Testing for the degree of commitment via set-identification

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Motivation

Kydland and Prescott (1977):

Time inconsistency of commitment-based optimal plans. Discretionary policy is time-consistent but sub-optimal.

Barro and Gordon (1983):

In a discretionary regime the inflation rate on average exceeds the level that would be optimal if commitment to history dependent policy rules was feasible.

Svensson (1997):

Lack of commitment implies a stabilization bias, because the monetary authority sets policy independently of the history of shocks. Thus, the joint path of inflation and output are different across policy regimes.

We describe a method for estimating and testing a model of optimal monetary policy that does not require a explicit choice of equilibrium concept.

The procedure considers a general specification of optimal monetary policy, nesting the commitment and the discretion characterizations as two special cases.

The approach is based on the derivation of bounds for inflation that are consistent with both forms of optimal policy and allow for partial identification of the economy's structural parameters.

We derive testable implications that allow for specification tests and discrimination between the monetary authority's modes of behavior.

We use the welfare function and the first order conditions for optimal policy, in the context of the linear-quadratic New-Keynesian model.

The first order conditions yield a general model that takes the form

$$\mathbb{E}\left[m\left(\theta\right)\right] \geq 0,$$

allowing for partial identification of θ, i.e., the identified set.

We estimate the identified set implied by optimal monetary policy and construct confidence regions covering the identified set with some pre-specified probability (Chernozhukov, Hong, and Tamer, 2007).

If the identified set is non-empty, we cannot reject the null of optimal policy. Next, we discriminate between commitment and discretion.

If the identified set is non-empty, we test if the moment restrictions implied by a specific policy regime are satisfied.

This requires a two-step procedure:

1. Under policy regime 1, θ^1 is identified. We estimate θ^1 by GMM.

2. With θ^1 in hand, we test the model:

$$\mathbb{E}\left[m\left(\theta\right)\right] \geq 0 \rightarrow \begin{cases} \mathbb{E}\left[m^1\left(\theta^1\right)\right] & = & 0 \\ \mathbb{E}\left[m^2\left(\theta^1\right)\right] & \geq & 0 \end{cases}$$

   If the model is rejected, policy regime 1 is rejected.

The method relies on Andrews and Soares (2010) GMS method.

Related Literature

Empirical Evaluation of Optimal Monetary Policy:

→ Baxter (1988), Ireland (1999), Givens (2010), Woodford and Gianonni (2010).

Partial Identification in Economics:

→ Manski and Tamer (2002), Haile and Tamer (2003). Blundell, Browning and Crawford (2008), Ciliberto and Tamer (2009), Moon and Schorfheide (2009).

OUTLINE

1. THE MODEL
2. BOUNDS FOR INFLATION
3. SET IDENTIFICATION
4. DISCRETION VS COMMITMENT
5. EMPIRICAL APPLICATION

The Model

The framework is that of the New-keynesian forward-looking model with monopolistic competition and Calvo price-setting

The welfare relevant output gap corresponds to the difference between the log of output and the log of natural output (fully flexible prices)

$$\begin{aligned} x_t &\equiv \left(\widehat{Y}_t - \widehat{Y}_t^n\right) \\ &= \left(\omega + \sigma^{-1}\right)^{-1} \widehat{s}_t, \end{aligned}$$

where $\widehat{s}_t$ is the aggregate real marginal cost.

The joint behavior of inflation π_t and the welfare relevant output gap is governed by the New-keynesian Phillips Curve (NKPC)

$$\pi_t = \beta E_t \left(\pi_{t+1}\right) + \kappa x_t,$$

and $(\beta, \omega, \sigma, \kappa)$ is a vector of structural parameters.

A quadratic approximation around the zero inflation equilibrium of the representative agent life-time utility is given by

$$\mathcal{W} = E_0 \sum_{t=0}^{\infty} -\frac{\beta^t}{2} \left\{ \pi_t^2 + \frac{\kappa}{\vartheta} \left[x_t + \left(\omega + \sigma^{-1} \right)^{-1} \delta_t \right]^2 \right\}.$$

where $\delta_t \leq 0$, is a composite exogenous disturbance representing time varying markups and tax-wedges.

Under commitment, the conditions that solve the monetary authority problem at some given period s are

$$\begin{aligned}
\frac{\kappa}{\vartheta} \left[x_t + \left(\omega + \sigma^{-1} \right)^{-1} \delta_t \right] + \kappa \lambda_t &= 0, \qquad t = s, s+1, \ldots \\
\pi_t + \lambda_{t-1} - \lambda_t &= 0, \qquad t = s, s+1, \ldots \\
\beta E_t \pi_{t+1} &= \pi_t - \kappa x_t,
\end{aligned}$$

where λ_t is the Lagrangian multiplier associated with the NKPC.

Assuming that the system has been initialized in period $s = 0$ and that $\lambda_{-1} = 0$, the commitment optimality conditions is

$$\pi_t^{(c)} = -\frac{1}{\vartheta}\left[x_t + \left(\omega + \sigma^{-1}\right)^{-1}\delta_t\right] + \frac{1}{\vartheta}\left[x_{t-1} + \left(\omega + \sigma^{-1}\right)^{-1}\delta_{t-1}\right].$$

But the commitment solution is time inconsistent: in each period t, the monetary authority is tempted to behave as if $\lambda_{t-1} = 0$, and ignores the impact of its current action on the private sector past decisions.

Thus, under discretion each period t, the policy maker ignores the past, and behaves as if $\lambda_{t-1} = 0$, yielding

$$\pi_t^{(d)} = -\frac{1}{\vartheta}\left[x_t + \left(\omega + \sigma^{-1}\right)^{-1}\delta_t\right].$$

We, therefore, have two alternative characterizations of the joint path of inflation and the output gap

$$\pi_t^{(c)} = -\phi^{-1}\left(\widehat{s}_t + \delta_t\right) + \phi^{-1}\left(\widehat{s}_{t-1} + \delta_{t-1}\right)$$

$$\pi_t^{(d)} = -\phi^{-1}\left(\widehat{s}_t + \delta_t\right),$$

where $\phi = \left(\omega + \sigma^{-1}\right)\vartheta > 0$, and $\delta_t \leq 0$.

How to lay down a general specification nesting these two special cases?

We derive bounds for inflation that exploit the state contingent inflation bias under discretion (Svensson's stabilization bias).

Bounds For Inflation

We make use of the following simple Lemma

Lemma

Let $\delta_t \leq 0$ for all t almost surely.

It follows that, $\Pr\left(\pi_t^{(d)} \geq \pi_t^{(c)} \middle| \widehat{s}_{t-1} \leq 0\right) = 1$*.*

Optimal monetary policy implies $\Pr\left(\pi_t^{(c)} \leq \pi_t \leq \pi_t^{(d)} \middle| \widehat{s}_{t-1} \leq 0\right) = 1$*.*

Furthermore, let measured inflation Π_t, be given by

$$\Pi_t = \pi_t + \nu_t,$$

with ν_t a measurement error. From Lemma 1 it follows that

$$\Pr\left(\pi_t^{(c)} + v_t \leq \Pi_t \leq \pi_t^{(d)} + v_t \middle| \widehat{s}_{t-1} \leq 0\right) = 1.$$

These inflation bounds yield a set of moment inequality conditions implied by optimal monetary policy

Proposition

Let observed inflation Π_t be given by the actual rate of inflation plus a measurement error v_t. Then the following moment inequalities

$$-\mathbb{E}\left[\left(\phi\Pi_t + \widehat{s}_t + \delta_t\right) 1\left(\widehat{s}_{t-1} \leq 0\right)\right] \geq 0,$$

$$\mathbb{E}\left[\left(\phi\Pi_t + \Delta\widehat{s}_t + \Delta\delta_t\right) 1\left(\widehat{s}_{t-1} \leq 0\right)\right] \geq 0,$$

are consistent with optimal monetary policy under both commitment and discretion.

This general specification nests commitment and discretion as two special cases, but also a continuum of levels of *soft-commitment*.

We define the following set of instruments

1. Z_t is strictly positive;

2. $\mathbb{E}\left[v_t\, 1\left(\widehat{s}_{t-1} \leq 0\right)\, Z_t\right] = 0$;

3. $\mathbb{E}\left[\delta_{t-r}\, 1\left(\widehat{s}_{t-1} \leq 0\right) Z_t\right] = \overline{\delta}\, \mathbb{E}\left[Z_t\right]$, for $r = 0, 1$.

The list of moment inequalities in Proposition 1 can be extended

$$-\mathbb{E}\Big[\,\big((\phi\Pi_t+\widehat{s}_t)\,1\,(\widehat{s}_{t-1}\leq 0)+\overline{\delta}\big)\,Z_t\Big] \quad \geq \quad 0, \tag{1}$$

$$\mathbb{E}\Big[\,(\phi\Pi_t+\Delta\widehat{s}_t)\,1\,(\widehat{s}_{t-1}\leq 0)\,Z_t\Big] \quad \geq \quad 0. \tag{2}$$

The set of values ϕ, for which the inequality condition (1) is satisfied increases linearly in $-\overline{\delta}$.

Heuristically, the higher the level of distortions, the higher the level of inflation under discretion and, hence, the larger the range of inflation rates consistent with optimal monetary policy.

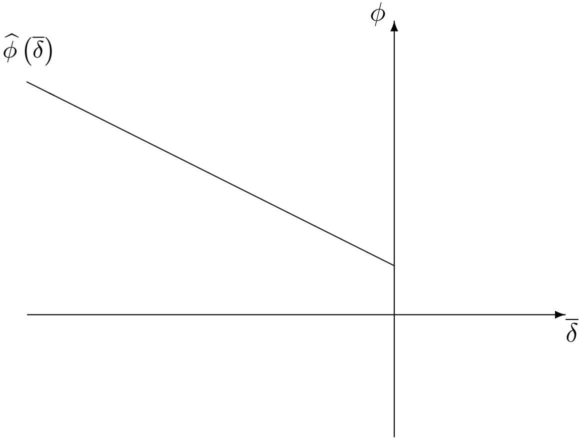


Figure: Discretion inequality condition in the $\overline{\delta}$ and ϕ space

Set Identification

The parameter vector $\theta = \left(\phi,\ \overline{\delta}\right)$ is partially identified.

Definition

Let $\theta = \left(\phi,\ \overline{\delta}\right) \in \Theta = \mathbb{R}^+ \times \mathbb{R}^-$. The identified set under optimal monetary policy is given by

$$\Theta^I = \{\theta \in \Theta : \text{ inequalities (1) and (2) hold}\}.$$

Our first objective is to construct an estimator of Θ^I.

If the latter is not empty, we construct a confidence region, at a given level.

If the identified set is non-empty, we have evidence that the monetary authority is implementing optimal monetary policy.

Define the $2p$ vector of moment functions associated with (1) and (2)

$$m_t(\theta) = \left[\; m^1_{d,t}(\theta) \;\ldots\; m^p_{d,t}(\theta)\;, m^1_{c,t}(\theta) \;\ldots\; m^p_{c,t}(\theta) \;\right],$$

with sample means $m_T(\theta)$ and with $\widehat{V}_T(\theta)$ the HAC estimator of the asymptotic variance of $\sqrt{T} m_T(\theta)$.

Following CHT (2007), we define the criterion function

$$Q_T(\theta) = \sum_{i=1}^{2p} \frac{\left[m^i_T(\theta)\right]^2_-}{\hat{v}^{i,i}_T(\theta)},$$

where $[x]_- = x\,1(x \leq 0)$. An estimator of the identified set is

$$\widehat{\Theta}^I_T = \left\{\theta \in \Theta \text{ s.t. } TQ_T(\theta) \leq d^2_T\right\}$$

where $d_T \propto \sqrt{\ln T}$.

To conduct inference in the moment inequality model, we construct a set $C_T^{1-\alpha}$, asymptotically containing Θ^I with probability at least $1-\alpha$. This constitutes the confidence region.

The $(1-\alpha)$ confidence region for the identified set, $C_T^{1-\alpha}$, satisfies

$$\lim_{T\to\infty} \inf P\left(\Theta^I \subseteq C_T^{1-\alpha}\right) \geq 1-\alpha,$$

where

$$C_T^{1-\alpha} = \left\{\theta \in \Theta : TQ_T(\theta) \leq c_{\alpha,T}\right\},$$

and $c_{\alpha,T}$ is the $(1-\alpha)$-percentile of the distribution of $\sup_{\theta\in\Theta^I} TQ_T(\theta)$.

To compute the critical value $c_{\alpha,T}$ of the distribution of $\sup_{\theta\in\Theta^I} TQ_T(\theta)$, we replace the unknown set Θ^I by its consistent estimator $\widehat{\Theta}^I_T$, as shown above, and we use bootstrap critical values.

1. For each $\theta \in \widehat{\Theta}^I_T$, construct $TQ^*_T(\theta)$ from block-bootstrapped data
2. Compute $\sup_{\theta\in\widehat{\Theta}^I} TQ^*_T(\theta)$
3. Repeat B times and construct the $(1-\alpha)$-percentile $c^*_{\alpha,T,B}$
4. The estimated confidence region is

$$\widehat{C}_T^{1-\alpha} = \left\{\theta \in \Theta : TQ_T(\theta) \leq c^*_{\alpha,T,B}\right\}$$

In the bootstrap procedure we use the Generalized Moment Selection (GMS) procedure introduced by Andrews and Soares (2010)

$$TQ_T^*(\theta) = \sum_{i=1}^{2p} \left(\sqrt{T} \left[\frac{m_{i,T}^*(\theta) - m_{i,T}(\theta)}{\sqrt{\widehat{v}_{i,i}^*(\theta)}} \right]_- \right)^2 \times$$
$$1 \left[m_{i,T}(\theta) \leq \sqrt{\widehat{v}_{i,i}(\theta)} \sqrt{2 \ln \ln T / T} \right]$$

→ The indicator function uses information about the slackness of the sample moment conditions to infer which population moment conditions are binding, and thus enter into the limiting distribution.

Discrete vs Commitment

We want to discriminate between two alternative equilibrium concepts, maintaining the assumption of optimal monetary policy.

→ Heuristically, this implies testing whether there is a θ in the identified set, for which the moment inequality conditions that are associated with the specific regime (discretion or commitment) hold as equalities.

The test is a two step procedure, requiring:

1. Estimation of the structural parameter under the null.
2. Construction of the test statistic using the criterion function approach.

Under discretion we formulate the model of optimal monetary policy as

$$\mathrm{E}\Big[\left(\left(\phi\Pi_t + \widehat{s}_t\right) 1\left(\widehat{s}_{t-1} \leq 0\right) + \overline{\delta}\right) Z_t\Big] \;\; = \;\; 0$$

$$\mathrm{E}\Big[\left(\phi\Pi_t + \Delta\widehat{s}_t\right) 1\left(\widehat{s}_{t-1} \leq 0\right) Z_t\Big] \;\; \geq \;\; 0,$$

We construct a test statistic for the following null hypothesis

$$H_0^d : \; \theta \in \Theta_d^I$$

against the alternative

$$H_1^d : \; \theta \notin \Theta_d^I$$

The test statistic is based on the criterion function

$$TQ_T^d\left(\theta^d\right) = T\left(\sum_{i=1}^{p}\frac{m_{d,T}^{i}\left(\theta^d\right)^2}{\hat{v}_T^{i,i}\left(\theta^d\right)} + \sum_{i=p+1}^{2p}\frac{\left[m_{c,T}^{i}\left(\theta^d\right)\right]_-^2}{\hat{v}_T^{i,i}\left(\theta^d\right)}\right).$$

To construct the test statistic, we first estimate $\widehat{\theta}_T^d$ by two-step GMM under discretion and then evaluate the criterion function at $\widehat{\theta}_T^d$.

Proposition

Under some regularity conditions

(i) under H_0^d, $\lim_{T,B\to\infty}\Pr\left(TQ_T^d\left(\widehat{\theta}_T^d\right) > c_{T,B,\alpha}^{*d}\right) = \alpha$

(ii) under H_1^d, $\lim_{T,B\to\infty}\Pr\left(TQ_T^d\left(\widehat{\theta}_T^d\right) > c_{T,B,\alpha}^{*d}\right) = 1.$

Testing commitment

Under commitment we formulate the model of optimal monetary policy as

$$-\mathrm{E}\Big[\left(\left(\phi\Pi_t + \widehat{s}_t\right) 1\left(\widehat{s}_{t-1} \leq 0\right) + \overline{\delta}\right) Z_t\Big] \quad \geq \quad 0$$

$$\mathrm{E}\Big[\left(\phi\Pi_t + \Delta\widehat{s}_t\right) 1\left(\widehat{s}_{t-1} \leq 0\right) Z_t\Big] \quad = \quad 0,$$

We construct a test statistic for the following null hypothesis

$$H_0^c : \; \exists\, \overline{\delta} \text{ such that } \phi \in \Theta_c^I\left(\overline{\delta}\right)$$

against the alternative

$$H_1^c : \; \nexists\, \overline{\delta} \text{ such that } \phi \in \Theta_c^I\left(\overline{\delta}\right)$$

For each $\overline{\delta}$, we construct the test statistic

$$TQ_T^c\left(\widehat{\phi}^c, \overline{\delta}\right) = T\left(\sum_{i=1}^{p} \frac{\left[m_{d,T}^i\left(\widehat{\phi}^c, \overline{\delta}\right)\right]_-^2}{\hat{v}_T^{i,i}\left(\widehat{\phi}^c, \overline{\delta}\right)} + \sum_{i=p+1}^{2p} \frac{m_{c,T}^i\left(\widehat{\phi}^c, \overline{\delta}\right)^2}{\hat{v}_T^{i,i}\left(\widehat{\phi}^c, \overline{\delta}\right)}\right),$$

where $\widehat{\phi}^c$ is the GMM estimator and $\overline{\delta}$ needs to be fixed.

Proposition

Under some regularity conditions

(i) *under* H_0^c, $\lim_{T,B\to\infty} \Pr\left(TQ_T^c\left(\widehat{\phi}^c, \overline{\delta}\right) > c_{T,B,\alpha}^{*c}\left(\overline{\delta}\right)\right) = \alpha$

(ii) *under* H_1^c, $\lim_{T,B\to\infty} \Pr\left(TQ_T^c\left(\widehat{\phi}^c, \overline{\delta}\right) > c_{T,B,\alpha}^{*c}\left(\overline{\delta}\right)\right) = 1.$

Empirical Application

We apply our methodology to quarterly time-series for the US economy.

Following Sbordone (1998) and Gali and Gertler (1999) we exploit the relationship between the theoretical output gap and real marginal cost, $\widehat{s}_t$, measured using the HP filtered unit labor cost (labor share).

Our measure of inflation is the percentage change in the GDP deflator.

The econometric framework developed in this paper relies on stationarity assumptions. Hence, the sample chosen spans 1983:q1–2008:q3.

This is consistent with the analysis in Clarida, Gali and Gertler (2000).

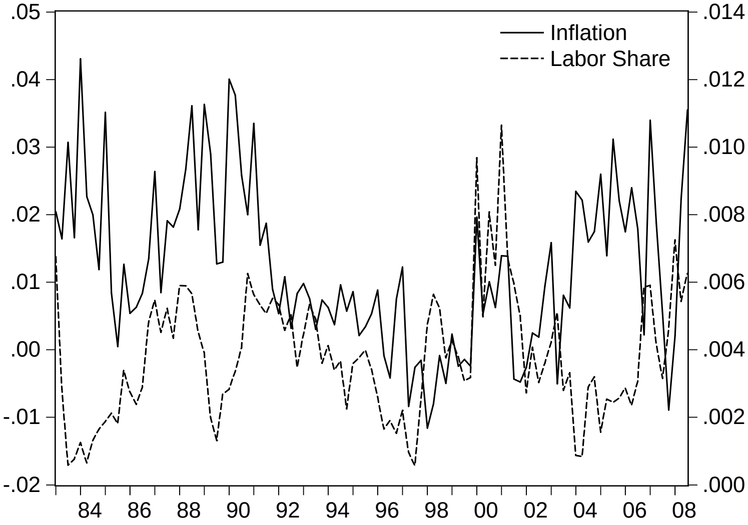
Table: Unit Root Tests For Inflation

	ADF test t-Statistic	Phillips–Perron test t-Statistic
1960:2 – 2008:3	−2.180 (0.18)	−2.933 (0.04)
1970:1 – 2008:3	−2.428 (0.14)	−2.652 (0.08)
1979:3 – 2008:3	−2.902 (0.05)	−3.482 (0.01)
1983:1 – 2008:3	−5.313 (0.00)	−5.142 (0.00)

p-value in parentheses.

Note: The ADF statistic is computed using the Schwarz information criteria to select the lag length. The Phillips–Peron statistic, is computed using Andrews' (1991) method to select the value for the lag truncation parameter q required to form the HAC estimator. A constant is included in both test regressions.

Figure: Labor Share and Inflation in the US, 1983:1–2008:3.



Instrumental variables

We use as instrument the variable *'Military Spending'*, given by the HP filtered log of real government expenditure in national defense.

As second instrument, we use the variable *'Oil Price Change'*, given by the log difference of the spot oil price.

The instrumental variables are adjusted using monotone transformations that guarantee positiveness: $Z_+ = Z - min(Z)$.

With the unit vector, the complete instrument list $p = 3$ instruments and 6 moment conditions overall.

It is possible to "test" the exclusion restriction.

Assume the representative agent has preferences

$$\frac{C_t^{1-1/\sigma}}{1-1/\sigma} - \int_0^1 \exp(\epsilon_t) \frac{H(j)^{1+\gamma}}{1+\gamma} dj,$$

where ϵ is a preference shifter

The labor wedge is approximately given by

$$\begin{aligned} \delta_t &= \sigma^{-1} c_t + \gamma h_t - w_t - \mu + \epsilon_t \\ &= \tilde{\delta} + \epsilon_t, \end{aligned}$$

we can proxy the labor wedge assuming log-log preferences and taking the cyclical component of $\tilde{\delta}$.

Figure: The Labor Wedge

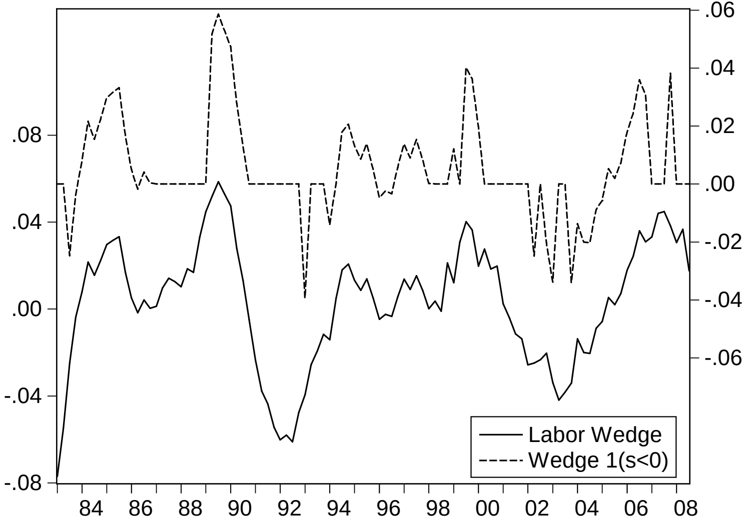


Table: Exclusion Restrictions

	Coefficient
Military Spending	-0.091
	(0.33)
Oil Price Change	-0.001
	(0.94)
R^2	0.01
F-stat	0.72
	(0.49)

p-value in parentheses.

Note: The dependent variable is $\widehat{\delta}_t 1\left[\widehat{s}_t < 0\right]$, and the regression includes a constant term (not reported).

Finally, concerning the instruments' relevance, we note that by regressing the real marginal cost $\widehat{s}$ on the instrumental variable *'Military Spending'*, we obtain an F statistic equal to 13.81.

If the variable *'Oil Price Change'* is also included, the F statistic is 7.01.

Thus the instruments are not too weak.

Moreover, the partial identification may help when instruments are weak as noticed by Moon and Schorfheide (2009).

Model specification tests

Table: Model Specification Tests

	Discretion	Commitment
$J-$test	1.178	0.735
p-val	(0.278)	(0.391)
TQ_T	13.417	26.963
p-val*	(0.164)	(0.022)

Note: The upper panel shows results from the Hansen test of overidentifying restriction based on the GMM model under discretion or commitment. The lower panel shows the results from the model specification tests based on the CHT criteria function.

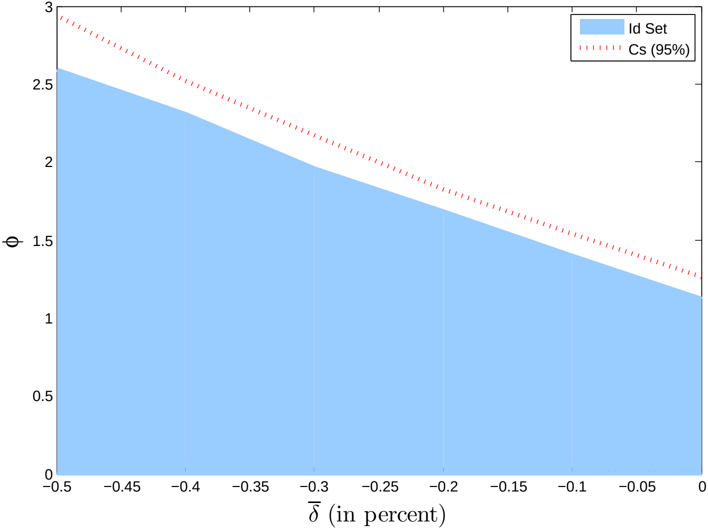


Figure: Identified Set and Confidence Set under Optimal Monetary Policy

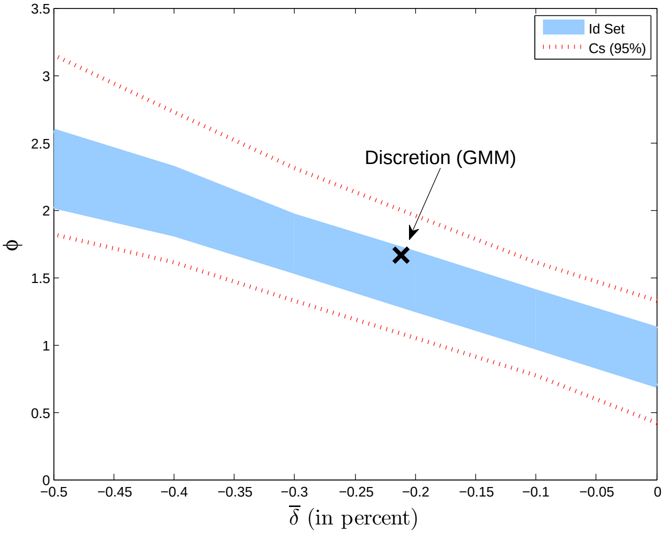


Figure: Identified Set and Confidence Sets under Discretion

Structural parameters estimated sets

Parameter	Definition	Value
α	Share of firms keeping prices fixed	0.6600
β	Discount factor	0.9914
ω	Output elasticity of real marginal cost	0.4400
ϑ	Price elasticity of demand	7.6600

There is a close connection between the parameters ϕ and the marginal cost elasticity of inflation, κ

$$\kappa = \left[\frac{(1-\alpha)(1-\alpha\beta)}{\alpha\vartheta\,(1+\omega\vartheta)}\right]\phi$$

By setting values for the parameters $(\alpha, \beta, \vartheta, \omega)$, the confidence region for ϕ implies a corresponding confidence region for,κ.

Table: Parameter Estimates and Confidence Regions

$100\times$ $\overline{\delta}$	Interval estimate for ϕ	Bootstrap 95% c.i.
0.000	$[0.70, 1.12]$	$[0.42, 1.33]$
-0.210[a]	$[1.26, 1.68]$	$[1.05, 1.96]$
-0.500	$[2.03, 2.59]$	$[1.82, 3.08]$
$100\times$ $\overline{\delta}$	Implied interval estimate for κ	Bootstrap 95% c.i.
0.000	$[0.0035, 0.0057]$	$[0.0021, 0.0067]$
-0.210[a]	$[0.0064, 0.0085]$	$[0.0053, 0.0099]$
-0.500	$[0.0103, 0.0131]$	$[0.0092, 0.0156]$

[a] GMM estimate for $\overline{\delta}$.

Conclusion

The paper establishes the following results:

(i) the estimated identified set is non-empty and, therefore, the time series of inflation and output gap are consistent with optimal monetary policy;

(ii) the estimated identified set under discretion is non-empty and contains the parameter vector estimated by GMM under discretion. Our test statistic does not reject discretion.

(iii) the estimated identified set under commitment is empty. Our test statistic rejects commitment.

(iv) for a given duration of price stickiness the estimated sets point to the existence of large strategic complementarities in price-setting, often interpreted as a high degree of real rigidity (Ball and Romer 1990).