Modeling Financial Crises Mutation

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Abstract

The recent financial events in Latin America and Europe have shown that a turmoil can lead to a concatenation of several aspects from currency, banking and sovereign debt crises. This paper proposes a multivariate model that encompasses the three types of crises (currency, banking and sovereign debt), hence allowing to investigate the potential causality between not only currency and banking crises but also sovereign debt ones. Besides, a methodological novelty is proposed consisting of an exact maximum likelihood method to estimate this multivariate dynamic probit model, thus extending Huguenin, Pelgrin and Holly (2009)'s method to dynamic models. Applied to a large sample of data for emerging countries, we show that in the bivariate case mutations from banking to currency (and vice-versa) are quite common. More importantly, the trivariate model turns out to be more parsimonious in the case of the two countries which suffered from the 3 types of crises. These findings are strongly confirmed by a conditional probability and an impulse-response function analysis, highlighting the interaction between the different types of crises and advocating hence the implementation of trivariate models whenever it is feasible.

J.E.L. Codes: *C33, F37*

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1 Introduction

Since the tulipmania,[1] economic literature has recorded numerous turmoils affecting the foreign exchange market (currency crisis), the banking market (banking crisis) and the government foreign debt (sovereign debt market). Nevertheless, recent episodes have proved that most of the time crises do not remain restricted to a single market, but tend to spill-over into another one. Analyzing the crisis events for a period of a hundred years in a sample of 56 countries, Bordo et al. (2001) shown that the $ex-post$ probability of twin crises (banking and currency crises) has strongly increased since WWII. Similarly, using data back to the XIX century, Kaminsky and Reinhart (2008) present evidence of a strong connection between debt cycles and economic crises in an analysis of both cross-country aggregates and individual country histories.

Nevertheless, some historical events showed that bilateral feed-back between crises were not always sufficient to have an exhaustive picture of a turmoil. For example, the Ecuadorian crisis in 1999 affected first the banking sector, impacting after simultaneously the Sucre[2], and the public finance. More recently, the European crisis emerged as a banking distress succeeding the collapse of the U.S. real estate bubbles. It took a sovereign debt dimension when some European countries (as Greece, Ireland, or Portugal), penalized by the recessive consequences of the banking credit crunch or by the public safety plans set up to stabilize the financial system, came close to default. A third dimension is now reached with the increase in volatility between the Euro and the Dollar as well as the rumors over a split of the Euro area. Theoretically, the potential spill-over from one crisis to another one can be analyzed using a balance sheet approach. Using such an accounting framework, Rosenberg et al. (2005) and, more recently, Candelon and Palm (2010) show how balance sheets are linked across sectors. Consecutively, the transmission of a shock to one country's economy (as the burst of the real estate market bubble in 2007) to that of another country will become visible in their balance sheet. The financial crisis takes then another shape.

It appears thus evident that an accurate financial crisis model has to take this feature into account. In a seminal paper, Glick and Hutchinson (1999) propose to model twin crises and to assess the extent to which each type of crisis provides information about the likelihood of the other one. Their approach relies in a first step on individual models for currency and banking crises. In a second step, the global model is estimated by using the instrumental variables method so as to tackle the potential endogeneity bias. Implemented on a pooled sample of 90 industrial and developing countries over the $1975-1997$ period, they find that the twin crisis phenomenon was most common in financially liberalized emerging markets

1. Kindelberger (2000) lists this event as the first financial crisis listed in the history. It has affected the Dutch tulip market in 1636.

2. The Ecuadorian currency has been replaced by the U.S. dollar on March, 13, 2000.

during the Asian crisis. Nevertheless, from a methodological point of view, the use of a two-step approach is not free of criticism with respect to the remaining endogeneity. Moreover, the use of a panel framework, driven by the shortness of the time dimension, will require some degree of homogeneity among countries. Finally, Glick and Hutchinson (1999) do not consider sovereign debt crises, focusing exclusively on twin crises.

This paper proposes to extend their study in several ways: First, it considers a multivariate model that encompasses the three types of crises (currency, banking and sovereign debt), thus allowing to investigate not only the potential mutation between currency and banking crises but also sovereign debt ones. Second, this paper introduces a methodological novelty by proposing an exact maximum likelihood approach to estimate this multivariate dynamic probit model. As shown by Huguenin, Pelgrin and Holly (2009) for a static model, a multivariate probit model cannot be precisely estimated using simulation methods. Its estimation requires hence to derive an exact maximum-likelihood function. We thus extend the univariate dynamic probit model developed by Kauppi and Saikkonen (2008) to a multivariate level and we derive its exact likelihood, allowing to obtain a converging and efficient estimate. Third, applied to a large sample of emerging countries, we show that in the bivariate case mutations of a banking crisis into a currency crisis (and vice-versa) are quite common, confirming hence Glick and Hutchinson (1999)'s results. More importantly, for the two countries which suffered from the 3 types of crises, the trivariate model turns out to be more parsimonious, thus supporting its implementation anytime when it is feasible.

The rest of the paper is organized as follows. Section 2 presents a multivariate dynamic probit model. In sections 3 we describe the Exact Maximum Likelihood method to estimate it as well as some numerical procedures. In section 4, the multivariate dynamic probit model is estimated for 18 emerging countries in its bivariate (twin crises) or multivariate form.

2 A Multivariate Dynamic Probit Model

Consider M latent continuous variables $y^*_{m,i,t}$ representing the pressure on the market m in country i, $m \in 1, 2, ..., M$, $i \in 1, ..., I$ at time period $t \in 1, ...T$. The observed variable $y_{m,i,t}$ takes the value 1 if a crisis occurs on market m, in country i at period T and the value 0 otherwise. For simplicity, the country index is removed in the sequel of the paper. Denote by $y^*_{m,t}$ and $y_{m,t}$ the $M \times 1$ vectors with elements y^*_t and y_t respectively. The general specification for the M-equation model would be

$$y^*_{m,t} = \pi_{m,t} + \epsilon_{m,t}, \tag{1}$$

and

$$y_{m,t} = \mathbb{1}(y^*_{m,t} > 0), \tag{2}$$

with $\pi_{m,t}$ being the expected value of $y_{m,t}$ that may depend on covariates which vary across markets, country and time and with $E(\epsilon_{m,t}|\pi_{m,t}) = 0$, $Var(\epsilon_{m,t}|\pi_{m,t}) = \Gamma$, $Cov(\epsilon_{m,t}, \epsilon_{m',t}|\pi'_{m,t}, \pi_{m',t'}) = \omega_{mm'}$ when $i = i'$, $t = t'$ and zero whenever $i \neq i'$ and $t \neq t'$. The multivariate probit model arises, when the $M \times 1$ vector $\epsilon_{i,t}$ with elements $\epsilon_{m,i,t}$ is assumed to have a multivariate normal distribution, with mean zero and covariance matrix $\Omega = [\omega_{mm'}]$. The multivariate version of the dynamic probit model proposed by Kauppi and Saikkonen (2008) is obtained when:

$$\pi_t = \alpha + x'_{t-1}\beta + y'_{t-1}\Delta + \Gamma'\pi_{t-1}, \tag{3}$$

where $\pi_t = (\pi_{1,t}, \pi_{2,t}, ..., \pi_{M,t})'$ and $\alpha = (\alpha_1, \alpha_2, ..., \alpha_M)'$. To simplify, we assume that the exogenous variables are specific to the type of crisis and thus $x'_{t-1} = diag(x'_{1,t-1}, x'_{2,t-1}, ..., x'_{M,t-1})$, where $x_{m,t-1}$ is a $(k_m \times 1)$ vector of explanatory variables corresponding to the m^{th} dependent variable at time $t-1$. Additionally, $\beta = (\beta_1, \beta_2, ..., \beta_M)'$, where β_m is a $(k_m \times 1)$ vector of parameters corresponding to the variables $x_{m,t-1}$. Besides, $y_{t-1} = (y_{1,t-1}, y_{2,t-1}, ..., y_{M,t-1})'$, and $\Delta = (\delta_1, \delta_2, ..., \delta_M)'$, where $\delta_m = (\delta_{m,1}, \delta_{m,2}, ..., \delta_{m,M})'$. Similarly, $\Gamma = (\gamma_1, \gamma_2, ..., \gamma_M)$, where $\gamma_m = (\gamma_{m,1}, \gamma_{m,2}, ..., \gamma_{m,M})'$.

Considering our initial goal which consists in jointly modeling the three types of crises (currency, banking and sovereign debt), a trivariate model for a particular country can be rewritten without loss of generality as (1), where the endogenous variable, *i.e.* $y_{m,t}$, corresponds to the type of crisis ($m \in [c; b; s]$ indicating the occurrence of a currency, banking or sovereign debt crisis), $x_{m,t}$ is a set of exogenous variables, and $M = 3$.

Remark 1. *This model can be equivalently expressed within a latent variable representation as follows*:

$$\begin{aligned} y^*_{m,t} &= \alpha_m + x'_{m,t-1}\beta_m + \sum_{m'} y_{m',t-1}\Delta_{m,m'} + \sum_{m'} \Gamma_{m,m'}\pi_{m',t-1} + \varepsilon_{m,t} \\ y_{m,t} &= \mathbb{1}(y^*_{m,t} > 0), \end{aligned} \tag{4}$$

where $m \in \{c, b, s\}$ *and* $m' \in \{c, b, s\}$ *in our trivariate case.* $y_{m,t}$ *equals 1 if* $y^*_{m,t} > 0$ *and* 0 *otherwise, while* $x_{m,t}$ *is a set of variables* $(1 \times k_m)$ *explaining the occurrence of a specific crisis.*

For the sake of simplicity we consider the case where each exogenous variable is specific to the type of crisis, so that X_t is a $3 \times K$ matrix, where $K = \sum_{m=1}^{3} k_m$. Besides, $\theta_m = [\alpha_m \ \beta'_m \ \tilde{\delta}_m \ \gamma_m]'$ is the vector of parameters for equation m and $\theta = [\theta'_c \ \theta'_b \ \theta'_s]'$. Finally, the disturbances $\varepsilon_t = [\varepsilon_{c,t} \ \varepsilon_{b.t} \ \varepsilon_{s,t}]'$ are trivariate normally distributed with a 3×3 symmetric matrix $\tilde{\Omega}$:

$$\tilde{\Omega} = \begin{pmatrix} \sigma_c^2 & \rho_{bc}\sigma_b\sigma_c & \rho_{sc}\sigma_c\sigma_s \\ \rho_{bc}\sigma_b\sigma_c & \sigma_b^2 & \rho_{sb}\sigma_b\sigma_s \\ \rho_{sc}\sigma_c\sigma_s & \rho_{sb}\sigma_b\sigma_s & \sigma_s^2 \end{pmatrix}, \tag{5}$$

where $\rho_{m,m-1}$ represents the correlation coefficients. It is also assumed that $\tilde{\varepsilon}_t$ is $i.i.d$ so that the covariance matrix for all T observations is given by $V(\tilde{\varepsilon}) = I_N \otimes \tilde{\Omega}$, $\tilde{\Omega}$ being a flexible covariance matrix.

Remark 2. *The matrices Δ and Γ provide useful information about the mutation of the crises*:

1. *The diagonal terms of Γ specify the persistence of each crisis. The closer they are to 1, the more persistent the crisis episode will be. It is noticeable that the diagonal elements of this matrix are constrained to be strictly inferior to 1. We exclude the case where the latent variable $y^*_{m,t}$ follows a random walk, which would be empirically counter-intuitive as financial crises are non-persistent.*
2. *The diagonal terms of Δ also deliver information about persistence of the crisis but somewhat different from this infer from Γ. Indeed, they indicate to what extent the probability of occurrence of a crisis depends on the regime prevailing the period before. It is important to notice that contrary to Γ the persistence is in all cases limited to one period.*
3. *The Granger-causal effects between the three crises are given by the off diagonal terms of Δ. In other words if $\Delta_{c,b} > 0$ it means that a banking crisis at time $t-1$ increases the probability of a currency turmoil at t.*

Remark 3. *As in the univariate case, several specifications can be obtained from the general multivariate model (3) and (4), by imposing particular restrictions on the parameters.*

1. *The first special case is a static trivariate model, as the one proposed by Huguenin, Pelgrin and Holly, (2009). Its corresponding index is given by*:[3]

$$\pi_t = \alpha + x'_{t-1}\beta. \tag{6}$$

2. *The second specification is a dynamic model including the lagged binary variable (y_{t-1}), which adds the lagged dependent variable to the previous model and thus takes the following form*:

$$\pi_t = \alpha + x'_{t-1}\beta + y'_{t-1}\Delta. \tag{7}$$

3. Note that the model can also be written in a latent variable representation similar to that of (4), as it is done in Huguenin, Pelgrin and Holly (2009).

It results that the persistence of the crisis regime is limited to one period.

3. *Third, the lagged index can be included into the model instead of the lagged binary variable. In such a case, the autoregressive trivariate dynamic probit model can be expressed as follows*:

$$\pi_t = \alpha + x'_{t-1}\beta + \Gamma'\pi_{t-1}, \tag{8}$$

In such a case the crisis regime may be quite persistent, but not infinite, as for any $m \neq m'$ $\gamma_{m,m'} < 1$.

4. *Finally, the most general model considered here, including both the lagged binary variable and the lagged index is given by the general model in (4).*

To simplify notation, in the next section $z_{m,t-1}$ will denote the vector of exogenous variables for equation m, corresponding to each of the particular cases resulting from (4) and (6) to (8) while $z_{t-1} = [z_{1,t-1}, z_{2,t-1}, ..., z_{m,t-1}]'$ is the vector of all the explanatory variables at time $t-1$. For instance, if we consider the static model, $z_{m,t-1} = [1 \; x_{m,t-1}]'$, and $z_{t-1} = [1 \; x_{1,t-1} 1 \; x_{2,t-1} \; 1 \; x_{3,t-1}]'$. θ is the vector of parameters ($\theta = [\alpha_m \; \beta_m]'$ in the example considered above), and Ω is the covariance matrix, so that the index at time t for equation m takes the general form of $\pi_{m,t} = z'_{m,t-1}\theta_m$ and the index at time t is given by $\pi_t = z'_{t-1}\theta$.

3 Exact Maximum Likelihood Estimation

The exact maximum likelihood estimator for the multivariate dynamic probit model cannot be obtained as a simple extension from the univariate model. For this reason, the simulated maximum likelihood method is generally considered. Nevertheless, Holly, Huguenin and Pelgrin (2009) prove that it leads to a bias in the estimation of the correlation coefficients as well as in their standard deviations. Therefore, they advocate exact maximum likelihood estimation. Since the correlations between the crisis binary variables, i.e. the contemporaneous transmission channels from one crisis to another one, constitute our main focus, asymptotic unbiased estimation of the correlations is of importance here and it calls for an explicit form of the likelihood. This section deals with this objective.

3.1 The Maximum Likelihood

Let us first notice that as in the univariate case, the slope and covariance parameters are not jointly identified. Similar to the univariate case, a first option would consist in standardizing the covariance matrix to an identity matrix. In such a case the new vector of disturbance $\varepsilon_t = \tilde{\Omega}^{-1/2}\tilde{\varepsilon}_t$ and it follows a standard normal multivariate distribution. Since it does not allow to estimate the correlation coefficients which constitute our objective, such an approach does not appear as appropriate. Alternatively, we opt for standardizing the residuals by their standard deviation. [4]

Following Greene (2002), the full information maximum-likelihood (FIML) estimates are obtained by maximizing the log-likelihood $\text{LogL}(Y|Z;\theta,\Omega)$, where θ is the vector of identified parameters and Ω is the covariance matrix. Under the usual regularity conditions [5] (Lesaffre and Kauffmann, 1992), the likelihood is given by the joint density of observed outcomes:

$$\text{L}(y|z,\theta;\Omega) = \prod_{t=1}^{T} L_t(y_t|z_{t-1},\theta;\Omega), \tag{9}$$

where $y_t = (y_{1,t}, y_{2,t}, y_{3,t})'$ and $y = [y_1, ..., y_T]$. The individual likelihood $L_t(.)$ is given in Lemma 1 as it is a well known result in the literature.

Lemma 1. *The likelihood of observation t is the cumulative density function, evaluated at the vector w_t of a 3-variate standardized normal vector with a covariance matrix $Q_t\Omega Q_t$:*

$$L_t(y_t|z_{t-1},\theta;\Omega) = Pr(y_1 = y_{1,t}, y_2 = y_{2,t}, y_{3,t} = y_{3,t}) = \Phi_{3,\varepsilon_t}(w_t; Q_t\Omega Q_t), \tag{10}$$

where Q_t is a diagonal matrix whose main diagonal elements are $q_{m,t} = 2y_{m,t} - 1$ and thus depends on the realization or not of the events ($q_{m,t} = 1$ if $y_{m,t} = 1$ and $q_{m,t} = -1$ if $y_{m,t} = 0$, $\forall\ m \in \{c, b, s\}$). Besides, the elements of the vector $w_t = [w_{1,t}, ..., w_{3,t}]$ are given by $w_{m,t} = q_{m,t}\pi_{m,t}$ (for a complete demonstration of Lemma 1, see Appendix 1).

Thus, the FIML estimates are obtained by maximizing the log-likelihood:

$$\text{LogL}(y|z,\theta;\Omega) = \sum_{t}^{T} \text{Log}\Phi_{3,\varepsilon}(w_t; Q_t\Omega Q_t) \tag{11}$$

with respect to θ and Ω [6].

4. We obtain the correlation matrix $\tilde{\Omega} = C\tilde{\Omega}C'$, where $C = diag(\sigma_{11}^{-1}, \sigma_{22}^{-1}, \sigma_{33}^{-1})$, corresponding to the identifiable parameters $\theta = (\tilde{\theta}_c', \tilde{\theta}_b', \tilde{\theta}_s')'$ where $\tilde{\theta}_m = \sigma_{m,m}^{-1}\theta_m$. For the sake of simplicity, $\tilde{\Omega}$ will be labeled as the covariance matrix hereafter. Moreover the index at time t is denoted by $\pi_t = z_{t-1}'\tilde{\theta}$.

5. If the parameters θ are estimated while the correlation coefficients are assumed constant, the log-likelihood function is concave. In this case the MLE exists and it is unique. Nevertheless, when θ and ρ are jointly estimated (as in our model), the likelihood function is not (strictly) log-concave as a function of ρ. Thus, the MLE exists only if the log-likelihood is not identically $-\infty$ and $E(z^T z|\rho)$ is upper semi-continuous finite and not identically 0. Furthermore, if no $\theta \neq 0$ fulfills the first order conditions for a maximum, the MLE of (θ, ρ) for the multivariate probit model exists and for each covariance matrix not on the boundary of the definition interval, the MLE is unique.

6. Besides, we tackle the autocorrelation problem induced by the binary crisis variable by considering a Gallant correction for the covariance matrix.

3.2 The Empirical Procedure

The main problem with FIML is that it requires the evaluation of high-order multivariate normal integrals while existing results are not sufficient to allow accurate and efficient evaluation for more than two variables (see Greene, 2002, page 714). Indeed, Greene (2002) argues that the existing quadrature methods to approximate trivariate or higher-order integrals are far from being exact. To tackle this problem in the case of a static probit, Huguenin, Pelgrin and Holly (2009) decompose the triple integral into simple and double integrals, leading to an Exact Maximum Likelihood Estimation (EML) that requires computing double integrals. Most importantly, they prove that the EML increases the numerical accuracy of both the slope and covariance parameters, which outperform the maximum simulated likelihood method (McFadden,1989) which is generally used for the estimation of multivariate probit models. Therefore, we extend the decomposition proposed by Huguenin, Pelgrin and Holly, (2009) in the case of our multivariate dynamic model so as to obtain a direct approximation of the trivariate normal cumulative distribution function.

The EML log-likelihood function is given by:

$$\mathrm{LogL}(y|z,\theta;\Omega) = \sum_{t=1}^{T} \mathrm{Log}\left[\prod_{m=1}^{3} \Phi(w_{m,t}) + G\right], \tag{12}$$

where $\Phi(w_t)$ is the univariate normal cumulative distribution function of w_t. Indeed, the log-likelihood function depends on the product of the marginal distributions (w_t) and the correction term G which captures the dependence between the m events analyzed.

The maximum likelihood estimators $\{\widehat{\theta};\widehat{\Omega}\}_{EML}$ are the values of θ and Ω which maximize:

$$\{\widehat{\theta};\widehat{\Omega}\}_{EML} = \mathrm{Arg}\max_{\theta;\Omega} \sum_{m=1}^{3} \mathrm{LogL}(.), \tag{13}$$

with $L(.)$ given in (11).

Under the regularity conditions of Lesaffre and Kaufman (1992), the EML estimator of a multivariate probit model exists and is unique. Besides, the estimates $\{\widehat{\theta};\widehat{\Omega}\}_{EML}$ are consistent and efficient estimators of the slope and variance-covariance parameters and are asymptotically normally distributed. It is worth noting that in a correctly specified model for which the error terms are independent across the m equations the EML function corresponds to $\sum_{t=1}^{T}\prod_{m=1}^{3}\Phi(w_{m,t})$, since the probability correction term G in eq. (12) tends toward zero.

We present here only the results for a bivariate and a trivariate model:

$$\Phi_2(w_t;Q\Omega Q) = \Phi(w_{1,t})\Phi(w_{2,t})\frac{1}{2\pi}\int_0^{\rho_{12}} \exp\left(-\frac{1}{2}\frac{w_{1,t}^2 + w_{2,t}^2 - 2w_{1,t}w_{2,t}}{1-\lambda_{12}^2}\right)\frac{d\lambda_{12}}{\sqrt{1-\lambda_{12}^2}} \tag{14}$$

for a bivariate model and

$$
\begin{aligned}
\Phi_3(w_t;Q\Omega Q) &= \prod_{m=1}^{3}\Phi(w_{m,t}) + G \\
&= \Phi(w_{1,t})\Phi(w_{2,t})\Phi(w_{3,t}) \\
&+ \Phi(w_{3,t})\int\limits_{0}^{\rho_{12}}\phi_2(w_{1,t},w_{2,t};\lambda_{12})d\lambda_{12} \\
&+ \Phi(w_{2,t})\int\limits_{0}^{\rho_{13}}\phi_2(w_{1,t},w_{3,t};\lambda_{13})d\lambda_{13} \\
&+ \Phi(w_{1,t})\int\limits_{0}^{\rho_{23}}\phi_2(w_{2,t},w_{3,t};\lambda_{23})d\lambda_{23} \\
&+ \int\limits_{0}^{\rho_{12}}\int\limits_{0}^{\rho_{13}}\frac{\partial\phi_3(w_t;\lambda_{12},\lambda_{13},0)}{\partial w_{1,t}}d\lambda_{12}d\lambda_{13} \\
&+ \int\limits_{0}^{\rho_{12}}\int\limits_{0}^{\rho_{23}}\frac{\partial\phi_3(\dot{w}_t;\lambda_{12},0,\lambda_{23})}{\partial w_{2,t}}d\lambda_{12}d\lambda_{23} \\
&+ \int\limits_{0}^{\rho_{13}}\int\limits_{0}^{\rho_{23}}\frac{\partial\phi_3(\ddot{w}_t;0,\lambda_{13},\lambda_{23})}{\partial w_{3,t}}d\lambda_{13}d\lambda_{23} \\
&+ \int\limits_{0}^{\rho_{12}}\int\limits_{0}^{\rho_{13}}\int\limits_{0}^{\rho_{23}}\frac{\partial^3\phi_3(\dot{\ddot{w}}_t;\lambda_{12},\lambda_{13},\lambda_{23})}{\partial w_{1,t}\partial w_{2,t}\partial w_{3,t}}d\lambda_{12}d\lambda_{13}d\lambda_{23}
\end{aligned} \tag{15}
$$

for a trivariate model, where ρ are the non-diagonal elements of the $Q_t\Omega Q_t$ matrix and λ are the non-diagonal elements of a theoretical 2×2 matrix and respectively a 3×3 matrix in which one of the correlation coefficients is null. Moreover, $\dot{w}_t$ is a vector of indices obtained by changing the order of the elements to $(w_{2,t},w_{3,t},w_{1,t})$. Similarly $\ddot{w}_t$ corresponds to a vector of indices of the form $(w_{3,t},w_{1,t},w_{2,t})$. Finally, $\dot{\ddot{w}}_t$ corresponds to w_t,$\dot{w}_t$ or $\ddot{w}_t$ respectively, depending on the way the last integral is decomposed. The computation of the last term is not trivial. However, this integral can be decomposed in a non-unique way as follows:

$$
\begin{aligned}
&\int_0^{\rho_{12}}\int_0^{\rho_{13}}\int_0^{\bar{\rho}_{23}} \frac{\partial^3 \phi_3(\dot{\ddot{w}}_t;\lambda_{12},\lambda_{13},\lambda_{23})}{\partial w_{1,t}\partial w_{2,t}\partial w_{3,t}} d\lambda_{12}d\lambda_{13}d\lambda_{23} \\
&\quad = \int_0^{\rho_{13}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\ddot{w}_t;\lambda_{12},\lambda_{13},\lambda_{23})}{\partial w_{3,t}} d\lambda_{13}d\lambda_{23} - \int_0^{\rho_{13}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\ddot{w}_t;0,\lambda_{13},\lambda_{23})}{\partial w_{3,t}} d\lambda_{13}d\lambda_{23} \\
&\quad = \int_0^{\rho_{12}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\dot{w}_t;\lambda_{12},\lambda_{13},\lambda_{23})}{\partial w_{2,t}} d\lambda_{12}d\bar{\rho}_{23} - \int_0^{\rho_{12}}\int_0^{\rho_{23}} \frac{\partial \phi_3(\dot{w}_t;\lambda_{12},0,\lambda_{23})}{\partial w_{2,t}} d\lambda_{12}d\lambda_{23} \\
&\quad + \int_0^{\lambda_{12}}\int_0^{\lambda_{13}} \frac{\partial \phi_3(w_t;\lambda_{12},\lambda_{13},\lambda_{23})}{\partial w_{1,t}} d\lambda_{12}d\lambda_{13} - \int_0^{\rho_{12}}\int_0^{\rho_{13}} \frac{\partial \phi_3(w_t;\lambda_{12},\lambda_{13},0)}{\partial w_{1,t}} d\lambda_{12}d\lambda_{13}.
\end{aligned}
\tag{16}
$$

These finite-range multiple integrals are numerically evaluated by using a Gauss-Legendre Quadrature rule [7] over bounded intervals. In such a context, two possibilities can be considered: whether the likelihood function is directly maximized, or the first order conditions [8] are derived so as to obtain an exact score vector. As stressed by Huguenin, Pelgrin and Holly (2009), the two methods may not lead to the same results if the objective function is not sufficiently smooth.

4 Empirical Application

This section aims at implementing the multivariate dynamic probit methodology presented above to a system composed by three types of crises, *i.e.* currency, banking and sovereign debt. We thus evaluate the probability of mutation of one type of crisis into another one. The existing literature offers some previous attempts to estimate the feed-back between currency and banking crises, [9] but as far as we know, no paper considers simultaneously the three types of crises. After a short data description and the presentation of the criteria implemented to detect the three types of crises, we estimate bivariate models by excluding sovereign debt crises. This constitutes a benchmark for the second part where the sovereign debt crises are included in the system.

7. Details about this quadrature are available in Appendix 2.
8. The score vector of the trivariate probit model is presented in Appendix 3.
9. The most advanced study being Glick and Hutchinson (1999).

4.1 Dating the crises

4.1.1 The Database

Monthly macroeconomic indicators expressed in US dollars covering the period from January 1985 to June 2010 have been extracted for 17 emerging countries [10] from the IMF-IFS database as well as the national banks of the countries under analysis via Datastream. The government bond returns are obtained via the JPMorgan EMDB database. More exactly, we have selected the main leading indicators used in the literature for the three types of crises we analyze (see Candelon et al., 2009, Jacobs et al., 2003, Glick and Hutchison, 1999, Hagen and Ho, 2004, Pescatori and Sy, 2007), namely, the one-year growth rate of international reserves, the growth rate of M2 to reserves ratio, one-year growth of domestic credit over GDP ratio, one-year growth of domestic credit, one-year growth of GDP, government deficit, debt service ratio and external debt ratio.

4.1.2 Dating the Crisis Periods

1. **The Currency Crises**

   Currency crises are generally identified using the market pressure index (MPI), which is a linear combination between exchange rate and foreign reserves changes. Hence if the pressure index exceeds a predetermined threshold [11] a crisis period is identified.

   As in Lestano and Jacobs (2004) and Candelon et al. (2009), a modified version of the pressure index proposed by Kaminski et al.(1998), which also incorporates the interest rate is used. It is denoted by (KLRm) and takes the following form:

$$\mathrm{KLRm}_{n,t} = \frac{\Delta e_{n,t}}{e_{n,t}} - \frac{\sigma_e}{\sigma_r}\frac{\Delta r_{n,t}}{r_{n,t}} + \frac{\sigma_e}{\sigma_i}\Delta i_{n,t}, \tag{17}$$

   where $e_{n,t}$ denotes the exchange rate (*i.e.,* units of country n's currency per US dollar in period t), $r_{n,t}$ represents the foreign reserves of country n in period t (expressed in $US\$$), while $i_{n,t}$ is the interest rate in country n at time t. We denote standard deviations σ_x are the standard deviations of the relative changes in the variables $\sigma_{(\Delta x_{n,t}/x_{n,t})}$, where x denotes each variable separately, including the exchange rate, foreign reserves, and the interest rate, with $\Delta x_{n,t} = x_{n,t} - x_{n,t-6}$. [12] For both subsamples, the currency crisis

10. Argentina, Brazil, Chile, Colombia, Ecuador, Egypt, El Salvador, Indonesia, Lebanon, Malaysia, Mexico, Panama, Peru, Philippines, South Africa, Turkey and Venezuela

11. Usually fixed to 2 or 3 times the sample's standard deviation as in Kaminski et al.(1998).

12. Additionally, we take into account the existence of higher volatility in periods of high inflation, and consequently the sample is split into high and low inflation periods. The cut-off corresponds to a six months inflation rate higher than 50%.

($CC_{n,t}$) threshold equals 1.5 standard deviations above the mean:

$$CC_{n,t} = \begin{cases} 1, & \text{if } \mathrm{KLRm}_{n,t} > 1.5\sigma_{\mathrm{KLRm}_{n,t}} + \mu_{\mathrm{KLRm}_{n,t}} \\ 0, & \text{otherwise.} \end{cases} \tag{18}$$

2. **The Banking Crises**

Banking crises are most commonly identified using the banking sector balance sheet, policy responses to bank runs and bank failures on a yearly basis (see the recent dating of Leaven and Valencia (2008). Nevertheless, our crisis dating requires a monthly frequency. Moreover, Eichengreen (1995, 1996) notices that banking crises are not always associated with a visible policy intervention. Indeed, some interventions may take place in the absence of a crisis in order to solve structural problems and perhaps to prevent a crisis. Besides, some measures can be taken only when the crisis has spread to the whole economy. Thus, Hagen and Ho (2004) propose a money market pressure index, accounting for the increasing demand for central bank reserves, to identify banking crises. Thus, it resembles a banking pressure index (BPI), available at monthly frequency:

$$BPI_{n,t} = \frac{\Delta\gamma_{n,t}}{\sigma_{\Delta\gamma}} + \frac{\Delta r_{n,t}}{\sigma_{\Delta r}}, \tag{19}$$

where γ is the ratio of reserves to bank deposits, r is the real interest rate, Δ is the six-months difference operator, and $\sigma_{\Delta\gamma}$ and $\sigma_{\Delta r}$ are the standard deviations of the two components. Sharp increases in the indicator (greater than the 90^{th} percentile denoted as $P_{BPI,90}$) signal a banking crises:

$$BC_{n,t} = \begin{cases} 1, & \text{if } \mathrm{IMP}_{n,t} > P_{BPI,90,n} \\ 0, & \text{otherwise.} \end{cases} \tag{20}$$

3. **The Sovereign Debt Crises**

Countries' 'default' does not constitute an adequate measure to characterize a sovereign debt crisis. Indeed a country may face debt-servicing difficulties or problems to refinance its debt on the international capital markets, without being in default. In order to overcome this problem, Pescatori and Sy (2007), consider a market-oriented measure of debt-servicing difficulties based on sovereign bond spreads.

In the line of this study, we consider that a sovereign debt crisis ($SC_{n,t}$) occurs if the CDS spreads exceed a critical threshold estimated by using kernel density estimation. More precisely, the existence of a mode around high spread values can be used to define crisis and calm periods, since whenever spreads are close to a limit that cannot be passed smoothly, the observations will concentrate around it until the limit is finally broken or the increasing pressure is reduced. Additionally, as expected, this threshold corresponds to a percentile between the 90^{th} and the 99^{th} percentiles, depending on the country (the number of crisis periods varies from one country to another), since crises are extreme events:

$$SC_{n,t} = \begin{cases} 1, & \text{if CDSspread}_{n,t} > Kernel\ Threshold_n \\ 0, & \text{otherwise.} \end{cases} \tag{21}$$

It is worth noting that most of the crisis periods we have identified by using the three aforementioned methods correspond to the ones reported in the literature on financial crises.

4.1.3 Remarks

1. As in Kumar (2003), we dampen every variable using the formula : $f(x_t) = sign(x_t)\log(1+|x_t|)$, so as to reduce the impact of extreme values. [13]
2. It should also be noted that the entire sample is used for the identification of currency and banking crises, while the identification of debt crises is realized by using data from December 1997 to date since the CDS spread series used for the identification of sovereign debt crises are not available before 1997 in the JPMorgan EMDB database. Consequently our empirical analysis will consist of two parts, the first one analyzing the case of twin crises (currency and banking) for which the entire database can be used, while the second part focuses on the interactions between the three types of crises and is thus based on data from 1997 onwards. The data sample actually used for each of the 17 countries and the two types of analyses is available in Table 1.
3. We only retain the countries for which the percentage of crisis periods is superior to 5% (See Table 2.) [14].
4. As mentioned in section 2, there are three dynamic multivariate specifications that can be used. However, as shown by Candelon et al. (2010), the dynamic model including the lagged binary variable (see (7)) seems to be the best choice for the Akaike information criterion. However, since we cannot expect a crisis to have a certain impact on the probability of emergence of another type of crisis from one month to another, which

13. Missing values of the series are replaced by cubic spline interpolation.
14. Argentina, Chile, Ecuador, Egypt, Indonesia, Lebanon, Mexico, South Africa and Venezuela are included in the bivariate analysis, whereas a trivariate model is specified for Ecuador and South Africa. Since the threshold has been arbitrarily set to 5%, we have also checked the borderline countries, like Colombia or Turkey in a bivariate analysis and Egypt in a trivariate analysis respectively, and similar results have been obtained.

would justify the notation $y_{m,t-1}$ from the theoretical part, in the empirical application we consider a response lag k of 3, 6 and respectively 12 months for the bivariate models and one of 3 or 6 months for the trivariate models [15]. Therefore, for each type of crisis we build a lagged variable $y_{m,t-k}$ which takes the value of one if there was crisis in the past k periods or at time t, and the value of 0 otherwise:

$$y_{m,t-k} = \begin{cases} 1, & \text{if } \sum_{j=0}^{k} y_{m,t-j} > 0 \\ 0, & \text{otherwise.} \end{cases} \tag{22}$$

5. The significance of the parameters of each model is tested by using simple t-statistics based on robust estimates of standard-errors. A special attention is given to the interpretation of cross-effects which stand for the transmission channels of the shocks/crisis. Besides, the joint nullity of the contemporaneous correlations between shocks is tested using a log-likelihood ratio test for the trivariate models.

4.2 Bivariate Analysis

Along the lines of Kaminsky et al. (1998) it is possible to find a large number of explanatory variables that may signal the occurrence of a crisis. Nevertheless, Candelon et al. (2010) showed that a univariate dynamic probit model presents the advantage of yielding plausible results while being more parsimonious. Indeed, a large part of the information is integrated either in the past state variable or in the lagged index and thus, only a few explanatory variables turn out to be significant. In this context, we expect ther multivariate (bivariate or trivariate) extension to be even more parsimonious. Therefore, we consider the two explanatory variables which are significant in Candelon et al. (2010), i.e. one-year growth of international reserves, one-year growth of M2 to reserves for currency crises as well as one-year growth of domestic credit over GDP and one-year growth of domestic credit for banking crises, resulting in four different specifications including one explanatory variable for each type of crisis. Moreover, three different lags (3 months, 6 months and 12 months) are considered for the lagged binary variable $y_{m,t-k}$. The dynamic probit model is estimated country-by-country. It is indeed a simplification as contagion (or spill-overs) from one country to another are not taken account. A panel version of the model would lead to several problems: First, as shown by Berg et al. (2008) heterogeneity due to country specificities would have to be accounted for. Second, the estimation of a fixed effect panel would be biased without a correction on the score vector. [16] Third, in a country by country analysis

15. A 12 months lag is not used in the case of trivariate models since it would significantly reduce the already small number of observations we have at our disposal

16. See Candelon et al., (2010) for a discussion about this point.

contagion has to be ignored. For all these reasons, we consider this extension beyond the scope of this paper and leave it for future research.

Each model is estimated via maximum-likelihood, the bivariate normal cumulative distribution function being approximated using the Gauss-Legendre quadrature, as proposed by Huguenin, Pelgrin and Holly, (2009). However, the quadrature specified in Matlab by default, *i.e.* the adaptive Simpson quadrature, has been considered as a benchmark.

Selection criteria, namely AIC and SBC, are used to identify the best model for each country and lag. It is nevertheless worth stressing that the results are generally robust to the choice of explanatory variables and even the choice of lags.

A summary of the results for the selected models is given in Table 3.

insert Table 3

First of all, it seems that most of the models exhibit dynamic and thus persistence, whatever the lag used to construct the 'past crisis' variable. This result confirms the findings of Candelon et al. (2010) and Bussière (2007), by showing that crises exhibit a regime dependence: if the country is proven to be more vulnerable than investors had initially thought, investors will start withdrawing their investments, thus increasing the probability of a new crisis. More precisely, most of the countries are found to have experienced banking and currency crises depending on their own past, *i.e.* Argentina, Egypt, Lebanon, Mexico, South Africa, Venezuela. Besides, only for a small number of cases, only one of the two types of crises is best reproduced by a dynamic model (currency crises in Chile (3 and 12 months), Mexico (6 and 12 months) ; banking crises in Argentina (6 and 12 months), Ecuador, Lebanon (6 months), South Africa (12 months) and Venezuela (12 months)). Actually, in Chile a past currency crisis had only a short term positive impact on the emergence of another currency crisis, whereas a past banking crisis has just a long term effect on the probability of occurrence of another banking crisis. Mexico, however, seems to be more prone to recurring currency crises than banking crises as the former type of crisis has a long-term impact on the probability of experiencing a new crisis, whereas the latter has a positive effect only in the short run. On the contrary, in Argentina, South Africa and Venezuela it is the impact of past banking crises on currency crises that is longer (up to one year) as opposed to that of past currency crises on banking ones (up to three and six months, respectively).

Second, for the majority of the countries (Argentina, Chile, Lebanon, Mexico and Venezuela), currency and banking crises are interconnected. This link between crises can take two forms. On the one hand, a certain type of crisis increases (or diminishes) the probability of occurrence of the other type of crisis. Such a causal linkage from banking to currency crisis was put in evidence by Glick and Hutchinson (1999) within a panel framework. Nevertheless, there is no reason for the transmission of shocks to be symmetric. Indeed, our country per country analysis reveals that some countries like Argentina (3 and 6 months) for which a banking crisis in the past k months increased the probability of a currency crisis at time t. At the same time, a banking crisis in Chile in the last 12 months reduced the probability of experiencing a currency crisis. Conversely, a currency crisis in Egypt and in Lebanon (3 months) diminished the probability of a banking crisis.

On the other hand, crisis shocks can be contemporaneously positively correlated. This feature seems to be very stable across models (independent of the lag used). The only exceptions are Egypt and Lebanon, for which there is no instantaneous correlation in the model with 3-months lagged binary variables and Mexico, for which such a correlation appears only for the 12-months lag. To sum up, but for Egypt, all countries are characterized by a positive instantaneous correlation between shocks of currency and banking crises variables, corroborating the previous findings of Glick and Hutchinson (1999).

Third, the macroeconomic variables are rarely significant [17]. These results corroborate our previous findings (see Candelon et al. 2010) that the dynamics of crises captures most of the information explaining the emergence of such phenomena. Furthermore, when these coefficients are significant, they have the expected sign (an increase in the growth of international reserves diminishes the probability of a crisis, while a suprise in the rest of indicators soars the probability of a crisis).

To summarize, these results confirm the presence of interaction between the banking and currency crisis. The twin crisis phenomenon is thus confirmed empirically. Besides, our findings are robust to the quadrature choice and the lags considered when constructing the dynamic binary variables.

4.3 Trivariate Analysis

But is it really enough ? This subsection extends the previous analysis to the trivariate case by modeling simultaneously currency, banking and debt crises. However, only two countries experienced these three events during a sufficiently long period. Ecuador presents for our sample an *ex-post* probability larger than 5% for whatever the type of crisis. Such a result is not surprising if one remembers that Ecuador faced a strong financial turmoil in the late 1990, affecting first the banking sector, [18] then the Sucre [19], and the government budget. Jacone (2004) showed that institutional weaknesses, rigidities in public finances, and high financial dollarization have amplified this crisis. South Africa constitutes a borderline case as the sovereign debt crisis probability is slightly below 5%.

Each of the models is then estimated for these countries using both the methodology

17. These results are available upon request.
18. 16 out of the 40 banks existing in 1997 faced liquidity problems.
19. The Ecuadorian currency has been replaced by the U.S. dollar on March, 13, 2000.

proposed by Huguenin et al. (2009) based on the Gauss-Legendre quadrature and the direct approximation of a triple integral based on the adaptive Simpson quadrature that Matlab uses by default. Similar results are obtained for the two methods [20]. However, the latter implies a significant gain in time without any loss in accuracy proving that recently developed quadrature methods are good approximations of the normal cumulative distribution function. Besides, 6 and 12 month-lags of the dynamic crisis variable are considered.

insert Table 4

In the case of Ecuador, the results corroborate our bivariate findings: the banking crises are persistent, while currency crises are not. Nevertheless, it is clear that the bivariate model is misspecified, since it cannot capture the impact of a banking crisis on the occurrence of a currency crisis when using the 6-months lagged binary variables to account for the dynamics of these phenomena (see Table 4).

Moreover, the trivariate model turns out to be more parsimonious since the index of past debt crisis has a positive effect on the probability of occurrence of both currency and debt crises. Therefore it supports the implementation of a trivariate crisis model whenever when it is feasible. We also observe that the contemporaneous correlation matrix is diagonal, ruling out the idea of common shocks. Crises in Ecuador turn out to be exclusively driven by transmission channels, as in the late 1990, when the banking distress was diffused to the currency and the government budget.

In the case of South-Africa, both currency and debt crises are persistent. There is no evidence of causality between the different types of crises, but significant contemporaneous correlation. It highlights the fact that contrary to Ecuador, South African crises did not mutate but they originated from a common shock. It is worth noting that the results are robust to the sensitivity analyses performed, namely the choice of macroeconomic variables and the use of different lags for the past crisis variables.

4.4 Further results

To grasp deeper information from the previous models, a conditional probability as well as the Impulse Response Functions (IRF) analyses are provided. For sake of space, we only report the results obtained for Ecuador. [21]

First, Figure 1 reports the conditional probabilities for each type of crisis obtained from both the bi- and trivariate models considering a forecast horizon of 3 and 6-months. To allow a fair comparison, both models are estimated on the same sample, *i.e.* from 1997 onwards.

20. The results for Ecuador when considering a 6-months lag have been obtained with Matlab's quadrature since the model based on the Gauss-Legendre Quadrature does not seem to converge.

21. Results for South Africa are available from the authors upon request.

It is without saying that the bivariate model does not provide any conditional probabilities for sovereign debt crisis.

It turns out that the trivariate model outperforms the bivariate one whatever the forecast horizon, i.e. conditional probabilities issued from the trivariate model are higher than those obtained with the bivariate model during observed crisis periods, while they appear to be similar in calm periods. Such results corroborate hence our previous findings, stressing that a crisis model should take into account the whole sequence of crises to be accurate. Besides, the conditional probabilities obtained from the trivariate model do not immediately collapse after the occurrence of the crisis, which is the case in the case of the bivariate model. It stresses hence the vulnerability of the economy after the exit of a turmoils in particular if it affects the foreign exchange market.

Second, to evaluate the effect of a crisis, considered her as a shock, an IRF exercise is performed for the trivariate model. As the order of the variables has been shown to be crucial, we consider the historical sequence of crises observed in Ecuador, *i.e.* Banking crises (the most exogenous ones), debt crises and currency crises (the most endogenous ones). Orthogonal impulse response functions are considered on the latent variable for a 6 month-horizon. The exogenous variables are fixed to the unconditional mean ($\bar{X}_{m,t}$). Departing from eq. 1, we express the IRF in term of latent model, i.e. the probability of being in a crisis state at time t and the binary crisis/calm variable as follows:

$$\begin{aligned} y_t^* &= \hat{\alpha} + \bar{y}_{t-1}\hat{\beta} + (\tilde{y}_{t-1}\hat{\Delta})' + \hat{\varepsilon}_t, \\ \Pr(\hat{y}_t = 1) &= \Phi_3(y_t^*), \\ \hat{y}_t &= \mathbb{1}(y_t^* > 0) = \mathbb{1}(\Pr(\hat{y}_t = 1) > 0.5), \end{aligned} \tag{23}$$

$\hat{\alpha}, \hat{\beta},\ \hat{\Delta}$ are obtained from the estimation of the trivariate model for Ecuador andthe correlated residuals $\hat{\epsilon}_{m,t}$ are transformed into orthogonal ones via a Choleski decomposition of the covariance matrix $\hat{\Omega}$. Therefore, a crisis is to arise at time t, if $\hat{y}_{m,t} = 1$.

Additionally, as in any non linear model, the IRF are calculated for two initial states: a tranquil one, $\tilde{y}_t = 0$, *i.e. "no type of crisis is observed at time* $t = 0$ *or in the previous 3 months"* and a turmoil regime, $\tilde{y}_t = 1$, *i.e. "all types of crisis are observed in* $t = 0$*"*. Confidence intervals are built taking the 2.5% and 97.5% percentiles of IRF's distribution obtained from 10,000 simulations of the model. The magnitude of the shock is fixed to 5 [22], allowing for a potential mutation of the crisis.

It is important to distinguish between a significant IRF and a significant shift from a calm to a crisis period. First, IRFs are demeaned, so that they are significant if the corresponding confidence interval does not include the value of 0. Second, the shift probability from calm to

22. Results for shocks of magnitude 10 are available upon request.

crisis or the probability of remaining in a crisis period is significant at time t if the confidence interval associated to the IRF_t remains in the grey area, *i.e.* the centered IRF_t is significantly lying above its unconditional mean ($\hat{\alpha} + \bar{x}_{m,t-1}\hat{\beta}_m$). While the first analysis is common to all vector autoregressive (VAR) models, the second one is specific to non-linear (threshold) time series models.

Figures 2 to 4 reports the diffusion of a banking, currency and debt crises.

First, it appears in Figure 2 that a banking crisis shock has almost no persistence in a calm initial state, as the IRF function comes back to mean after a single period. On the contrary the persistence jumps to 5 months for an initial crisis state. Similarly the diffusion of a banking crisis shock to another types of turmoil is exclusively observed in a crisis initial states. Besides, the shift probability from calm to crisis is significant only for the banking crisis and up to the second period (see the left part of figure 2), whereas the probability of remaining in a crisis period is significant for all three types of crises until $t = 2$ (see the right part of figure 2). This underlines the uncertainty encompassing the duration of a crisis beyond one month after the shock. Overall these first results clearly exhibit the crisis sequence faced by Ecuador in the late 90's. Figure 3 reports the response of the three latent variables to a debt crisis shock. In such a case, the shock on the banking and currency crises vanishes almost instantaneously in the case of a calm initial state, while it disappears after 4 or 5 months, if the economy is facing initially a joint crisis. As for the debt crisis, the impact of the shock lasts at least 5 months even though we are certain of being in a crisis period during the first two periods (the confidence interval is in the grey area at that time). Finally, Figure 4 presents the IRF after currency crisis shock. As in the previous cases, the impact on the banking crisis is not important if we depart from a calm situation, while it becomes significant during 4 periods for an initial crisis period. At the same time, the response of the debt crisis is slowly dumped towards the baseline for a calm initial state, whereas it is significant during the first 4 periods if we introduce the shock while being in a crisis state. It seems that the persistence this shock is around two months for a calm initial period while it dies away only after 5 months in the alternative.

Overall, the conditional probability and the IRF analyses stress the superiority of the trivariate model to encounter for the diffusion mechanisms that occurred in Ecuador after the banking crisis at the end of the 1990. Strong interactions between the three types of crises are clearly present in particular between banking and other crises.

5 Conclusion

This paper is the first attempt to model simultaneously the three types of crises (currency, banking and sovereign debt), thus allowing to investigate the potential mutations between not only the currency and the banking crises but also the sovereign debt one. It is actually an extension of the previous papers which investigate the twin crises phenomenon (in particular Glick and Hutchinson, 1999). To achieve this objective, a methodological novelty is introduced by proposing an exact maximum likelihood approach to estimate the multivariate dynamic probit model, extending hence the Huguenin, Pelgrin and Holly (2009)'s method to dynamic models. Applied to a large sample of emerging countries, we show that in the bivariate case causality from banking to currency (and vice-versa) are quite common. More importantly, for the two countries, Ecuador and South Africa, which suffer from the 3 types of crises, the trivariate model turns out to be the most performing in term of conditional probability and to understand why a specific crisis mutes to another one : this can be due to common shocks (as in South Africa) or to a strong causal structure (as in Ecuador). More generally, this paper advocates the use of trivariate probit crisis models whenever it is possible, to have a better insight on the financial turmoils.

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Appendix 1: Proof of lemma 1

By definition, the likelihood of observation t is given by:

$$\begin{aligned}
L_t(y_t|z_{t-1},\theta;\Omega) &= \Pr((-q_{1,t}y^*_{1,t} \leq 0), ..., (-q_{M,t}y^*_{M,t} \leq 0)) \\
&= \Pr(-q_{1,t}\varepsilon_{1,t} \leq q_{1,t}\pi_{1,t}, ..., -q_{M,t}\pi_{M,t} \leq q_{M,t}\pi_{M,t}) \\
&= \Phi_{M,-Q_t\varepsilon_t}(w_t|0_M;\Omega) \\
&= \int_{-\infty}^{w_{M,t}} ... \int_{-\infty}^{w_{1,t}} \phi_{M,-Q_t\varepsilon_t}(Q_t\varepsilon_t,\Omega) \prod_{m=1}^{M} d\varepsilon_{m,t}.
\end{aligned}$$

Since each $q_{m,t}$ takes only the values $\{-1,1\}$, it is straightforward to show that $Q_t = Q_t^{-1}$ and $|Q_t\Omega Q_t| = |\Omega|$. Moreover, the density of an M-variate standardized normal vector $-Q_t\varepsilon_t$ with covariance matrix Ω may be re-written as the density of an M-variate standardized normal vector ε_t with variance-covariance matrix $Q_t\Omega Q_t$:

$$\begin{aligned}
\phi_{M,-Q_t\varepsilon_t}(Q_t\varepsilon_t;\Omega) &= |2\pi\Omega|^{\frac{-1}{2}} \exp\left\{\frac{-1}{2}(-Q_t\varepsilon_t)'\Omega^{-1}(-Q_t\varepsilon_t)\right\} \\
&= |2\pi(Q_t\Omega Q_t)|^{\frac{-1}{2}} \exp\left\{\frac{-1}{2}\varepsilon_t'(Q_t\Omega Q_t)^{-1}\varepsilon_t\right\} \\
&= \phi_{M,\varepsilon_t}(\varepsilon_t;Q_t\Omega Q_t).
\end{aligned}$$

Therefore, the likelihood of observation t is given by :

$$\begin{aligned}
L_t(y_t|Z_{t-1},\theta;\Omega) &= \int_{-\infty}^{q_{M,t}\pi_{M,t}} ... \int_{-\infty}^{q_{1,t}\pi_{1,t}} \phi_{M,\varepsilon_t}(\varepsilon_t;Q_t\Omega Q_t) \prod_{m=1}^{M} d\varepsilon_{m,t} \\
&= \Phi_{M,\varepsilon_t}(Q_t\pi_t;Q_t\Omega Q_t).
\end{aligned}$$

Appendix 2: The Gauss-Legendre Quadrature rule

The goal of the Gauss-Legendre Quadrature rule is to provide an approximation of the following integral:

$$\int_a^b f(x)dx. \tag{24}$$

In a first step, the bounds of the integral must be changed from $[a, b]$ to [-1,1] before applying the Gaussian Quadrature rule:

$$\int_a^b f(x)dx = \frac{b-a}{2} \int_{-1}^{1} f(z)dz, \tag{25}$$

where $z_i = \frac{b-a}{2} abs_i + \frac{b+a}{2}$ and the nodes abs_i, $i \in \{1, 2, ..., p\}$ are zeros of the Legendre polynomial $P_p(abs)$.

Definition 1. *Then, the standard p-point Gauss-Legendre quadrature rule over a bounded arbitrary interval [a,b] is given by the following approximation*:

$$\int_a^b f(x)dx \approx \frac{b-a}{2} \sum_{i=1}^{p} v_i f(z_i) + R_p, \tag{26}$$

where v_i *are the corresponding weights,* $v_i = \frac{2}{(1-abs_i^2)\left(\frac{\partial P_p(abs)}{\partial abs}|_{abs_i}\right)^2}$, $\sum_{i=1}^{p} v_i = 2$, *and* R_p *is the error term,* $R_p = Q_p f^{(2p)}(\xi) = \frac{(b-a)^{2p+1}(p!)^4}{(2p+1)(2p!)^3} f^{2p}(\xi)$, *with* $\xi \in (a, b)$.

Appendix 3: The EML score vector for a trivariate dynamic probit model

For ease of notation, let us denote by $\rho_{i,j}$, $i, j = \{1, 2, 3\}$, $i \neq j$ the correlation coefficients associated to the Ω matrix. The likelihood of observation t may be written as:

$$\begin{aligned} P_t &= \Phi_3(q_1\pi_{1,t}, q_2\pi_{2,t}, q_3\pi_{3,t}, q_1q_2\rho_{12}, q_1q_3\rho_{13}, q_2q_3\rho_{23}) \\ &= \Phi(q_1\pi_{1,t})\Phi(q_2\pi_{2,t})\Phi(q_3\pi_{3,t}) \\ &+ q_1q_2\Phi(q_3\pi_{3,t})\Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\ &+ q_1q_3\Phi(q_2\pi_{2,t})\Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\ &+ q_2q_3\Phi(q_1\pi_{1,t})\Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\ &+ q_1q_2q_3\Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\ &+ q_1q_2q_3\Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\ &+ q_1q_2q_3\Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}), \end{aligned} \tag{27}$$

where

$$\Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) = \int_0^{\rho_{12}} \psi_2(\pi_{1,t}, \pi_{2,t}, \lambda_{12})d\lambda_{12}$$

$$\Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) = \int_0^{\rho_{13}} \psi_2(\pi_{1,t}, \pi_{3,t}, \lambda_{13})d\lambda_{13}$$

$$\Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) = \int_0^{\rho_{23}} \psi_2(\pi_{2,t}, \pi_{3,t}, \lambda_{23})d\lambda_{23},$$

and

$$\Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) = \int_0^{\rho_{13}}\int_0^{\rho_{23}} \frac{-\pi_{3,t} + \lambda_{13}\pi_{1,t} + \lambda_{23}\pi_{2,t}}{1 - \lambda_{13}^2 - \lambda_{23}^2}\psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \lambda_{13}, \lambda_{23}, 0)d\lambda_{13}d\lambda_{23}$$

$$\Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) = \int_0^{\rho_{23}}\int_0^{\rho_{12}} \frac{-\pi_{2,t} + \lambda_{23}\pi_{3,t} + \lambda_{12}\pi_{1,t}}{1 - \lambda_{23}^2 - \lambda_{12}^2}\psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \lambda_{23}, \lambda_{12}, 0)d\lambda_{23}d\lambda_{12}$$

$$\begin{aligned} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}) = \int_0^{\rho_{12}}\int_0^{\rho_{13}} & \frac{-(1-\rho_{23}^2)\pi_{1,t} + (\lambda_{12} - \lambda_{13}\rho_{23})\pi_{2,t} + (\lambda_{13} - \lambda_{12}\rho_{23})\pi_{3,t}}{1 - \lambda_{12}^2 - \lambda_{13}^2 - \rho_{23}^2 + 2\lambda_{12}\lambda_{13}\rho_{23}} \\ & \times \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \lambda_{13}, \rho_{23})d\lambda_{12}d\lambda_{23}. \end{aligned}$$

Therefore, the first order partial derivatives can be obtained as follows :

$$\begin{aligned}
\frac{\partial}{\partial \pi_1} P_t = {} & q_1 \psi(\pi_{1,t}) \Phi(q_2 \pi_{2,t}) \Phi(q_3 \pi_{3,t}) \\
& + q_1 q_2 \Phi(q_3 \pi_{3,t}) \frac{\partial}{\partial \pi_1} \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\
& + q_1 q_3 \Phi(q_2 \pi_{2,t}) \frac{\partial}{\partial \pi_1} \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\
& + q_1 q_2 q_3 \psi(\pi_{1,t}) \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_1} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_1} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_1} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned} \tag{28}$$

$$\begin{aligned}
\frac{\partial}{\partial \pi_2} P_t = {} & q_2 \psi(\pi_{2,t}) \Phi(q_1 \pi_{1,t}) \Phi(q_3 \pi_{3,t}) \\
& + q_1 q_2 \Phi(q_3 \pi_{3,t}) \frac{\partial}{\partial \pi_2} \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\
& + q_1 q_2 q_3 \psi(\pi_{2,t}) \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\
& + q_2 q_3 \Phi(q_1 \pi_{1,t}) \frac{\partial}{\partial \pi_2} \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_2} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_2} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_2} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned} \tag{29}$$

$$
\begin{aligned}
\frac{\partial}{\partial \pi_3} P_t = {} & q_1 \psi(\pi_{3,t}) \Phi(q_1 \pi_{1,t}) \Phi(q_2 \pi_{2,t}) \\
& + q_1 q_2 q_3 \psi(\pi_{3,t}) \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\
& + q_1 q_3 \Phi(q_2 \pi_{2,t}) \frac{\partial}{\partial \pi_3} \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\
& + q_2 q_3 \Phi(q_1 \pi_{1,t}) \frac{\partial}{\partial \pi_3} \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_3} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_3} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \pi_3} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned} \tag{30}
$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{12}} P_t = {} & q_1 q_2 \Phi(q_3 \pi_{3,t}) \frac{\partial}{\partial \rho_{12}} \Psi_2(\pi_{1,t}, \pi_{2,t}, \rho_{12}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{12}} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{12}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned} \tag{31}
$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{13}} P_t = {} & q_1 q_3 \Phi(q_2 \pi_{2,t}) \frac{\partial}{\partial \rho_{13}} \Psi_2(\pi_{1,t}, \pi_{3,t}, \rho_{13}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{13}} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{13}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned} \tag{32}
$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{23}} P_t = {} & q_2 q_3 \Phi(q_1 \pi_{1,t}) \frac{\partial}{\partial \rho_{23}} \Psi_2(\pi_{2,t}, \pi_{3,t}, \rho_{23}) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{3,t}, \pi_{1,t}, \pi_{2,t}, \rho_{13}, \rho_{23}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) \\
& + q_1 q_2 q_3 \frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}),
\end{aligned} \tag{33}
$$

where

$$\frac{\partial}{\partial \pi_1}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{23}}\int_0^{\rho_{13}} \frac{\partial}{\partial \lambda_{13}}\psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\lambda_{23},0)d\lambda_{13}\lambda_{23}$$
$$= \int_0^{\rho_{23}} \psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\lambda_{23},0)d\lambda_{23},$$

$$\frac{\partial}{\partial \pi_2}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{13}}\int_0^{\rho_{23}} \frac{\partial}{\partial \lambda_{23}}\psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\lambda_{23},0)d\lambda_{23}\lambda_{13}$$
$$= \int_0^{\rho_{13}} \psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\rho_{23},0)d\lambda_{13},$$

$$\frac{\partial}{\partial \pi_3}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{13}}\int_0^{\rho_{23}} [(\pi_{3,t}-\lambda_{13}\pi_{1,t}-\lambda_{23}\pi_{2,t})^2-(1-\lambda_{13}^2-\lambda_{23}^2)]$$
$$\times \frac{1}{(1-\lambda_{13}^2-\lambda_{23}^2)^2}\psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\lambda_{13},\lambda_{23},0)d\lambda_{13}d\lambda_{23},$$

$$\frac{\partial}{\partial \rho_{13}}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{23}} \frac{-\pi_{3,t}+\rho_{13}\pi_{3,t}+\lambda_{23}\pi_{2,t}}{1-\rho_{13}^2-\lambda_{23}^2}\psi_3(\pi_3,\pi_1,\pi_2,\rho_{13},\lambda_{23},0)d\lambda_{23},$$

$$\frac{\partial}{\partial \rho_{23}}\Psi_3(\pi_{3,t},\pi_{1,t},\pi_{2,t},\rho_{13},\rho_{23},0) = \int_0^{\rho_{13}} \frac{-\pi_{3,t}+\lambda_{13}\pi_{3,t}+\rho_{23}\pi_{2,t}}{1-\lambda_{13}^2-\rho_{23}^2}\psi_3(\pi_3,\pi_1,\pi_2,\lambda_{13},\rho_{23},0)d\lambda_{13},$$

$$\frac{\partial}{\partial \pi_1}\Psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{23}}\int_0^{\rho_{12}} \frac{\partial}{\partial \lambda_{12}}\psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\lambda_{23},\lambda_{12},0)d\lambda_{12}\lambda_{23}$$
$$= \int_0^{\rho_{23}} \psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t,},\lambda_{23},\rho_{12},0)d\lambda_{23},$$

$$\frac{\partial}{\partial \pi_2}\Psi_3(\pi_{2,t},\pi_{3,t,},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{23}}\int_0^{\rho_{12}} [(\pi_{2,t}-\lambda_{23}\pi_{3,t}-\lambda_{12}\pi_{1,t})^2-(1-\lambda_{23}^2-\lambda_{12}^2)]$$
$$\times \frac{1}{(1-\lambda_{23}^2-\lambda_{12}^2)^2}\psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\lambda_{23},\lambda_{12},0)d\lambda_{23}d\lambda_{12},$$

$$\frac{\partial}{\partial \pi_3}\Psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\rho_{12},0) = \int_0^{\rho_{12}}\int_0^{\rho_{23}} \frac{\partial}{\partial \lambda_{23}}\psi_3(\pi_{2,t},\pi_{3,t,},\pi_{1,t},\lambda_{23},\lambda_{12},0)d\lambda_{23}\lambda_{12}$$
$$= \int_0^{\rho_{12}} \psi_3(\pi_{2,t},\pi_{3,t},\pi_{1,t},\rho_{23},\lambda_{12},0)d\lambda_{12},$$

$$\frac{\partial}{\partial \rho_{12}} \Psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) = \int_0^{\rho_{23}} \frac{-\pi_{2,t} + \lambda_{23}\pi_{3,t} + \rho_{12}\pi_{1,t}}{1 - \lambda_{23}^2 - \rho_{12}^2} \psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \lambda_{23}, \rho_{12}, 0) d\lambda_{23},$$

$$\frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{2,t},, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \rho_{12}, 0) = \int_0^{\rho_{12}} \frac{-\pi_{2,t} + \rho_{23}\pi_{3,t} + \lambda_{12}\pi_{1,t}}{1 - \rho_{23}^2 - \lambda_{12}^2} \psi_3(\pi_{2,t}, \pi_{3,t}, \pi_{1,t}, \rho_{23}, \lambda_{12}, 0) d\lambda_{12},$$

$$\begin{aligned} \frac{\partial}{\partial \pi_1} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}) = & \int_0^{\rho_{12}} \int_0^{\rho_{13}} \{[(1-\rho_{23}^2)\pi_{1,t} - (\lambda_{12} - \lambda_{13}\rho_{23})\pi_{2,t} - (\lambda_{13} - \lambda_{12}\lambda_{23})\pi_{3,t}]^2 \\ & - (1-\rho_{23}^2)(1 - \lambda_{12}^2 - \lambda 13^3 - \rho_{23}^2 + 2\lambda_{12}\lambda_{13}\rho_{23})\} \times \\ & \frac{1}{(1 - \lambda_{12}^2 - \lambda_{13}^2 - \rho_{23}^2 + 2\lambda_{12}\lambda_{13}\rho_{23})^2} \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \lambda_{13}, \rho_{23}) d\lambda_{12} d\lambda_{13}, \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial \pi_2} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}) &= \int_0^{\rho_{13}} \int_0^{\rho_{12}} \frac{\partial}{\partial \lambda_{12}} \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t},, \lambda_{12}, \lambda_{13}, \rho_{23}) d\lambda_{12}\lambda_{13} \\ &= \int_0^{\rho_{13}} \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \lambda_{13}, \rho_{23}) d\lambda_{13}, \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial \pi_3} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}) &= \int_0^{\rho_{12}} \int_0^{\rho_{13}} \frac{\partial}{\partial \lambda_{13}} \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \lambda_{13}, \rho_{23}) d\lambda_{13}\lambda_{12} \\ &= \int_0^{\rho_{12}} \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \rho_{13}, \rho_{23}) d\lambda_{12}, \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial \rho_{12}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}) = & \int_0^{\rho_{13}} \frac{(1-\rho_{23}^2)\pi_{1,t} + (\rho_{12} - \lambda_{13}\rho_{23})\pi_{2,t} + (\lambda_{13} - \rho_{12}\rho_{23})\pi_{3,t}}{1 - \rho_{12}^2 - \lambda_{13}^2 - \rho_{23}^2 + 2\rho_{12}\lambda_{13}\rho_{23}} \\ & \times \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \lambda_{13}, \rho_{23} d\lambda_{13}), \end{aligned}$$

$$\begin{aligned} \frac{\partial}{\partial \rho_{13}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}) = & \int_0^{\rho_{12}} \frac{(1-\rho_{23}^2)\pi_{1,t} + (\lambda_{12} - \rho_{13}\rho_{23})\pi_{2,t} + (\rho_{13} - \lambda_{12}\rho_{23})\pi_{3,t}}{1 - \lambda_{12}^2 - \rho_{13}^2 - \rho_{23}^2 + 2\lambda_{12}\rho_{13}\rho_{23}} \\ & \times \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \rho_{13}, \rho_{23} d\lambda_{12}), \end{aligned}$$

$$
\begin{aligned}
\frac{\partial}{\partial \rho_{23}} \Psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \rho_{13}, \rho_{23}) &= \int_0^{\rho_{12}} \int_0^{\rho_{13}} \frac{\partial^2}{\partial \pi_{2,t} \partial \lambda_{13}} \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \lambda_{13}, \rho_{23}) d\lambda_{12} d\lambda_{13} \\
&= \int_0^{\rho_{12}} \frac{-(1-\rho_{13}^2)\pi_{2,t} + (\lambda_{12} - \rho_{13}\rho_{23})\pi_{1,t} + (\rho_{23} - \lambda_{12}\rho_{13})\pi_{3,t}}{1 - \lambda_{12}^2 - \rho_{13}^2 - \rho_{23}^2 + 2\lambda_{12}\rho_{13}\rho_{23}} \\
&\times \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \rho_{13}, \rho_{23}) d\lambda_{12} \\
&= \int_0^{\rho_{12}} \int_0^{\rho_{13}} \frac{\partial^2}{\partial \pi_{3,t} \partial \lambda_{12}} \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \lambda_{12}, \lambda_{13}, \rho_{23}) d\lambda_{12} d\lambda_{13} \\
&= \int_0^{\rho_{13}} \frac{-(1-\rho_{12}^2)\pi_{3,t} + (\lambda_{13} - \rho_{12}\rho_{23})\pi_{1,t} + (\rho_{23} - \rho_{12}\lambda_{13})\pi_{2,t}}{1 - \rho_{12}^2 - \lambda_{13}^2 - \rho_{23}^2 + 2\rho_{12}\lambda_{13}\rho_{23}} \\
&\times \psi_3(\pi_{1,t}, \pi_{2,t}, \pi_{3,t}, \rho_{12}, \lambda_{13}, \rho_{23}) d\lambda_{13}.
\end{aligned}
$$

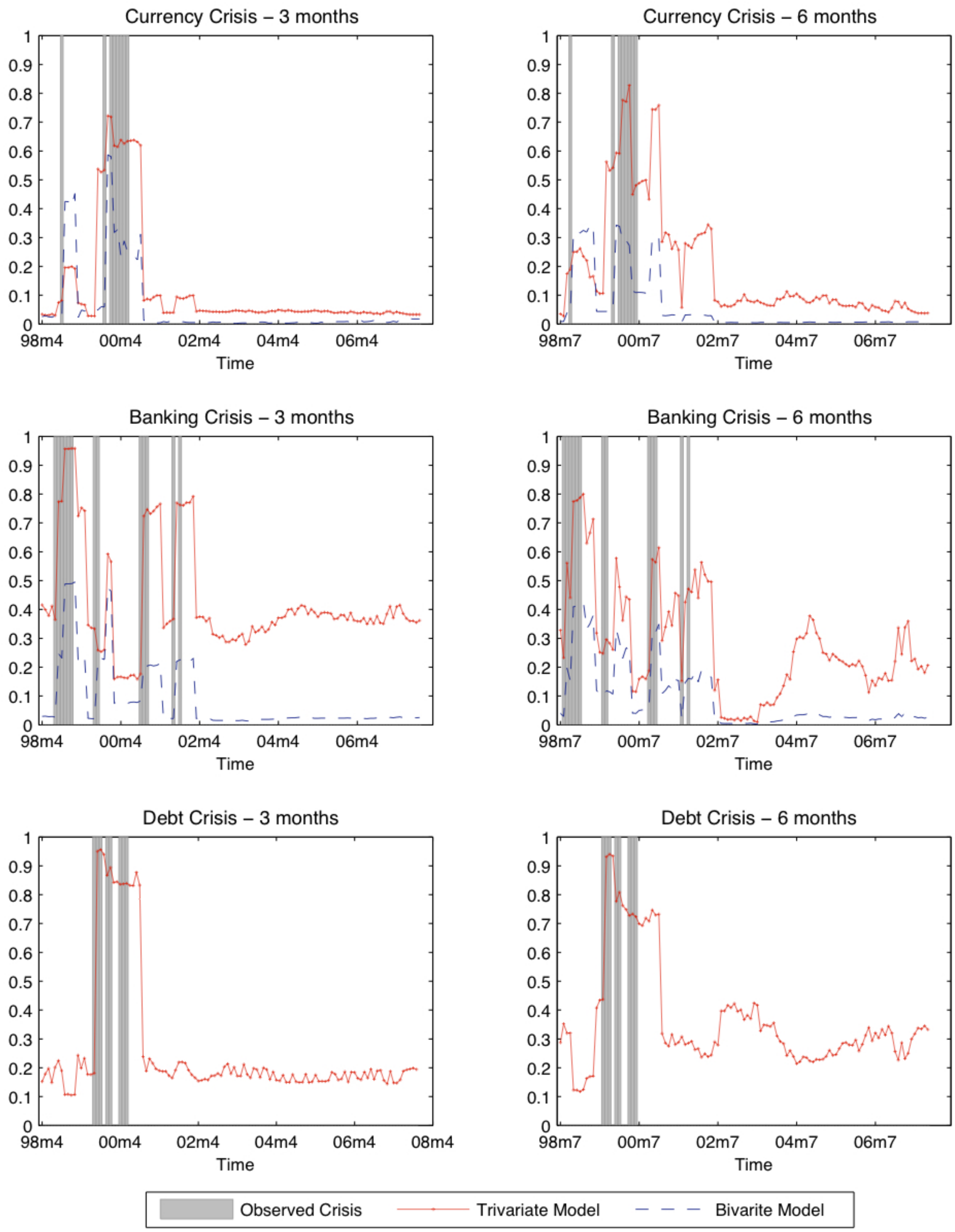

FIGURE 1 – Conditional crisis probabilities - Ecuador

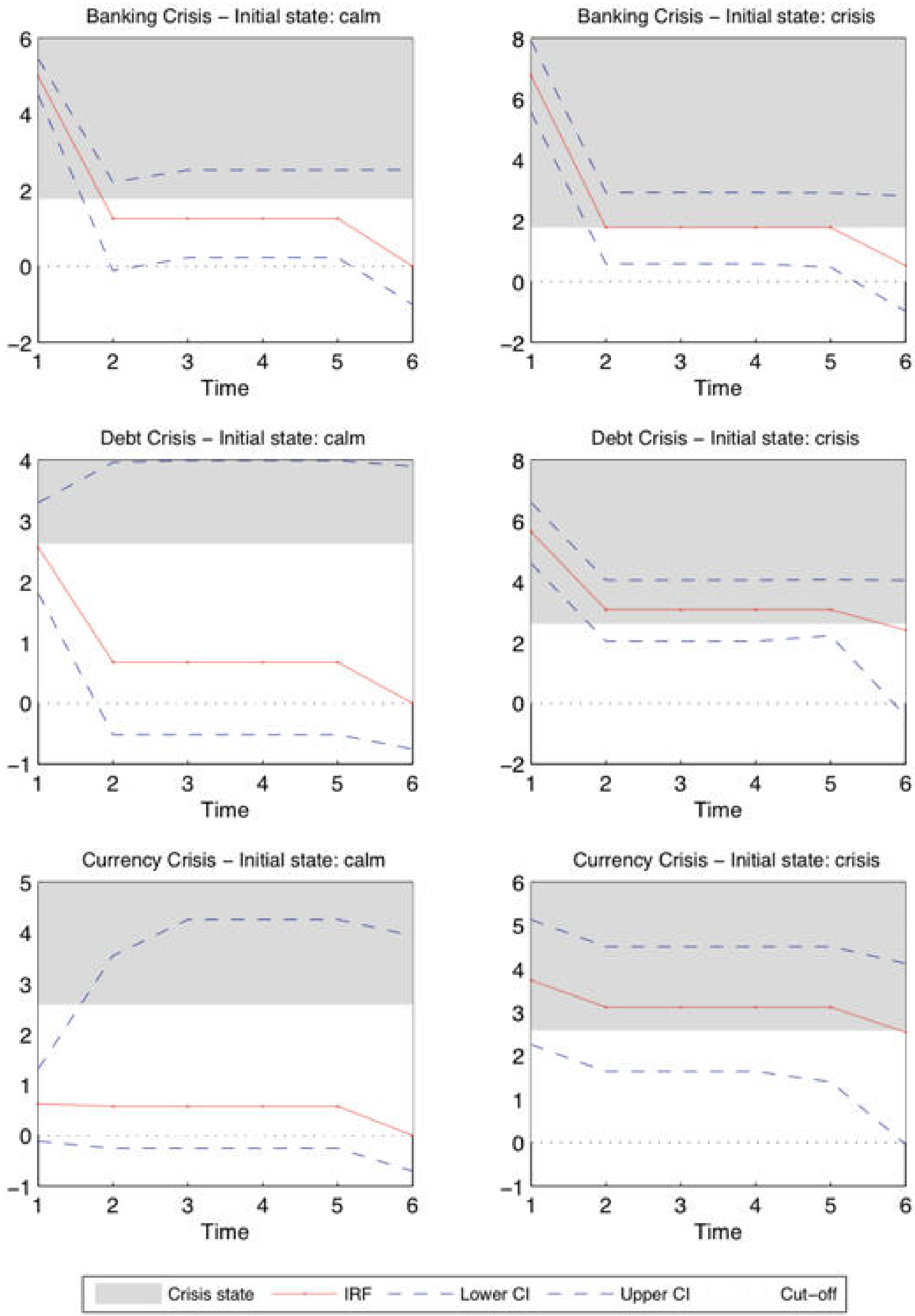

FIGURE 2 – IRF after a banking crisis shock - Ecuador 3 months

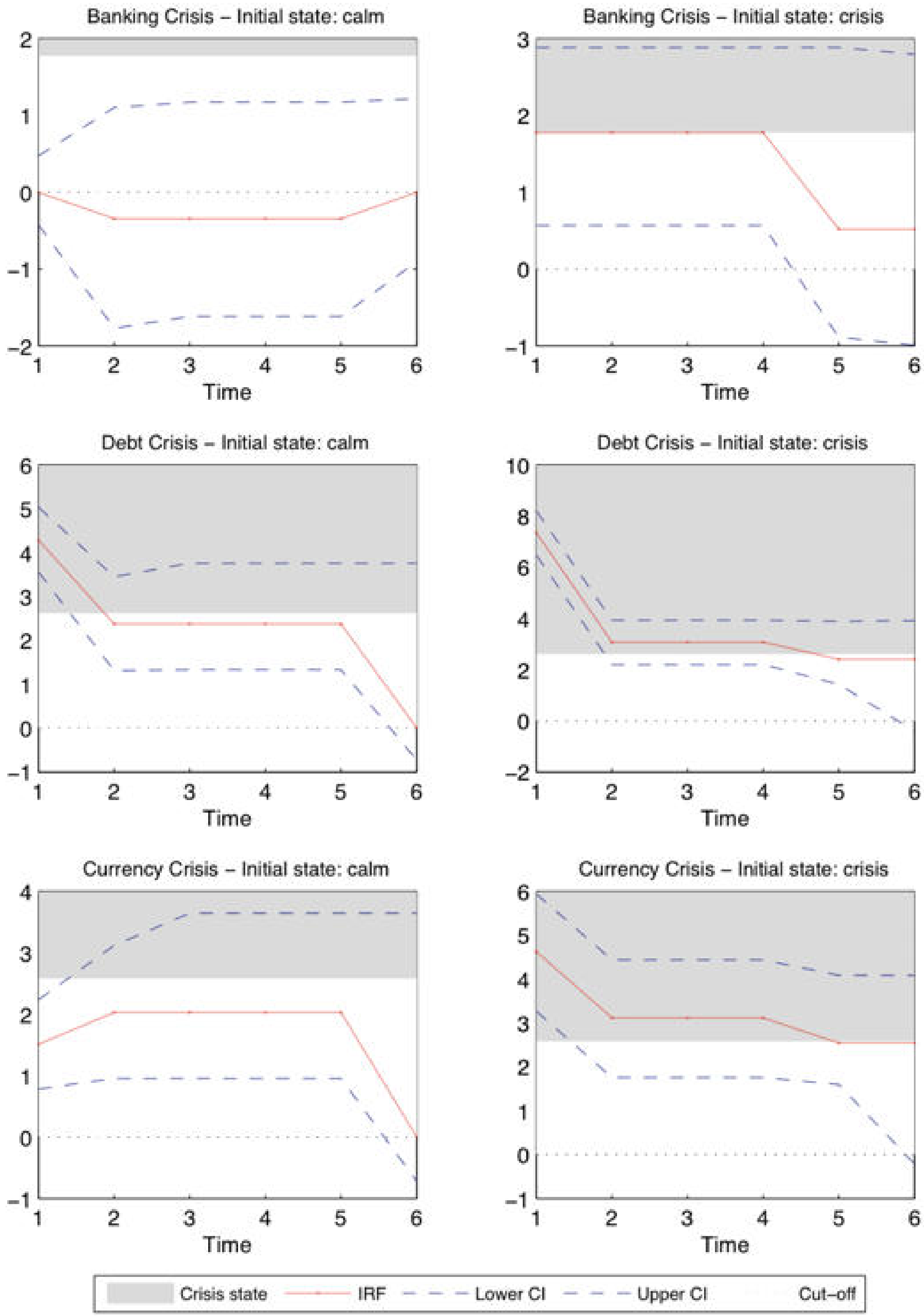

FIGURE 3 – IRF after a debt crisis shock - Ecuador 3 months

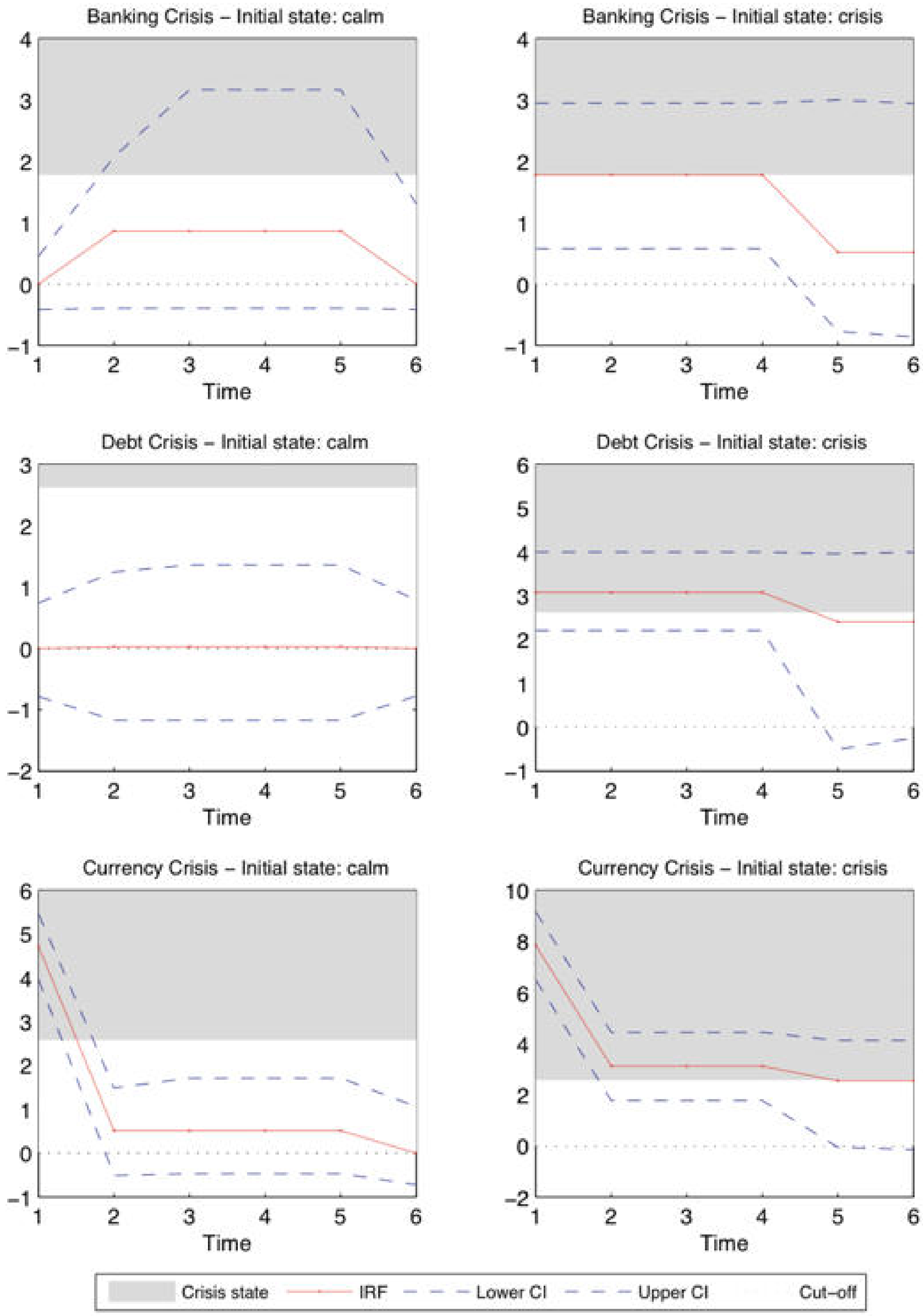

FIGURE 4 – IRF after a currency crisis shock - Ecuador 3 months

TABLE 1 – Database

Country	Bivariate model	Trivariate model
Argentina	February 1988 - May 2010	December 1997 - May 2010
Brazil	September 1990 - May 2010	December 1997 - May 2010
Chile	January 1989 - May 2009	May 1999 - May 2010
Colombia	February 1986 - August 2009	December 1997 - August 2009
Ecuador	January 1994 - November 2007	December 1997 - November 2007
Egypt	February 1986 - June 2009	July 2001 - June 2009
El Salvador	January 1991 - November 2008	April 2002 - November 2008
Indonesia	January 1989 - August 2009	May 2004 - August 2009
Lebanon	January 1989 - April 2010	April 1998 - April 2010
Malaysia	January 1988 - March 2010	December 1997 - March 2010
Mexico	January 1988 - May 2010	December 1997 - May 2010
Peru	January 1990 - May 2010	December 1997 - May 2010
Philippines	January 1995 - February 2008	December 1997 - February 2008
South Africa	January 1988 - August 2009	December 1997 - August 2009
Turkey	January 1988 - May 2010	December 1997 - May 2010
Venezuela	February 1986 - November 2009	December 1997 - November 2009

Note: Data availability.

Table 2 – Percentage of crisis periods

	Bivariate model		Trivariate model		
	Currency crisis	Banking crisis	Currency crisis	Banking crisis	Debt crisis
Argentina	**5.13**	**8.90**	4.00	**6.67**	**10.0**
Brazil	3.77	**7.19**	0.00	3.33	2.67
Chile	**6.07**	**10.0**	**5.79**	**5.79**	3.31
Colombia	4.95	**9.90**	**9.22**	**12.8**	0.00
Ecuador	**5.73**	**9.93**	**6.67**	**10.8**	**6.67**
Egypt	**6.76**	**9.96**	4.17	**7.30**	**7.30**
El Salvador	3.65	**9.85**	0.00	0.00	2.50
Indonesia	5.30	**9.90**	0.00	**14.0**	**6.25**
Lebanon	**9.62**	**9.96**	1.38	**8.97**	2.76
Malaysia	3.10	**10.0**	4.05	**6.08**	4.73
Mexico	**6.50**	**9.93**	0.00	**9.33**	0.00
Panama	0.00	**9.89**	0.00	**6.38**	0.00
Peru	4.45	**8.22**	0.00	**10.7**	0.00
Phillipines	4.90	**9.80**	**5.69**	**6.50**	3.25
South Africa	**6.71**	**9.89**	**7.09**	**7.80**	4.26
Turkey	4.80	**8.56**	4.00	**6.67**	0.00
Venezuela	**7.33**	**10.1**	4.17	**7.64**	2.78

Note: A percentage of crisis superior to 5% is represented in bold.

Table 3 – Bivariate Analysis

Country		3 months θ	3 months Ω	6 months θ	6 months Ω	12 months θ	12 months Ω
Argentina	currency banking	$\begin{bmatrix} + & + \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} . & + \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$
Chile	currency banking	$\begin{bmatrix} + & . \\ . & . \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & . \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} . & - \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$
Ecuador	currency banking	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$
Egypt	currency banking	$\begin{bmatrix} + & . \\ - & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ - & + \end{bmatrix}$	$\begin{bmatrix} 1 & - \\ - & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & - \\ - & 1 \end{bmatrix}$
Lebanon	currency banking	$\begin{bmatrix} + & . \\ - & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$
Mexico	currency banking	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & . \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & . \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$
South Africa	currency banking	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & . \\ . & 1 \end{bmatrix}$
Venezuela	currency banking	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} + & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$	$\begin{bmatrix} . & . \\ . & + \end{bmatrix}$	$\begin{bmatrix} 1 & + \\ + & 1 \end{bmatrix}$

Note: Three different lags of the dependent variable are used, namely 3, 6 and 12 months. 'θ' stands for the parameters of the lagged crisis variables, while Ω represents the covariance matrix. A'+'/'-' sign means that the coefficient is significant and positive/ negative, while a '.' indicates its non-significance. For example, in the case of Argentina, 3 months, all the parameters are positive and significative except for the impact of a currency crisis on the probability of occurrence of banking crises. Similarly, the correlation coefficient between currency and banking crises is significative.

Table 4 – Trivariate Analysis

Country		3 months θ	3 months Ω	6 months θ	6 months Ω
Ecuador	currency	. . +	1 . .	. + +	1 . .
	banking	. + .	. 1 .	. + .	. 1 .
	sovereign	. . +	. . 1	. . +	. . 1
South Africa	currency	+ . .	1 . +	+ . .	1 . +
	banking	. . .	. 1 .	. . .	. 1 .
	sovereign	. . +	+ . 1	. . +	+ . 1

Note: Two different lags of the dependent variable are used, namely 3 and 6 months. 'θ' stands for the parameters of the lagged crisis variables, while Ω represents the variance-covariance matrix. A'+'/'-' sign means that the coefficient is significant and positive/ negative, while a '.' indicates its non-significance. For example, in the case of Ecuador, 3 months, sovereign debt crises have a positive and significative impact on the probability of occurrence of currency crises.