dybvig (1983).

market economies in the 1990s and 2000s. Their central banks continue interventions in the foreign exchange market to mitigate the fluctuations in the nominal exchange rate.

We extend the analysis of financial contagion in the context of a two-country multi-region model a la Allen and Gale (2000) with open-economy monetary features of Chang and Velasco (2000). We assume that one of the regions in the Allen-Gale framework is a separate country with its own currency and a central bank. In this economy, both banking and exchange rate crises may develop. In contrast to Allen and Gale, we find that increased globalization, i.e. a more complete structure of links among regions, increases financial fragility, if the smaller country maintains a floating exchange rate regime. Furthermore, in a globalized world a higher level of financial fragility may result from a switch to the floating exchange rate regime in the smaller country. The intuition behind these results is that the smaller country with floating exchange rate regime 're-exports' negative shocks to the neighboring region(s) of the larger economy via the exchange rate depreciation, rather than absorbs them. These regions cannot follow suit, and so are more likely to suffer from the financial meltdown. These findings rationalize the phenomenon of fear of floating in many emerging market economies in 1990s and 2000s.

Our results are striking, because they are obtained in a framework 'most favorable' to the floating exchange rate regime. The setup rules out a financial crisis originating in the economy with the floating exchange rate regime, and there are no other negative effects of the exchange-rate instability present in the 'real world' (including the real sector disruptions due to the balance sheet effects, the lack of nominal anchor, etc.). Furthermore, the run-avoidance under the floating exchange rate regime yields a *lower* exchange-rate depreciation than under the fear-of-floating regime when an external shock hits the economy. However, we show that even under such extreme conditions, a switch from the fear-of-floating regime to the floating regime increases the financial fragility when the system of interbank links is complete. The result is even more striking as it does not depend on the functional form of the utility function of the representative agent.

This paper is related to the three different streams of literature. The first one focuses on the interbank linkages and contagion in the context of closed-economy Diamond-Dybvig-style models. Allen and Gale (2000) consider an economy that consists of a number of regions with a representative bank in each of them. Banks attract deposits and invest them not only in liquid and illiquid assets but also exchange deposits within the banking system given an exogenous interbank market structure. The main finding of their model is that higher degree of market completeness reduces the fragility of the banking system, in other words it reduces the probability of financial contagion. Contrary to the model of Allen and Gale, Brusco and Castiglionesi (2007) show that the market structure where every region is linked only to another region is less fragile than the one where every region is connected with all other regions. Moreover, Degryse and Nguyen (2007) prove this result empirically using the data on the Belgian banking system. In particular, they find that a change from a complete market structure towards incomplete one has reduced the severity of contagion. However, Mistrulli (2005) points out that due to the lack of data and some estimation bias Degryse and Nguyen have underestimated the risk of contagion under incomplete market structure. Based on a unique data set on bilateral exposures Mistrulli investigate how the structural changes of the Italian interbank market in 1990-2003 have affected the probability of contagion. The results indicate that the probability of contagion has increased as a result of the decline in market completeness.

The second stream extends the Diamond-Dybvig framework to an open economy and analyzes the international aspects of financial contagion and the impact of exchange rate regime on financial fragility. In the monetary open-economy model of Chang and Velasco (2000) it is shown that the switch from the fixed exchange rate regime to the flexible one makes the country less financially fragile. Particularly, if the Central Bank of this country acts as a lender of last resort then under the fixed exchange rate regime only currency crisis is possible. If not then the currency crisis is substituted be the banking crisis. Under the flexible exchange rate regime the Central Bank has the opportunity to devalue the national currency. This allows the Central Bank to retain its reserves and eliminate the possibility of crises of any type. Kawamura (2007) presents a small-open-economy two-good version of the Diamond-Dybvig model. He introduces the assumption of cash-in-advance constraints according to which both tradable and non-tradable goods can be purchased only with domestic currency issued by the Central Bank. He also establishes a comprehensive mechanism of exchange of domestic currency for tradables and non-tradables through the assumption of a time separation of market sessions. Kawamura demonstrates that under expansionary monetary policy a flexible exchange rate regime has multiple equilibria, one of which is a partial currency run.

Finally, our paper complements the third stream of literature that studies the 'fear of floating' phenomenon. It has been argued that many emerging market economies mitigate exchange rate fluctuations, because they lack a developed financial system that would help them to cope with the exchange rate variability and its adverse impact on balance sheets of firms and the government. In particular, many agents in developing economies have only limited ability to borrow long-term in their national currency and to hedge their exchange rate risks. Hausmann et al. (2001) find a very strong and robust relationship between the ability of a country to borrow internationally in its own currency and its willingness to tolerate exchange rate volatility vis--vis the interest rate volatility. Calvo and Reinhart (2002) develop a model where fear of floating arises from the combination of the lack of credibility (as manifested by risk-premium shocks), high exchange rate pass-through and inflation targeting. Caballero and Krishnamurthy (2001) relate fear of floating to the inelastic supply of external funds in times of financial crisis which causes an exchange rate overshooting.

The rest of the paper is structured as follows. Section 2 outlines the basic model, i.e. the model without an interbank deposit market. Section 3 describes the model of interbank deposit market and contrasts complete and incomplete market structure. Section 4 is the core of the paper. It contrasts conditions for an economy-wide contagion under different assumptions about the exchange-rate regime and completeness of interbank links. Section 5 concludes the paper. All proofs are placed in Appendix.

2 The Basic Model

2.1 Model Setup

Consider a world economy that consists of four virtually identical regions, A, B, C and D. Regions A, B and C are the parts of 'domestic' economy, while region D is a 'foreign' economy. Each region is populated by a continuum of ex ante identical agents. There are three dates 0, 1, 2. Each agents is born in period 0 and has an endowment of one unit of a tradable consumption good. The price of consumption good in the world market is constant and equals one dollar. We assume that any agent can freely exchange dollars for goods at any moment of time. Consumption good can be invested in a long-run constant-returns-to-scale technology which yields $r < 1$ units of consumption good in period 1 and $R > 1$ units of consumption good in period 2. Alternatively, agents can invest their endowment in the world market. In this case one unit of consumption good invested in period 0 produces one unit of consumption good either in period 1 or in period 2.[2] In all regions agents have Diamond-Dybvig preferences. When born in period 0 agents do not know their type. In period

[2] Instead of investment in the world market, we can assume a storage technology with the gross rate of return equal to 1.

1 each agent discovers his type. With probability λ he is impatient, i.e. derives utility only from consumption in period 1, and his utility function is $u(x)$, where x is his first-period consumption. Function $u(.)$ is smooth, strictly increasing, strictly concave and satisfies Inada conditions. With probability $1-\lambda$ he is patient and derives utility from the real value of holdings of the currency between periods 1 and 2 and from consumption in period 2. Each economy, the domestic one and the foreign one, has its own central bank (to be described later). Dollars are the currency of the domestic economy, while pesos are the currency of the foreign economy. We assume the absolute purchasing power parity, but the nominal peso-dollar exchange rate, and hence, the price level in the foreign economy, may vary. Without loss of generality we assume that in period 0 the peso-dollar exchange rate is equal to one, and the price level in both countries is equal to one. We assume that the utility of a patient agent is $u(\chi(m)+y)$, where y is his consumption in period 2, and m is the real money holdings between periods 1 and 2 (deflated by the price level of period 2). Function $\chi(.)$ is smooth, strictly concave and satisfies $\chi(0)=0, \lim_{m\to 0}\chi'(m)=\infty$ and $\chi'(\bar{m})=0$ for some $\bar{m}>0$. $\bar{m}$ can be regarded as a satiation level of money holdings.

The realization of each agent's type is private information to that agent. Domestic residents can invest but not borrow in the world market.

2.2 The Social Planner's Problem

The Social Planner's Problem is essentially identical for any of the four regions, so we present the problem for an arbitrary 'representative' region.

The Social Planner maximizes the expected utility of the representative agent:

$$U = \lambda u(x) + (1-\lambda)u(\chi(m)+y) \tag{1}$$

subject to:

$$k + b \leq 1 \tag{2}$$

$$\lambda x \leq b + rl \tag{3}$$

$$(1-\lambda)y \leq R(k-l) \tag{4}$$

$$x \leq \chi(m) + y \tag{5}$$

$$x, y, m, k, l, b \geq 0$$

Optimization is done with respect to x, y, m, k, b and l, where x is the consumption of an impatient agent in period 1, y is the consumption of a patient agent in period 2, m is the real money balances provided to a patient agent in period 1, b is the per-capita investment in the world market, k is the per-capita investment in long-term technology and l is the first-period termination of the illiquid technology. Equation (2) is the aggregate resource constraint in period 0 in per-agent terms. It shows that the sum of investment in long-term technology, k, and world market, b, cannot exceed the initial endowment. Equations (3) and (4) are the aggregate resource constraint in periods 1 and 2, respectively. Equation (3) shows that consumption of an impatient agent comes from the return on storage, b, and period 1 termination of the long-term investment, l. It also takes into account that the share of impatient agents in the population is λ. Equation (4) shows that consumption of the patient agents comes from the return on remaining illiquid technology and takes into account that the share of patient agents is $1 - \lambda$. Finally, equation (5) is the incentive-compatibility constraint. It shows that a patient agent has no incentive to misrepresent himself in period 1 and claim he is impatient. It is worth noting that m is not present in the left-hand side of any constraint. This is because money is costless to produce, and hence the Social planner can create money up to the satiation point, $\bar{m}$. The complete analysis of the problem (1)-(5) is given in the Appendix. Here we would like to note that it is never optimal to interrupt the long-term technology in period 1, and it is never optimal to leave any resources unused. Therefore, in equilibrium $l = 0$ and inequalities (2)-(4) are satisfied as equalities. Furthermore, the social planner is able to provide unlimited amount of money balances to agents. Without loss of generality we assume that $m = \bar{m}$. Finally, the assumptions about the utility function ensure that the solution to the problem (1)-(5) does exist, and is unique.

2.3 Competitive Equilibrium and Central Banks

Similar to the original Diamond-Dybvig model (and its numerous derivatives), the first-best allocation can be decentralized in a competitive equilibrium with banks. Commercial banks arise endogenously to provide liquidity and insure agents against preference shocks. The complete characterization of the equilibrium will depend upon the central bank policy. We consider two different cases: the case of a central bank acting as a lender of last resort in the foreign country, and the case of a central bank merely supplying the currency for the impatient agents in the domestic country. We assume that the central bank of the domestic country lends dollars interest-free to banks in period 1 and allows banks to use these dollars only for withdrawals of reportedly patient agents. In period 2 banks (if solvent) repay the loan.[3] It is convenient to assume that the Central bank provides exactly $h = (1-\lambda)\bar{m}$ in real per-agent terms. This limited role of the central bank rules out depreciation of dollars vis-a-vis the consumption good.

Next consider the case of the foreign economy. We assume that its central bank also lends the local currency (i.e., pesos) to the commercial bank to satisfy the real money demand of the patient agents, but it can also serve as a lender of last resort, i.e. in the case of bank run it can provide liquidity support to the commercial bank. In particular, if more than λ customers claim to be impatient, the Central Bank lends as many pesos as necessary to meet the demand of reportedly impatient depositors. In the latter case, the Central Bank obtains control over the long-term asset in period 1 and liquidates it as needed to sell the dollars to agents claiming impatience.[4] All transactions between the commercial bank and its depositors are done in the local currency - pesos.

2.4 Timeline

The sequence of events is as follows (exchanges of dollars for consumption good are omitted for brevity):

Period 0

1. Agents are born with their endowments.

2. Foreign-country agents exchange their endowments for pesos at the central bank of the foreign country.

3. All agents make deposits at the commercial bank of their region. Foreign-country agents deposit pesos, Domestic-country agents deposit dollars.

4. Commercial bank of the foreign country exchanges all of its pesos at the central bank for dollars, invests a part of its funds into domestic long term technology, and another part - in the world market. Commercial banks of the domestic country also invest a part of their funds into domestic long term technology, and another part - in the world market.

[3] We borrow this assumption from Chang and Velasco (2000). It allows commercial banks to satisfy the money demand of reportedly patient agents without diverting real resources.

It is convenient to assume that the central bank of the domestic country is the only agent that can borrow dollars on the international market. It can borrow interest-free, because world-market investors earn zero net return on their investment, and in equilibrium the central bank always repays the debt.

[4] It is convenient to assume that the central bank appropriates the share of the long-term investment which is equal to the share of patient agents claiming impatience.

Period 1

5. All agents learn their types.

6. All agents report their types to their banks.

7. After observing the share of reportedly impatient agents each commercial bank in the domestic economy liquidates all of its world market investment and in case it faces a higher fraction of impatient agents (a bank run) it also terminates the long-term investment to satisfy the withdrawal requests of depositors. Each commercial bank in the domestic economy requests a loan from the Central Bank to satisfy money demand of the reportedly patient depositors. Commercial bank of the foreign country also observes the share of reportedly impatient agents, and if it is higher than λ requests an emergency credit from the Central bank. The commercial bank of the foreign country requests pesos to satisfy the money demand of reportedly patient depositors.

8. Central bank of the foreign country issues pesos and provides a loan to the commercial bank. Central bank of the domestic country borrows dollars in the world market and lends them to the commercial banks.

9. Commercial banks make payments to their depositors.

10. Reportedly impatient agents of the foreign country exchange pesos obtained from the commercial bank for dollars at the CB.

11. Truly impatient agents consume, patient agents who claim impatience invest in the world market.

Period 2

12. Commercial banks liquidate all their long-term investment. Reportedly impatient, but truly patient agents liquidate their world-market investments.

13. Commercial banks repay their debts to central banks.

14. The Central bank of the Domestic country repays the debt to the world-market investors.

15. Foreign commercial bank exchanges dollars for pesos at the CB of the foreign country.

16. These pesos are used to pay off foreign-country reportedly patient agents. Commercial banks of the domestic country pay off patient agents.

17. Foreign-country reportedly patient agents exchange their pesos for dollars.

18. All patient agents consume.

2.5 Commercial Banks and Multiplicity of Equilibrium

Because of the perfect competition among banks each representative bank strives to offer agents a deposit contract that maximizes the expected utility of the representative agent (1) subject to the following constraints:

$$k + b \leq 1 \tag{6}$$

$$\lambda x + (1-\lambda)M \leq b + h + rl \tag{7}$$

$$(1-\lambda)y - (1-\lambda)M \leq R(k-l) - h \tag{8}$$

$$\chi(M) + y \geq x \tag{9}$$

$$X, y, M, k, l, b \geq 0,$$

where M is the nominal money balances (in local currency terms) lent to commercial banks in period 1. Inequalities (6)-(8) are the budget constraints of the commercial bank in periods 0-2, respectively. Inequalities (7)-(8) differ from the inequalities (3)-(4) in the Social Planner's problem by the term $(1-\lambda)M$ in the left-hand side and the term h in the right-hand side. h is the amount of loan from the central bank (in per-agent terms) to provide for the money demand of the patient agents. This loan is repaid in period 2. $(1-\lambda)M$ is the payment of the bank to the reportedly patient agents in period 1.

It is straightforward to see that the system (1), (6)-(9) yields the same values for x, y, k and b as the system (1)-(5) if $M = \bar{m}$, i.e. the real value of the currency does not change, and $h = (1-\lambda)\bar{m}$.[5] This implies that the competitive equilibrium allocation coincides with the command optimum. It is important to note that the bank is unable to make the payment to a depositor conditional on his type, because the type is private information. Instead, the bank has to offer a demand-deposit contract: any depositor can claim that he is impatient and withdraw x in period 1. On the other hand, if a depositor claims he is patient, he can withdraw M dollars in period 1. Following the bulk of the literature, we assume the sequential service constraint: the bank cannot condition the first-period payouts on the number of agents claiming impatience, and pays x to every agent claiming impatience and M to every agent claiming patience as long as it has any resources left. However, like in the standard Diamond-Dybvig framework, the demand-deposit arrangement yields

[5] Henceforth, we will use these variables for their optimal values to simplify notations.

multiplicity of equilibria. There is an 'honest' equilibrium, in which allocation coincides with social (command) optimum. Because of the incentive-compatibility constraint (9) patient agents have no incentive to misrepresent their type when other patient agents do the same. But there is another equilibrium, a bank run. If, for any reason, a sufficient number of patient agents believe that the commercial bank will be insolvent in period 2, they claim impatience and attempt to withdraw their deposits in period 1. The commercial bank has not enough resources to pay all the depositors, if

$$b + rk < x \tag{10}$$

Assuming that inequality (10) is satisfied, the bank will interrupt all its long-term investment and exhaust all its resources before it is able to pay all the depositors. Hence, the patient depositors who wait until period 2, will get noting in period 2, and it is indeed optimal for all patient depositors to attempt to withdraw in period 1.

Another condition for the run of patient agents is

$$M < x \tag{11}$$

If inequality (11) is violated, impatient agents have an incentive to pretend they are patient. This would be a 'reversed' run.

The representative bank of the foreign country strives to offer agents a deposit contract that maximizes the expected utility of the representative agent (1) subject to the same constraints (6)-(9). The only difference is that all contracts are denominated in pesos. In case of no run the command optimum allocation is achieved. Furthermore, emergency credit from the central bank of the foreign country prevents the bank run. There is a currency crisis instead: it is the central bank that has to terminate the long-term investment to satisfy the demand for dollars of the reportedly impatient agents. As inequality (10) is satisfied, the central bank has not enough resources to exchange pesos for dollars at the initial exchange rate equal to 1, and has to devalue the peso.

3 The Model of Interbank Deposit Market

We now move to a model of interbank deposit market. Following Allen and Gale (2000) we assume that the probability of being an impatient can vary across regions. There are three states of nature. In state S_1 the probability of being impatient is the same from region to region and equals λ. In states S_2 and S_3 there are two possible values of this probability for each region, a high value and a low value, denoted by w_H and w_L, respectively, where $0 < w_L < w_H < 1$ and $\lambda = (w_H + w_L)/2$. States S_2 and S_3 are equally likely, in each region an agent has the same *ex ante* probability of being impatient, namely λ. Realizations of liquidity shocks are given in Table 1.

Table 1

State	A	B	C	D	Probability of the State
S_1	λ	λ	λ	λ	p
S_2	w_H	w_L	w_H	w_L	$0.5(1-p)$
S_3	w_L	w_H	w_L	w_H	$0.5(1-p)$

Given that the aggregate demand for liquidity in foreign and domestic countries is the same in each state, the world social planner problem is the same as in section 2. But to implement the command optimum in a decentralized setting, banks have to make interbank deposits in other regions. We compare two alternative arrangements: a complete interbank market, where all regions are linked to all other regions, and an incomplete interbank market.

3.1 Complete Interbank Deposit Structure

Commercial banks are allowed to exchange deposits in period 0. The case of complete market is illustrated in figure 1 (see Appendix).

Each bank exchanges demand deposits with three other banks, i.e. it makes deposits in all the banks, and received deposits from all other banks. Deposits in the foreign economy are denominated in pesos, while all deposits of the domestic economy are denominated in dollars. We assume that if some commercial bank of domestic country wants to make a deposit in the commercial bank of foreign country, it will exchange dollars for pesos at the Central Bank of the foreign country at an exchange rate of one dollar per peso. This will lead to the increase of the dollar reserves of the Central Bank of the foreign country. These reserves will be just enough when the commercial bank of the foreign country in turn decides to put its deposit to the commercial bank of the domestic country considered above. Assume that every commercial bank in region j holds $z/2 = (w_H - \lambda)/2$ demand deposits in each region $i \neq j$. This is the smallest amount that banks have to deposit in order to fully insure against region-specific liquidity shock. Banks from other regions have the same rights as private depositors from the region, and the deposit rates are also the same. It is straightforward to show that such an arrangement allows banks to attain the first-best allocation. Every bank offers the same terms of the demand deposit contracts in period 0 and makes the same investment decision as in autarky. When a region faces a positive liquidity shock (it has a higher share of impatient deposits, w_H), it withdraws demand deposits from other regions in period 1 and satisfies liquidity needs of the depositors without having to interrupt long-term technology. In the second period, it has a lower need for funds because of the smaller number of patient depositors, and hence it can pay off banks with the lower share of impatient/higher share of patient agents. Furthermore, it is also easy to show that different currencies do not affect the feasibility of first-best (no-run) allocation. For example, when a domestic-country bank withdraws the peso-denominated deposit from the foreign country bank, the latter bank exchanges dollars for pesos in the Central bank, pays them to the domestic-country bank, which in turn goes to the foreign Central bank and exchanges pesos back for dollars. As a result, neither foreign central bank reserves, nor amount of pesos in circulation are affected.

3.2 Incomplete Interbank Deposit Structure

An alternative, i.e. incomplete, system of interbank links is shown in figure 2 (see Appendix). We assume that bank in each region can hold deposits only in one neighboring region.

Commercial bank from region A can hold deposits only in region B, bank in region B can hold deposits only in region C and so on. We assume that the amount of each interbank deposit is $z = w_H - \lambda$. Again, this is the smallest amount of interbank deposit that allows banks to attain the command optimum allocation for their depositors. The way banks acquire and redeem deposits denominated in different currencies is quite similar to the case of complete interbank market structure.

4 Analysis of Financial Fragility

To compare financial fragility under different interbank deposit arrangements and different exchange rate regimes, we assume that there is a financial crisis in one region, i.e., a bank run in a region of the domestic economy, or a currency crisis in the foreign economy. We analyze and compare conditions under which the crisis will spread to all other regions, in particular, under which in all regions of the domestic economy patient agents will attempt to withdraw in period 1.[6] We focus on the case when the share of impatient agents in all regions is λ.[7] If under certain conditions a

[6]The behaviour of the patient agents in the foreign economy depends on the exchange rate regime. Under the flexible exchange rate regime they never misrepresent their type, and hence there is no run.

[7]We follow Allen and Gale by focusing on this case. To justify this focus one can assume that p, the probability of the state S_1, is sufficiently close to 1.

wider range of parameter values ensures the spread of the crisis throughout the world economy, we conclude that under these conditions the global financial system is more fragile.

4.1 Pecking Order of Asset Liquidation

In period 1 each commercial bank can be in one of the three conditions. It is solvent if it can satisfy the demands of every depositor who wish to withdraw (including banks in other regions) by using only its world-market short-term assets and deposits in other regions. The bank is insolvent if it can carry out its obligations either by liquidating some proportion of its long- term asset (in case of domestic country) or drawing on the emergency credit of the Central Bank (in case of foreign country). The commercial bank in domestic country is said to be bankrupt if it can not satisfy the demands of its depositors by liquidating all of its assets. The commercial bank in foreign country can not be bankrupt due to unlimited emergency credit of the Central Bank. There exists a pecking order of liquidation of assets in period 1. According to this pecking order banks liquidate their short-term assets first, then they liquidate deposits in other regions and finally they liquidate long-term investments. This pecking order is ensured by the following condition:

$$\frac{R}{r} > \frac{y}{x} > 1 \tag{12}$$

The opportunity cost of period 1 consumption (in terms of future consumption) is different across assets. For the short-term asset it is unity because one unit of short-term asset is worth one unit of consumption good today and if it is reinvested it will be worth one unit of consumption good tomorrow. If we withdraw one unit of demand deposit in period 1 we get x units of consumption goods and give up y units of consumption good in period 2. So, the opportunity cost of getting liquidity by liquidating deposits is y/x. Finally, if the bank liquidates one unit of the long term asset, it gives up R units of consumption good in period 2 and gets r units of consumption in period 1. So the opportunity cost of liquidating the long term asset is R/r.[8] This pecking order is violated in case of bankruptcy. If the bank in some region is bankrupt then all its depositors including banks in other regions will rush to withdraw their deposits regardless of their own pecking order.

[8] Inequality (12) is not particularly restrictive. $y/x > 1$ follows from the first-order condition of the problem (1),(6)-(9), $u'(x) = Ru'(\chi(m) + y)$ assuming that $\chi(m)$ is sufficiently small compared with y. Inequality $R/r > y/x$ holds for a wide variety of utility functions and parameter values. In particular, it holds for any logarithmic utility function $u(x) = \log(x)$ as well as for any constant relative risk-aversion utility function $u(x) = x^{1-\gamma}/(1-\gamma)$ if the CRRA parameter $\gamma > 1$.

4.2 Bank Buffer

Following Allen and Gale (2000) we introduce the notion of buffer. Buffer is defined as the maximum amount of dollars that can be obtained by liquidating the long-term asset in period 1 without causing a run by patient depositors. When a bank is insolvent, patient depositors should be given at least $x - \chi(m)$ in order to withdraw their deposits in period 2. Otherwise it will be better for them to (attempt to) withdraw in period 1. A bank with a fraction λ of impatient consumers must keep at least $(1 - \lambda)(x - \chi(m))/R$ units of long-term asset in order to meet the demand of the patient agents in period 2. Thus the buffer is

$$g(\lambda) = r\left[k - \frac{(1-\lambda)(x - \chi(m))}{R}\right] \tag{13}$$

4.3 Monetary Regimes

Financial fragility in the world economy critically depends on the monetary regime adopted by the central bank of the foreign country. What does the central bank do when patient agents report impatience and withdraw their deposits from commercial bank in period 1: does it attempt to keep the exchange rate and liquidate the long-term investment in order to get dollars to intervene in the foreign-exchange market? Or does it allow the peso to depreciate and keep the long-term investment until period 2? We denote the share of the long-term investment that the central bank is willing to terminate as v and contrast two polar cases: when the central bank is willing to terminate all the long-term investment in order to maintain the exchange rate, ($v = 1$), and when it does not terminate at all, ($v = 0$). The former case is comparable to the 'fear of floating' regime, when the central bank intervenes in the foreign exchange market, and is willing to use the foreign exchange reserves in order to prevent excessive fluctuations of the exchange rate. The latter case is comparable to the flexible exchange rate regime, when the central bank does not intervene.

4.4 Liquidation Values

Liquidation value of a commercial bank deposit is the value of bank assets per unit of deposit in dollar terms when the bank is bankrupt (liquidated) in the domestic country, or when the currency is devalued in the foreign country. Consider, for example, incomplete interbank deposit market and bank run in region A that does not spread to region B. If all depositors in region A decide to withdraw their deposits the bank's dollar liabilities will be $(1 + z)$ because the bank from region

D holds z units of deposit and patient and impatient agents in region A together hold one unit of deposit. Its dollar assets consist of b dollars obtained from liquidation of the short term asset, rk dollars obtained from early liquidation of long term investment and zx dollars obtained from the liquidation its deposit in the bank in region B. The equilibrium value of q^A is determined as

$$q^A = \frac{b + rk + zx}{1 + z} \tag{14}$$

4.5 Financial Fragility in the Case of Incomplete Market Structure

4.5.1. The Fear-of-Floating Exchange Rate Regime

We begin with the case of incomplete market structure shown in figure 2. We assume that the central bank of the foreign country pursues 'fear of floating' policy and stands ready to terminate the long-term technology in period 1 in order to get dollars for allegedly impatient agents. We proceed as follows: we start with the financial crisis in region D (the foreign country) and derive conditions for the crisis to spread to all other regions. Then we show that the conditions are exactly the same for the crisis originating in region A to spread to the rest of the world. Finally, we argue that the conditions are the same if a crisis begins in regions B and C.

If all depositors in the foreign country (including foreign banks) withdraw their deposits, they get $(1 + z)x$ pesos, which is equal to the demand for dollars from the foreign-country central bank at the initial exchange rate of 1 peso per dollar. However, the dollar reserves at the central bank will be at most $b + rk + zx$. Under condition $b + rk < x$, the central bank has to devalue the peso, and the new exchange rate is

$$E^1 = \frac{(1 + z)x}{b + rk + zx} > 1 \tag{15}$$

After devaluation the bank in region C will suffer losses because it will get only zx/E^1 dollars while it should pay zx dollars to the bank in region B. So, the loss will be $zx(1 - 1/E^1)$. The bank in region C will be bankrupt if its loss exceeds the buffer:

$$zx \left(1 - \frac{1}{E^1}\right) > g(\lambda) \tag{16}$$

It turns out that condition (16) is a necessary and sufficient condition for the global run originating in region D.

Lemma 1. A run in region D ensures runs in regions A, B, and C if and only if conditions (10) and (16) are satisfied.

Conditions (10) and (16) ensure runs in regions B and A, because of the spillover effect. More regions are already bankrupt, more losses have accumulated due to early liquidation of the long term asset and the liquidation value of the interbank deposit in each subsequent bank gets smaller and smaller.

What will happen if all depositors in region A decide to withdraw their deposits? The bank in region A is bankrupt and maximum liquidation value of its assets given that the bank in region B is not bankrupt is

$$q^A = \frac{b + rk + zx}{1 + z} \tag{17}$$

Bankruptcy of the bank in region A will lead to the loss in the bank of the foreign country. If the loss exceeds the buffer

$$z(x - q^A) > g(\lambda) \tag{18}$$

then all depositors of the foreign country will rush to withdraw their deposits, so the Central Bank will be forced to devalue the ruble and the lower bound of the new exchange rate will be

$$E^2 = \frac{(1+z)x}{b + rk + zq^A} > 1 \tag{19}$$

It is easy to show that $E^2 > E^1$ so that the foreign country will experience a higher devaluation than in case when the liquidity shock starts in the foreign country itself. This happens because of the loss on deposits held in region A.

It turns out that condition (18) is equivalent to (16) and Lemma 2 verifies that (10) coupled with (16) are the necessary and sufficient conditions for the run affecting regions D, B and C.

Lemma 2. A run in region A ensures runs in regions B, C and D, if and only if conditions (10) and (16) are satisfied.

Furthermore, it turns out that conditions for the global run originating in regions B and C are exactly the same. In other words, for the 'fear-of-floating' case $v = 1$ we can say that all banks are in some sense completely identical. Since the Central Bank of a small country will liquidate all of its long-term assets in case of a run then liquidation value of small country's deposits in dollar terms will be the same as the liquidation value of deposits of any big country's bank (in case of a run on this particular bank). This is due to the devaluation rule adopted by the Central Bank of the small country.

4.5.2. Floating Exchange Rate Regime

Consider now the case of $v = 0$, i.e. the case of the floating exchange rate in the small country, when its central bank does not terminate the illiquid technology in order to minimize the exchange rate depreciation. In this case the currency crisis equilibrium in region D is not possible even if the implicit short term peso-denominated obligations are higher than the dollar denominated assets of the whole country under the exchange rate of one peso per dollar. The commitment of the Central Bank not to liquidate long term investments in period 1 can be considered as insurance for the patient agents that they will get a promised amount of pesos and will be able to exchange them for dollars at a fixed exchange rate of one peso per dollar. So, this devaluation rule eliminates currency crisis equilibrium in foreign country. However, if a run occurs somewhere in the domestic country it can lead to devaluation in foreign country. Let us consider a case when suddenly all depositors in region A decide to withdraw their deposits. The bank in region A will be bankrupt and maximum liquidation value of its assets given that the bank in region B is solvent will be determined by the following equation (20).

$$q^A = \frac{b + rk + zx}{1 + z} \tag{20}$$

The dollar denominated assets of the bank in region D whose maximum value is equal to $b + zq^A$ will be lower than its peso-denominated liabilities $\lambda x + zx$ under the exchange rate equal to unity. The bank will ask for emergency credit from the Central Bank to cover the difference and pass the control over long term assets on the Central Bank. Since the early liquidation of the long term assets is not allowed, the Central Bank will devalue the currency without liquidation of the long term asset and the lower bound of the new exchange rate will be equal to

$$E^3 = \frac{(\lambda + z)x}{b + zq^A} \tag{21}$$

The bank in region C will suffer a loss due to this devaluation. It will be bankrupt if and only if its minimum possible loss exceeds the buffer:

$$zx\left(1 - \frac{1}{E^3}\right) > g(\lambda) \tag{22}$$

Lemma 3 verifies that this is the main condition for the global crisis originating in A.

Lemma 3. Under the assumptions of incomplete market structure and flexible exchange rate regime a run in region A ensures runs in regions B and C, if and only if conditions (10) and (22) are satisfied.

As Lemma 4 below asserts, the main condition for the global contagion originating in region B is as follows:

$$zx\left[1-\frac{b+\frac{z}{1+z}\left(b+rk+zq^B\right)}{(\lambda+z)x)}\right]>g(\lambda), \tag{23}$$

where

$$q^B=\frac{b+rk+zx}{1+z} \tag{24}$$

Lemma 4. Under the assumptions of incomplete market structure and flexible exchange rate regime a run in region B ensures runs in regions A and C, if and only if conditions (10) and (23) are satisfied.

The last equation shows the liquidation value of the deposits in the bank of region B after it suffers from a run. Inequality (23) shows the condition under which the bank in region C is bankrupt. The proof of Lemma 4 shows that inequality (23) ensures that bank in region A is bankrupt as well.

Finally, Lemma 5 specifies conditions under which the run in region C spreads to all other regions of the domestic economy. It turns out that the conditions are the same as in the case of the fear-of-floating exchange rate regime.

Lemma 5. Under the assumptions of incomplete market structure and flexible exchange rate regime a run in region C ensures runs in regions A and B, if and only if conditions (10) and (16) are satisfied.

The result is not unexpected given that the contagion spreads from region A to B and then to C, i.e. the regions of the domestic economy, and hence the exchange rate regime in the foreign economy does not affect the contagion.

The Proposition 1 below contrasts conditions for global run under flexible and fear-of-floating regimes.

Proposition 1. *Under incomplete market structure, conditions for a global run are at least as stringent under $v=0$ as under $v=1$.*

Proposition 1 states that the global economy is more fragile under the fear-of-floating regime than under the flexible exchange rate regime. The proof compares the condition for the global run originating in each of the 4 regions. If the crisis starts in region C, the conditions are identical. If the crisis starts in regions A or B, the crisis can spread to all other regions of the domestic economy, but the conditions are more stringent (i.e., the global run is less likely) under the flexible exchange rate regime. Finally, a run can originate in region D only under the fear-of-floating regime.

Proposition 1 is fully consistent with findings of Chang and Velasco (2000). In both models, the flexible exchange rate arrangement allows the monetary authority to refrain from termination of the long-term technology. This, in turn, ensures that the patient agents have no incentive to run and mitigates (in this model) or prevents (in the model of Chang and Velasco) the financial crisis.

4.6 Financial Fragility when Interbank Deposit Market is Complete

4.6.1. The Case of the Fear-of-floating regime

Under complete market structure the bank in each region holds $z/2 = (w_H - \lambda)/2$ deposits in each other regions. The claim on any region is smaller than in case of incomplete interbank market structure but the total claim is larger. Consider what happens if all depositors in foreign country decide to withdraw their deposits. The argument is the same as before. The bank's dollar denominated assets become lower than its peso-denominated liabilities under fixed exchange rate, as

$$(1 + 3z/2)x > b + rk + 3zx/2$$

The Central Bank will devalue the ruble and the lower bound of new exchange rate will be

$$E^4 = \frac{(1 + 3z/2)x}{b + rk + 3zx/2} \tag{25}$$

It is easy to show that $E^1 > E^4$, i.e. under the complete market structure the Central Bank devalues the peso by less than under the incomplete one. Since devaluation and the claim on any bank are lower than under the incomplete market structure, the loss of each other will be also lower. Banks in regions A, B and C will be bankrupt if the following inequality holds for every bank:

$$\frac{zx}{2}\left(1 - \frac{1}{E^4}\right) > g(\lambda) \tag{26}$$

Because the complete market structure is symmetric, the condition is the same for every region of the domestic economy.

Lemma 6. Under the assumptions of complete market structure and fear-of-floating exchange rate regime a run in region D ensures runs in regions A, B and C, if and only if conditions (10) and (26) are satisfied.

If the liquidity shock occurs in some region i of the domestic country (the place of origin does not matter now since the market is complete) then banks in all other regions will be bankrupt if the following inequality holds for each bank:

$$\frac{z}{2}(x - q_1^i) > g(\lambda) \tag{27}$$

where

$$q_1^i = \frac{b + rk + 3zx/2}{1 + 3z/2} \tag{28}$$

Is the liquidation value of the unit of deposit in region i provided there is no run in other regions. It turns out that conditions (26) and (27) are equivalent.

Lemma 7. Under the assumptions of complete market structure and fear-of-floating exchange rate regime a run in any region of the domestic economy ensures runs in all other regions, if and only if conditions (10) and (26) are satisfied.

We are now in a position to compare conditions for the global crisis under complete and under incomplete market structure, provided that the foreign country has a fear-of-floating exchange rate regime.

Proposition 2. *Under the fear-of-floating exchange rate regime, conditions for a global run are more stringent under the complete market structure than under incomplete one.*

The Proposition 2 states that the global contagion is more likely when the structure of interbank links is incomplete than when it is complete. This is essentially the original Allen and Gale (2000) result. We have already seen that under the fear-of-floating regime the difference between the domestic and the foreign country becomes immaterial.

4.6.2. The Case of the Floating Exchange Rate regime

If the Central Bank in the foreign country commits not to liquidate the long term assets in period 1 the financial crisis is not possible, as patient agents will never have an incentive to get dollars in period 1. Consider what will happen if a run occurs in some region i of the domestic country. The run will make the liquidation value of the deposit in region i fall below x, and cause an immediate devaluation in region D. Both effects will reinforce each other, because banks i and D hold deposits in each other. However, two other banks in the domestic economy will not be bankrupt as long as the buffer in each of them exceeds (or equals) the net withdrawals of interbank deposits.

Let q^i denote the liquidation value of the deposit in region i (provided that the banks in two other regions are not bankrupt) and E^5 denote the nominal exchange rate in region D after the devaluation. Then the following two equations determine their values:

$$q^i = \frac{b + rk + zx + 0.5zx/E^5}{1 + 3z/2} \tag{29}$$

and

$$E^5 = \frac{(\lambda + 3z/2)x}{b + zx + zq^i/2} \tag{30}$$

Then the main condition for the overall run will be:

$$\frac{z}{2}\left(2x - q^i - \frac{x}{E^5}\right) > g(\lambda) \tag{31}$$

The left-hand side of the last inequality is the sum of two terms, $z(x - x/E^5)/2$, and $z(x - q^i)/2$. The first term is the loss of value of deposit in region D (because of devaluation). The second term is the loss of value of deposit in region i because of the run. If the total loss exceeds the buffer, the global run becomes inevitable.

Lemma 8. Under the assumptions of complete market structure and flexible exchange rate regime a run in any region of the domestic economy ensures runs in all other regions of the domestic country, if and only if conditions (10) and (31) are satisfied.

The following proposition states a counterintuitive result: under the fear-of-floating regime, the peso is devalued by more than under the flexible regime.

Proposition 3 *If a financial crisis originates in a region of the domestic country under the complete system of interbank links, the peso-dollar exchange rate is higher if* $v = 1$ *than if* $v = 0$*, i.e.,* $E^4 > E^5$.

The intuition behind proposition 3 is as follows: when the foreign country has the floating exchange rate, the truly patient depositors have no incentive to join the run. Therefore the demand for dollars is smaller than under the fear-of-floating regime, and the demand effect dominates the supply effect.

The following proposition states the main result of the section:

Proposition 4. *Under complete market structure, conditions for a global run are more stringent under $v = 1$ than under $v = 0$.*

Proposition 4 states that the Chang and Velasco (2000) result is reversed when the structure of interbank links is complete, i.e. the global economy is more financially fragile, if the small economy (the foreign country) has a flexible exchange rate regime. It is important to note that this result is independent of the functional form of the utility function of the representative agent.

This result is striking, because it is obtained in a framework 'most favorable' to the floating exchange rate regime. The setup rules out a financial crisis originating in the economy with the floating exchange rate regime, and there are no other welfare-reducing effects of the exchange-rate instability present in the 'real world. ' Furthermore, the run-avoidance under the floating exchange rate regime yields a *lower* exchange-rate depreciation than under the fear-of-floating regime when an external shock hits the economy (Proposition 3). However, we show that even under such extreme conditions, a switch from the fear-of-floating regime to the floating regime increases the financial fragility when the system of interbank links is complete. The result is even more striking as it does not depend on the functional form of the utility function of the representative agent.

Why does a combination of flexible exchange rate regime in one country and complete system of interbank links contribute to the global financial fragility? The reason is that the country with flexible exchange rate regime immediately 're-exports' negative shocks (including the shock of a bank run in one of the regions of the domestic country) to the other region(s) of the domestic economy via the exchange rate depreciation rather than absorbs them. These regions are under double pressure, as each of them loses a part of the value of the deposit in the region affected by the bank run, and a part of the value of the deposit in the foreign country. However, these regions cannot follow the foreign country and devalue, and so they are more likely to suffer from the financial meltdown.

4.7 Complete versus Incomplete Structure: A Comparison

Finally we compare conditions for contagion under the flexible exchange rate regime. Is the incomplete structure more prone to contagion than the complete one, as is the case under the fear-of-floating exchange rate regime? It turns out the answer is generally ambiguous, and it depends on the starting point of the financial crisis. If the contagion begins in region C, then the result of Allen and Gale (2000) still holds and the incomplete market structure is more fragile.

Lemma 9. If the foreign country has a flexible exchange rate regime and the bank run originates in region C, then the conditions for the global contagion are more stringent under the complete market structure.

However, if the initial bank run happens in regions A, or B, then the comparison of contagion conditions depends on the functional form of the utility function and parameters of the model. In particular, assume that the utility function with constant relative risk aversion:

$$U(x) = \frac{x^{1-\theta} - 1}{1 - \theta}$$

and the utility-from-holding-money function is

$$\chi(m) = \sqrt{\bar{m}^2 - (m - \bar{m})^2}$$

For $m \leq \bar{m}$. Then there exists a set of parameter values for which the complete market structure is more fragile, i.e. the left-hand side of inequality (31) is greater than the left-hand side of inequality (22). An example of such a set is $R = 1.5, r = 0.8, \bar{m} = 0.2, \lambda = 0.5, \theta = 2, z = 0.1$.

5 Concluding Remarks

We analyze financial contagion in the context of a two-country multi-region model a la Allen and Gale (2000) with open-economy monetary features of Chang and Velasco (2000). We assume that one of the regions in the Allen-Gale framework is a separate country with its own currency and a central bank. In this framework, the major results of Allen and Gale and Chang and Velasco are obtained as special nested cases. Under incomplete structure of interbank links, a switch from the fear-of-floating to the flexible exchange rate regime reduces financial fragility, which is fully consistent with the Chang and Velasco findings. Under the fear-of-floating regime a switch from the incomplete system to complete system of interregional links, interpreted as globalization, also reduces financial fragility. This is essentially the original Allen and Gale (2000) results, as the fear-of-floating regime makes the difference between the currencies immaterial for the financial contagion.

However, the combination of complete system of interregional links and the flexible exchange rate regime yields two novel results. First, in contrast to Allen and Gale, we find that a switch to the complete structure of links among regions, may increase financial fragility, if the smaller country maintains a floating exchange rate regime. Also, in a globalized world a higher level of financial fragility results from a switch to the floating exchange rate regime in the smaller country.

These findings complement the existing explanations of the fear of floating phenomenon in many emerging market economies. Financial links of countries with different exchange rate regimes may be a source of financial fragility, and the importance of this source rises with globalization.

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Figure 1. Complete Market Structure

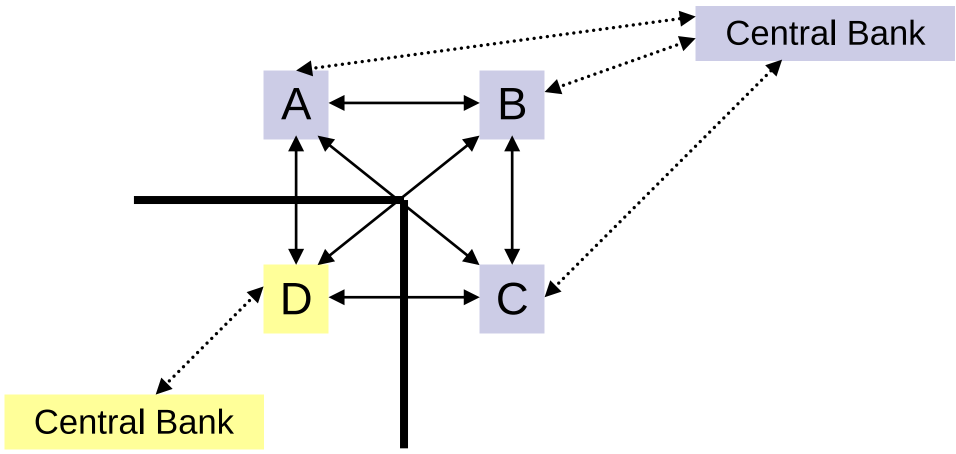
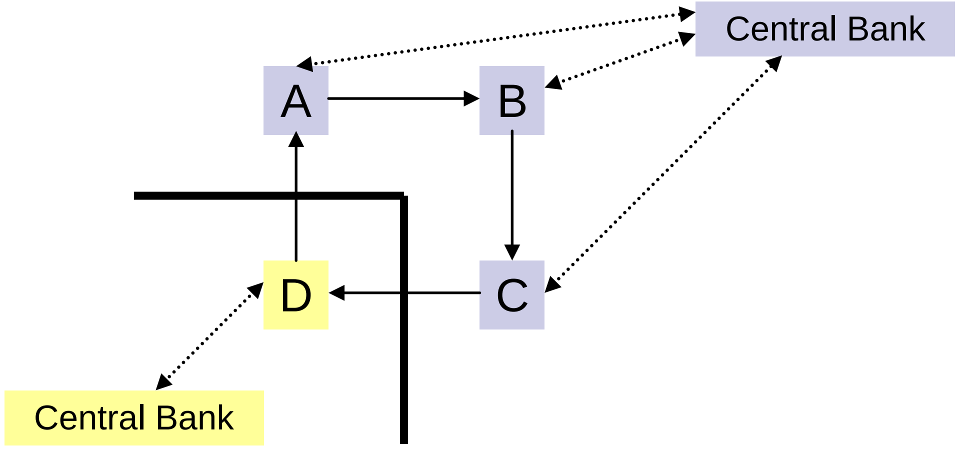


Figure 2. Complete Market Structure



Appendix

Solution to the Social Planner's problem (1)-(5)

Max $\lambda u(x)+(1-\lambda)u(\chi(m)+y)$ subject to

$\beta 1: k+b\leq 1$ (2)

$\beta 2: \lambda x\leq b+rl$ (3)

$\beta 3: (1-\lambda)y\leq R(k-l)$ (4)

$\beta 4: x\leq \chi(m)+y$ (5)

$x,y,m,k,l,b\geq 0$

$$\frac{\partial L}{\partial x}=\lambda u'(x)-\beta 2\lambda-\beta 4=0$$
$$\frac{\partial L}{\partial y}=(1-\lambda)u'(\chi(m)+y)-\beta 3(1-\lambda)+\beta 4=0$$
$$\frac{\partial L}{\partial M}=(1-\lambda)u'(\chi(m)+y)\chi'(m)+\beta 4\chi'(m)=0$$
$$\frac{\partial L}{\partial k}=-\beta 1+\beta 3R=0$$
$$\frac{\partial L}{\partial b}=-\beta 1+\beta 2=0$$
$$\frac{\partial L}{\partial l}=\beta 2r-\beta 3R<0$$

If we assume that incentive compatibility constraint does not bind, then $\beta 4=0$. From the first order conditions we have that $\beta 1=\beta 3R$, $\beta 1=\beta 2$. The first order condition for *l* is strictly negative given that $\beta 2=\beta 3R$ and *r<1*. Therefore $l=0$ - there is no liquidation of the long term investment in period 1. The first three conditions then transform to the following:

$$\lambda u'(x)-\beta 3R\lambda=0$$
$$(1-\lambda)u'(\chi(m)+y)-\beta 3(1-\lambda)=0$$
$$(1-\lambda)u'(\chi(m)+y)\chi'(m)=0$$

From the last equation we have that

$$\chi'(m)=0\Leftrightarrow m=\overline{m}$$

From the three binding restrictions above we can derive the following

$$\lambda x=b$$
$$\lambda x=1-k$$
$$k=\frac{(1-\lambda)y}{R}$$
$$R\lambda x+(1-\lambda)y=R$$

From the first order conditions we can obtain

$$\frac{\lambda u'(x)}{(1-\lambda)u'(\chi(\overline{m})+y)} = \frac{R\lambda}{1-\lambda} \text{ or } u'(x) = Ru'(\chi(\overline{m})+y)$$

Solution to the Problem of the Representative Commercial Bank

Max $\lambda u(x)+(1-\lambda)u(\chi(M)+y)$ subject to

$\beta 1: k+b \leq 1$ (6)

$\beta 2: \lambda x+(1-\lambda)M \leq b+h+rl$ (7)

$\beta 3: (1-\lambda)y-(1-\lambda)M \leq R(k-l)-h$ (8)

$\beta 4: \chi(M)+y \geq x$ (9)

$\beta 5: \lambda x \leq b$ (7)

$x, y, M, k, l, b, h \geq 0$

$$\frac{\partial L}{\partial x} = \lambda u'(x) - \beta 2\lambda - \beta 4 - \beta 5\lambda = 0$$
$$\frac{\partial L}{\partial y} = (1-\lambda)u'(\chi(M)+y) - \beta 3(1-\lambda) + \beta 4 = 0$$
$$\frac{\partial L}{\partial M} = (1-\lambda)u'(\chi(M)+y)\chi'(M) - \beta 2(1-\lambda) + \beta 3(1-\lambda) + \beta 4\chi'(M) = 0$$
$$\frac{\partial L}{\partial k} = -\beta 1 + \beta 3R = 0$$
$$\frac{\partial L}{\partial b} = -\beta 1 + \beta 2 + \beta 5 = 0$$
$$\frac{\partial L}{\partial h} = \beta 2 - \beta 3 = 0$$
$$\frac{\partial L}{\partial l} = \beta 2r - \beta 3R < 0$$

If we assume that incentive compatibility constraint does not bind, then $\beta 4 = 0$. We also assume that the commercial bank does not hold excess reserves or in other words, $\lambda x = b$. From the first order conditions we have that $\beta 1 = \beta 3R$, $\beta 2 = \beta 3$ and $\beta 5 = \beta 3(R-1)$. The first order condition for *l* is strictly negative given that $\beta 2 = \beta 3$ and *r*<*R*. Therefore $l = 0$ - there is no liquidation of the long term investment in period 1. The first three conditions then transform to the following:

$$\lambda u'(x) - \beta 3R\lambda = 0$$
$$(1-\lambda)u'(\chi(M)+y) - \beta 3(1-\lambda) = 0$$
$$((1-\lambda)u'(\chi(M)+y)\chi'(M) = 0$$

From the last equation we have that

$$\chi'(M) = 0 \Leftrightarrow M = \overline{m}$$

From our assumption that $\lambda x = b$, we have that

(1.5) $(1-\lambda)\overline{m} = h$

From the three binding restrictions above we can derive the following

$\lambda x = b$

$\lambda x = 1-k$

$k = \frac{(1-\lambda)y}{R}$

(1.6)$R\lambda x + (1-\lambda)y = R$

From the first order conditions we can obtain

$$\frac{\lambda u'(x)}{(1-\lambda)u'(\chi(\overline{m})+y)} = \frac{R\lambda}{1-\lambda} \text{ or } u'(x) = Ru'(\chi(\overline{m})+y)$$

Proof of Lemma 1

The bank in region C is bankrupt, therefore the liquidation value of its assets is

(A1.1) $q^C = \frac{b+rk+zx/E^1}{1+z}$

The bank in region B will be bankrupt if its loss is greater than its buffer. The following condition should hold:

(A1.2) $z(x-q^C) > g(\lambda)$

It is already given that the bank in region C is bankrupt and its buffer is lower than its loss:

(A1.3) $zx\left(1-\frac{1}{E^1}\right) > g(\lambda)$

We need to prove that $q^C < x/E^1$. Then the inequality (A1.2) will be satisfied. The proof is as follows. By using the definition of E^1 we have that

$$\frac{x}{E^1} = \frac{b+rk+zx}{1+z}$$

We know that E^1 is bigger than one. From this it follows that

$$q^C = \frac{b+rk+zx/E^1}{1+z} < \frac{x}{E^1} = \frac{b+rk+zx}{1+z}.$$

We have proved that the bank in region B is bankrupt. The liquidation value of its assets is

(A1.4) $q^B = \frac{b+rk+zq^C}{1+z}$

The bank in region A will be bankrupt if its loss is greater than its buffer. The following condition should hold:

(A1.5) $z(x - q^B) > g(\lambda)$

Again, we need to prove that $q^B < q^C$. We have already proved that $q^C < x/E^1$. Thus,

$$q^B = \frac{b + rk + zq^C}{1+z} < q^C = \frac{b + rk + zx/E^1}{1+z}$$

Proof of Lemma 2

The proof is completely analogous to the one of Lemma 1and is omitted for brevity.

Proof of Lemma 3

First, we show that $E^3 < E^1$.

It is easy to prove that

$$E^1 > E^3 \Leftrightarrow \frac{(1+z)x}{b+rk+zx} > \frac{(\lambda+z)x}{b+zq^A} \text{ given that } q^A = \frac{b+rk+zx}{1+z}.$$

$$\frac{(1+z)x}{b+rk+zx} > \frac{(\lambda+z)(1+z)x}{b+bz+z(b+rk+zx)}$$

$$\frac{1}{b+rk+zx} > \frac{(\lambda+z)}{b+bz+z(b+rk+zx)}$$

$$b + bz + zb + zrk + z^2 x > \lambda b + \lambda rk + \lambda zx + zb + zrk + z^2 x$$

$$b + bz > \lambda b + \lambda rk + \lambda zx$$

From the solution of the Social Planner's problem we have that $\lambda x = b$, therefore

$$\lambda x > \lambda b + \lambda rk$$

So the expression $E^1 > E^3$ simplifies to the following

$$x > b + rk$$

which is true as it coincides with (10).

The bank in region B will be bankrupt if its minimum possible losses exceed the buffer

$z(x - q^C) > g(\lambda)$ where $q^C = \dfrac{b + rk + zx/E^3}{1+z}$.

It is already given that the bank in region C is bankrupt and its buffer is lower than its loss:

$$zx\left(1 - \frac{1}{E^3}\right) > g(\lambda) \text{ where } E^3 = \frac{(\lambda+z)x}{b+zq^A}$$

We need to prove that $q^C < x/E^3$. The proof is as follows

$$q^C = \frac{b + rk + zx/E^3}{1+z} \vee \frac{x}{E^3}$$

$$(b+rk)E^3 + zx \vee x + zx$$

$$E^3 \vee \frac{x}{b+rk}$$

We know already that $E^3 < E^1$. It can be proved that $E^1 < \frac{x}{b+rk}$

$$\frac{(1+z)x}{b+rk+zx} < \frac{x}{b+rk}$$

$$b + rk + z(b+rk) < b + rk + zx$$

$$b + rk < x$$

Thus, we have obtained that $E^3 < E^1 < \frac{x}{b+rk}$

Proof of Lemma 4

Bank in region B is bankrupt if the condition (10) is satisfied.

Bank in region A will be bankrupt if $z(x - q^B) > g(\lambda)$ where $q^B = \frac{b+rk+zx}{1+z}$

If bank in region A is bankrupt then the Central bank of the region D is forced to devalue the peso and new exchange rate will be $\frac{(\lambda+z)x}{b+z\bar{q}^A}$ where $\bar{q}^A = \frac{b+rk+zq^B}{1+z}$

Bank in region C will be bankrupt if

(A4.1) $zx\left(1 - \frac{b+z\bar{q}^A}{x(\lambda+z)}\right) > g(\lambda)$

It can be proved that $z(x - q^B) > zx\left(1 - \frac{b+z\bar{q}^A}{x(\lambda+z)}\right)$. Therefore for global run conditions (10) and (A4.1) have to be satisfied.

Proof of $z(x - q^B) > zx\left(1 - \frac{b+z\bar{q}^A}{x(\lambda+z)}\right)$

(A4.2) $q^B < \frac{b+z\bar{q}^A}{(\lambda+z)}$

$\lambda q^B + zq^B < b + z\bar{q}^A$ where $\lambda x = b$

$$\lambda q^B + zq^B < \lambda x + z\bar{q}^A$$

$$z(q^B - \bar{q}^A) < \lambda(x - q^B)$$

$$\frac{z}{1+z}(q^B + zq^B - b - rk - zq^B) < \frac{\lambda}{1+z}(x + xz - b - rk - zx)$$

$$z(q^B - b - rk) < \lambda(x - b - rk)$$

$$zq^B < \lambda(x - b - rk) + z(b+rk)$$

$$\frac{z}{1+z}(b + rk + zx) < \lambda(x - b - rk) + z(b+rk)$$

$$zb + rkz + z^2x < \lambda x - \lambda b - \lambda rk + zb + zrk + \lambda xz - \lambda bz - \lambda rkz + zbz + zrkz$$

$$z^2(x-b-rk) < \lambda x - \lambda b - \lambda rk + \lambda xz - \lambda bz - \lambda rkz$$
$$(z^2 - \lambda - \lambda z)(x-b-rk) < 0 \text{ where } x > b + rk$$
$$z^2 - \lambda - \lambda z < 0 \text{ where } \lambda = w_H - z$$
$$z^2 - w_H + z - w_H z + z^2 < 0$$
$$2z^2 - w_H + z - w_H z < 0 \text{ where } z = \frac{w_H - w_L}{2} \text{ and } w_H > w_L$$
$$\frac{w_H{}^2 - 2w_H w_L + w_L{}^2 - 2w_H + w_H - w_L - w_H{}^2 + w_H w_L}{2} < 0$$
$$-w_H w_L + w_L{}^2 - w_H - w_L < 0$$
$$w_L(w_L - w_H) - (w_H + w_L) < 0 \text{ - holds because } w_H > w_L$$

Proofs of Lemmas 5-8 are straightforward and are omitted for brevity.

Proof of Proposition 1

If a run starts in region A then it is easy to prove that contagion is more likely for v=1. To prove this we have to show that LHS(22)<LHS(18).

$$zx\left(1 - {}^{1}\!/_{E^3}\right) \vee z(x - q^A) \text{ where } q^A = \frac{b + rk + zx}{1+z} \text{ and } E^3 = \frac{(\lambda + z)x}{b + zq^A}$$

$$zx\left(1 - {}^{1}\!/_{E^3}\right) \vee z(x - q^A) \Leftrightarrow q^A \vee {}^{x}\!/_{E^3} \Leftrightarrow E^3 \vee E^1$$

We have already proved that $E^1 > E^3$

If a run starts in region B then it can be proved that LHS (A4.1) is smaller than LHS(18), in other words the global run is more likely for v=1.

$$zx\left(1 - \frac{b + z\bar{q}^A}{x(\lambda + z)}\right) < z(x - q^A) \text{ where } q^A = q^B = \frac{b + rk + zx}{1+z} \text{ and } \bar{q}^A = \frac{b + rk + zq^B}{1+z}$$

$$\frac{b + z\bar{q}^A}{(\lambda + z)} > q^A \Leftrightarrow \text{(L4.2) } q^B < \frac{b + z\bar{q}^A}{(\lambda + z)}$$

If a run starts in region C then conditions for a global run are the same under v=0 and v=1, namely (10) and (18).

Proof of Proposition 2

To prove this proposition we need to show that LHS(18) is larger than LHS(27)

$$z(x - q^A) \vee \frac{z}{2}(x - q_1^i) \text{ where } q^A = \frac{b + rk + zx}{1+z} \text{ and } q_1^i = \frac{b + rk + 3zx/2}{1 + 3z/2}$$

$$\frac{x}{2} + q_1^i \vee q^A$$

It can be proved that $q_1^i > q^A$

$$\frac{b + rk + 3zx/2}{1 + 3z/2} > \frac{b + rk + zx}{1+z}$$

$$b + rk + 3zx/2 + zb + zrk + 3z^2x/2 > b + rk + zx + 3zb/2 + 3zrk/2 + 3z^2x/2$$

$3zx/2 + zb + zrk > zx + 3zb/2 + 3zrk/2$

$zx/2 > zb/2 + zrk/2 \Leftrightarrow x > b + rk$

Therefore $\dfrac{x}{2} + q_1^i > q^A$

Proof of Proposition 3

$$E^4 = \frac{(1+3z/2)x}{b+rk+3zx/2}$$

$$E^5 = \frac{x(\lambda + 3z/2 + 3z\lambda/2 + 2z^2)}{(b+zx)(1+2z)+rk(z/2)}$$

$$\frac{(1+3z/2)}{b+rk+3zx/2} \vee \frac{(\lambda + 3z/2 + 3z\lambda/2 + 2z^2)}{(b+zx)(1+2z)+rk(z/2)}$$

$$b + zx + 2zb + 2z^2x + 0{,}5zrk + 1{,}5zb + 1{,}5z^2x + 3z^2b + 3z^3x + 0{,}75z^2rk \vee$$

$$\vee\ \lambda b + \lambda rk + 1{,}5z\lambda x + 1{,}5zb + 1{,}5zrk + 2{,}25z^2x + 1{,}5z\lambda b + 1{,}5z\lambda rk + 2{,}25z^2\lambda x + 2z^2b + 2z^2rk + 3z^3x$$

$$b + zx + 2zb + 2z^2x + 1{,}5z^2x + z^2b \vee$$

$$\vee\ \lambda b + \lambda rk + 1{,}5z\lambda x + zrk + 2{,}25z^2x + 1{,}5z\lambda b + 1{,}5z\lambda rk + 2{,}25z^2\lambda x + 1{,}25z^2rk$$

$$b + zx + 2zb + 2z^2x + 1{,}5z^2x + z^2b \vee$$

$$\vee\ \lambda b + \lambda rk + 1{,}5zb + zrk + 2{,}25z^2x + 1{,}5z\lambda b + 1{,}5z\lambda rk + 2{,}25z^2b + 1{,}25z^2rk$$

$$b + zx + 0{,}5zb + 1{,}25z^2x \vee$$

$$\vee\ \lambda b + \lambda rk + zrk + 1{,}5z\lambda b + 1{,}5z\lambda rk + 1{,}25z^2b + 1{,}25z^2rk$$

$$\lambda x + zx + 0{,}5zb + 1{,}25z^2x \vee$$

$$\vee\ \lambda(b+rk) + zrk + 1{,}5z\lambda b + 1{,}5z\lambda rk + 1{,}25z^2(b+rk)$$

$$(\lambda + 1{,}25z^2 + 0{,}5z\lambda)(x-b-rk) + zx \vee zrk + z\lambda b + z\lambda rk$$

$$(\lambda + 1{,}25z^2 + 0{,}5z\lambda)(x-b-rk) + zx + z\lambda x \vee zrk + z\lambda b + z\lambda rk + z\lambda x$$

$$(\lambda + 1{,}25z^2 + 0{,}5z\lambda)(x-b-rk) + (z + z\lambda)(x-b-rk) > 0 \Leftrightarrow \text{E4>E5}$$

Proof of Proposition 4

To prove that under complete market structure a global run is more likely under v=0 than under v=1 we need to show that LHS(31) is higher than LHS(27).
After solving the system (29)-(30) we get the following values for q_0^i and E^5.

$$q_0^i = \frac{(b+zx)(\lambda+2z)+rk(\lambda+3z/2)}{\lambda + 3z/2 + 3z\lambda/2 + 2z^2}$$

$$E^5 = \frac{x(\lambda + 3z/2 + 3z\lambda/2 + 2z^2)}{(b+zx)(1+2z)+rk(z/2)}$$

We have to prove that

$$x-q^{A}+x-\frac{x}{E^{5}}>x-\frac{b+rk+3zx/2}{1+3z/2}$$

Step 1

$$x>\frac{x}{E^{5}}$$

$$\lambda x+3zx/2+3zx\lambda/2+2z^{2}x>b+2zb+zx+2z^{2}x+zrk/2$$

$$3zx/2+3zx\lambda/2+>2zb+zx+zrk/2$$

$$zx/2>zx\lambda/2+zrk/2\Leftrightarrow x>b+rk$$

Step 2

$$\frac{b+rk+3zx/2}{1+3z/2}>q^{A}$$

$$\frac{b+rk+3zx/2}{1+3z/2}>\frac{(b+zx)(\lambda+2z)+rk(\lambda+3z/2)}{\lambda+3z/2+3z\lambda/2+2z^{2}}$$

$$\lambda b+3zb/2+3z\lambda b/2+2z^{2}b+rk\lambda+3zrk/2+3z\lambda rk/2+2z^{2}rk+3zx\lambda/2+9z^{2}x/4+9z^{2}x\lambda/4+3z^{3}x>$$

$$>b\lambda+2zb+zx\lambda+2z^{2}x+rk\lambda+3rkz/2+3zb\lambda/2+3z^{2}b+3z^{2}x\lambda/2+3z^{3}x+3z\lambda rk/2+9z^{2}rk/4$$

$$3zb/2+2z^{2}b+2z^{2}rk+3zx\lambda/2+9z^{2}x/4+9z^{2}x\lambda/4>$$

$$>2zb+zx\lambda+2z^{2}x+3z^{2}b+3z^{2}x\lambda/2+9z^{2}rk/4$$

$$zx\lambda/2+z^{2}x/4+3z^{2}x\lambda/4>zb/2+z^{2}b+z^{2}rk/4$$

$$z^{2}x/4+3z^{2}x\lambda/4>z^{2}b+z^{2}rk/4$$

$$x/4+3b/4>b+rk/4$$

$$x>b+rk$$

Proof of Lemma 9

To prove this lemma we need to show that LHS(18) is larger than LHS(27). This is already done in the proof of Proposition 2.