A bargaining theory of trade invoicing

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Abstract

We develop a theoretical model of international trade pricing where the price and exposure to exchange rate fluctuations are set through a bargaining between individual exporters and importers. We find that the choice of price and invoicing currency reflect the market structure, such as the extent of fragmentation and heterogeneity on both sides of the market. A situation where importers have a low effective bargaining weight is characterized by a high price and a limited exposure of importers to exchange rate movements. The impact of the market structure is not limited to the outcome for specific exporter-importer pairs but also affects aggregate variables such as the average price and invoicing, and the correlation between invoicing and the value of transactions.

VERY PRELIMINARY AND INCOMPLETE

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1 Introduction

What determines the currency used in the invoicing of international trade? This question is the subject of an extensive research agenda in international economics, including both empirical and theoretical contributions, as it plays a central role in determining who between exporters and importers bears the cost of exchange rate fluctuations.

This paper addresses a substantial shortcoming of the existing theoretical literature, specifically that the invoicing currency is chosen unilaterally by exporters taking the downwards sloping demand they face from importers into account. The theoretical literature has identified a host of determinants, including the "coalescing" motive leading exporters to limit the fluctuations of their prices relative to their competitors, the "hedging" motive favoring a currency that brings marginal revenue in line with marginal costs, the transaction cost in foreign exchange markets, and the role of macroeconomic conditions such as exchange rate pegs to name a few.[1] The existing contributions however consider that the choice lies solely with the exporter, an assumption that is at odds with growing evidence that the invoicing choice reflects a bargaining between exporters and importers (see Friberg and Wilander 2008).

We develop a simple model of bargaining between exporter and importers. A range of exporters produce goods at a cost and sell them to a range of importers who resell them in the domestic market. Each exporter - importer pair bargains over the pricing which consists of a preset component and a degree of exposure to exchange rate movements between the time of invoicing choice and the time of the actual transaction. In equilibrium all exporters transact with all importers. The payoff for an agent is the profits made on all transactions that she is part of. The outside option of a party during a specific exporter-importer bargaining is the profits she makes on the transactions with all the other counterparts. A key assumption is that exporters and importers have a concave valuation of profits. This implies that failing to reach an agreement with a large counterpart is more costly, at the margin, than failing to reach one with a smaller party.

We consider a Nash bargaining outcome where the weight put on the importer's surplus, referred to as the formal bargaining weight, is the same

[1] See for instance Bacchetta and van Wincoop (2005), Devereux, Engel and Storgaard (2004), and Goldberg and Tille (2008).

for all exporter-importer pairs. We derive a measure of effective bargaining weight that reflects the share of the exporter and importer in each other's profits and the concavity of their valuations of payoffs. The importer has a larger weight when she accounts for a large share of the exporter's sales, the exporter accounts for a small share of the importer's purchases, the importer's valuation of payoffs is not too concave, and the exporter's valuation is more so. This effective bargaining weight directly translates into the preset level of the price and the exposure to the exchange rate. Specifically, an importer with a higher effective weight secures a lower preset price, thus shifting the surplus of the match her way. This lowers her marginal valuation of payoffs, implying that she cares relatively little about exchange rate volatility and thus accepts a higher exchange rate exposure.

We illustrate our results with two numerical examples. We first focus on the number of exporter and importers, assuming homogeneity within each group, and show that a higher degree of fragmentation among importers reduces their effective bargaining weight, raises the preset component of the price and limits their exchange rate exposure. We then focus on heterogeneity between exporters and between importers by considering the interaction between two exporters of different sizes and two importers of different sizes. We show that the extent of heterogeneity on the two sides of the markets affects the effective weights, preset price and exposure to exchange rates across all the four exporter-importer pairs. The effect is not limited to specific pairs and also applies to aggregate variables, such as the average price and exposure as well as their cross sectional dispersion. We find that higher exporter (importer) heterogeneity raises (lowers) the average preset price and the average exporter's exchange rate exposure, and leads to a positive (negative) correlation between transaction value and use of the importer's currency in the invoicing. The impact is robust to alternative parametrizations of the model.

Our work fits in the literature on invoicing choice and price adjustments in international economics. Goldberg and Tille (2009) consider a highly disaggregated data set of Canadian imports and finds a robust link between the size of individual transactions and invoicing choice. Gopinath, Itskhoki, and Rigobon (2010) show that invoicing choice is closely related to the pass-through of cost fluctuations into final prices in the United States, which much higher pass-through for import prices set in currencies other than the dollar. As our model assumes that prices are fully preset, up to the exchange rate exposure, we cannot consider the relation between the choice of invoicing and the response of prices to cost fluctuations. Our assumption is motivated by our focus on invoicing in a novel pricing framework, and extending it to include price adjustment is left for future work. The relevance of market structure for invoicing is likely to extend to price adjustment as well.

Our paper is also related to the industrial organization literature on bargaining games between suppliers and retailers. To our knowledge, this paper is the first contribution introducing bargaining between exporters and importers with concave valuation of payoffs in an uncertain environment. DeGraba (2005) considers a model where the valuation of goods varies across buyers Sellers make price offers that the buyers can accept or refuse. As the seller cannot observe the true valuation of her counterpart, she has an incentive to offer better conditions to larger buyers as loosing a large customer is more costly than loosing a larger one. While the model includes uncertainty, it does so in the form of idiosyncratic valuations and thus abstract from aggregate risk such as exchange rate movements. Normann and al. (2003) consider a framework where a seller with increasing marginal costs of production makes take it or leave it offers to sellers. The seller offers a lower price to large buyers as large sales take place at a point on the curve schedule where the marginal cost his low. The setting however does not include uncertainty. Inderst and Wey (2001) develop a framework where prices are set between two retailers and two producers, and focuses on the incentives for horizontal mergers. They however assume that all agents share the same marginal valuation of resources, in contrast to our setting of concave valuations that can differ between agents. Similarly, Chipty and Snyder (1999), who focus on the incentives for mergers among buyers and seller, assume that buyers and sellers share the same marginal valuation of the price between them. Horn and Wolinsky (1988) analyze a setting with two buyers and two sellers where the marginal valuation of the price can differ between buyers and sellers, and focus on the incentives of agents to merge and form a monopoly, showing that this is not necessarily an optimal choice because of the impact on bargaining power. The model however considers that each buyer only buys from one seller, and thus abstract from the ability of buyers to play one seller against another to gain a better bargaining position. Dowbson and Waterson (1997) consider a larger number of identical buyers, but abstract from uncertainty and heterogeneity in payoffs' valuation.

Our setting differs from the existing contributions in the industrial organization literature in that we consider uncertainty, allow for the bargaining weight of different agents to vary through their market share, and consider concave valuation of payoffs that can differ across agents.

The paper is organized as follows. We present the main features of the model in Section 2, and the solution method in Section 3. Section 4 presents the solution in a case focusing on the numbers of exporters and importers, assuming intra-group heterogeneity. Section 5 focuses on intra-group heterogeneity. Section 6 concludes. Throughout the paper we focus on an intuitive presentation of the main points. The key technical aspects are presented in the appendix, and the detailed derivations are in a technical appendix available on request.

2 An exporter-importer bargaining model

2.1 Structure and payoffs

There are two types of agents in the model: importers and exporters. There are B importers indexed by b, and X exporters indexed by x. Exporters sell goods to importers, who in turn resell them to customers in the destination country.

Importer b purchases Q_{xb} units of goods from exporter x and resells them at a price Z_b in her currency. The production cost of exporter x, in her currency, is denoted by C_x. Each importer can purchase goods from all exporters, and each exporter can sell goods to all importers. Consider that transactions occur for all exporter-importer pairs (which is the case in equilibrium). The payoff of importer b is a concave valuation of her profits:

$$U_b = \frac{1}{1-\gamma_b} E\left(\sum_{i=1}^{X}\left(Z_b - P_{ib}^b\right) Q_{ib}\right)^{1-\gamma_d} \tag{1}$$

where P_{ib}^b is the price charged by exporter i to importer b, with the b subscript denoting that it is expressed in the importer's currency. The payoff of exporter x is a concave valuation of her profits:

$$U_x = \frac{1}{1-\gamma_x} E\left(\sum_{j=1}^{B}\left(P_{xj}^x - C_x\right) Q_{xj}\right)^{1-\gamma_x} \tag{2}$$

where P_{xj}^x is the price charged by exporter x to importer j, with the x subscript denoting that it is expressed in the exporter's currency.

Our analysis focuses on the price charged by exporter x to importer b. It entails two components: a preset price P_{xb}^{fix} that is fixed before shocks are realized, and the extent to which the price in the importer's currency moves with ex-post fluctuations in the exchange rate. Specifically, we denote the percentage of exchange rate movements that are transmitted to the importer's price by $1-\beta_{xb}$, where $\beta_{xb} \in (0,1)$. We interpret β_{xb} as the extent of local currency pricing (LCP). If $\beta_{xb}=1$ the importer is shielded from exchange rate fluctuations, corresponding to full local currency pricing. If $\beta_{xb}=0$ the exporter is shielded from exchange rate fluctuations, a case referred to as producer currency pricing (PCP) in the literature. The exchange rate S is defined as units of exporter's currency per unit of importer's currency, so that an increase corresponds to a depreciation of the exporter's currency. We assume that the log exchange rate s is normally distributed around zero, without loss of generality.

The price paid by the importer in her currency is (lower case letters denote logs):

$$P_{xb}^{b} = \exp\left[p_{xb}^{fix} - (1-\beta_{xb})\, s\right]$$

Similarly the price received by the exporter in her currency is:

$$P_{xb}^{x} = P_{xb}^{b} S^{d} = \exp\left[p_{xb}^{fix} + \beta_{xb} s\right]$$

The price between exporter x and importer b is determined through a bilateral bargaining. A key element of such a setting is the surplus that each counterpart gains from a successful match. In equilibrium there are transactions between all importer-exporter pairs, as such transactions generate a surplus. The surplus generated by the xb transaction is then the value of the importer's (exporter's) payoff from holding transactions with all exporters (importers), minus the payoff from holding transactions with all exporters except x (all importers except b).

Specifically, the surplus that b derives from a match with x is:

$$\begin{aligned} SB^{bx} &= \frac{1}{1-\gamma_b} E\left(\sum_{i=1}^{X}\left(Z_b - \exp\left[p_{ib}^{fix} - (1-\beta_{ib})\, s\right]\right) Q_{ib}\right)^{1-\gamma_b} \\ &\quad - \frac{1}{1-\gamma_b} E\left(\begin{array}{c} \sum_{i=1}^{X}\left(Z_b - \exp\left[p_{ib}^{fix} - (1-\beta_{ib})\, s\right]\right) Q_{ib} \\ -\left(Z_b - \exp\left[p_{xb}^{fix} - (1-\beta_{xb})\, s\right]\right) Q_{xb} \end{array}\right)^{1-\gamma_b} \end{aligned} \quad (3)$$

Similarly the surplus for x is:

$$\begin{aligned} SX^{bx} &= \frac{1}{1-\gamma_x} E\left(\sum_{j=1}^{B}\left(\exp\left[p_{xj}^{fix}+\beta_{xj}s\right]-C_x\right)Q_{xj}\right)^{1-\gamma_x} \\ &\quad -\frac{1}{1-\gamma_x} E\left(\begin{array}{c} \sum_{j=1}^{B}\left(\exp\left[p_{xj}^{fix}+\beta_{xj}s\right]-C_x\right)Q_{xj} \\ -\left(\exp\left[p_{xb}^{fix}+\beta_{xb}s\right]-C_x\right)Q_{xb} \end{array}\right)^{1-\gamma_x} \end{aligned} \tag{4}$$

We allow for the quantity Q_{xb} to be price sensitive. Specifically, it is inversely related to the ratio between the price P_{xb}^{b} and a reference price denoted by R_{xb}. We consider a constant price elasticity of demand ρ. The reference price R_{xb} entails a fixed component and a sensitivity to the exchange rate: $R_{xb} = \exp\left[r_{xb}^{fix} - (1-\eta_{xb})\,s\right]$. The quantity sold from x to b is then written as:

$$Q_{xb} = \exp\left[q_{xb}^{set} - \rho\left(p_{xb}^{fix} - r_{xb}^{fix} + (\beta_{xb} - \eta_{xb})\,s\right)\right] \tag{5}$$

where q_{xb}^{set} is the exogenous component of demand.

The quantity Q_{xb} is produced using a decreasing returns to scale technology using an input L_{xb}. Specifically the technology is $Q_{xb} = A_{xb}\,(L_{xb})^{\lambda}$ where A_{xb} is a constant parameter and $\lambda \leq 1$. We allow for the unit cost of the input to be affected by the exchange rate and denote it by $\exp\left[w_x + \zeta_x s\right]$ where w_x is exogenous and ζ_x is the elasticity of the cost with respect to the exchange rate. Under this specification the marginal cost of production is:

$$MC_{xb} = \frac{1}{\lambda}\exp\left[w_x + \frac{1-\lambda}{\lambda}\left(q_{xb}^{set} - \rho\left(p_{xb}^{fix} - r_{xb}^{fix} + (\beta_{xb} - \eta_{xb})\,s\right)\right) + \zeta_x s - \frac{1}{\lambda}a_{xb}\right] \tag{6}$$

The average cost is $AC_{xb} = \lambda MC_{xb}$.

2.2 Determination of price

The two components of the price between x and b, namely p_{xb}^{fix} and β_{xb}, are set to maximize a combination of the exporter's and importer's surpluses (3) and (4). We consider a Nash bargaining where the combination is a geometric averages of the surpluses, with weights representing the formal bargaining weights of the parties. This approach is standard in the literature, being used by Chipty and Snyder (1999), Dowbson and Waterson (1997), and

Horn and Wolinsky (1988). Specifically we define the joint surplus of a match between x and b as:

$$SJ^{bx} = \left[SB^{bx}\right]^{\delta}\left[SX^{bx}\right]^{1-\delta} \tag{7}$$

where δ captures the formal weight of the importer in the bargaining, and is assumed to be the same for all xb pairs.

The components of the price between x and b are chosen to maximize (7) leading to the two following conditions:

$$0 = \delta\frac{\partial SB^{bx}}{\partial p_{xb}^{fix}}SX^{bx} + (1-\delta)\frac{\partial SX^{bx}}{\partial p_{xb}^{fix}}SB^{bx} \tag{8}$$

$$0 = \delta\frac{\partial SB^{bx}}{\partial \beta_{xb}}SX^{bx} + (1-\delta)\frac{\partial SX^{bx}}{\partial \beta_{xb}}SB^{bx} \tag{9}$$

The exact expression of the various derivatives in (8) and (9) are complex and given in the appendix.

3 Solution method

3.1 Steady state

The first-order conditions (8) and (9) are non-linear functions of the pricing components not only between x and b, but also between x and the other importers, as well as between b and the other exporters. This reflects the fact that the surpluses (3)-(4) are affected by all exporter-importer transactions.

As there is no closed-form solution of the system (8) and (9), we solve it by considering approximations around a steady state. In the steady state there is no uncertainty, and we denote variables with an upper bar. Without loss of generality that the exchange rate is equal to unity, so that $\bar{s} = 0$.

A convenient way to capture the relevance of importer b for exporter x is to compute the share of x's total profits that are accounted for by sales to b:

$$shbforx = \frac{\left(\bar{P}_{xb} - \bar{C}_x\right)\bar{Q}_{xb}}{\sum_{j=1}^{B}\left(\bar{P}_{xj} - \bar{C}_x\right)\bar{Q}_{xj}} \tag{10}$$

Similarly the relevance of exporter x for importer b is the share of b's profits that come from sales of goods provided by x:

$$shxforb = \frac{\left(\bar{Z}_b - \bar{P}_{xb}\right)\bar{Q}_{xb}}{\sum_{i=1}^{X}\left(\bar{Z}_b - \bar{P}_{ib}\right)\bar{Q}_{ib}} \tag{11}$$

It is also useful to define the following function $H(s,\gamma)$ of $s \in [0,1]$ and $\gamma > 0$:

$$H(s,\gamma) = \frac{1}{1-\gamma}\frac{1}{s}\left[1-(1-s)^{1-\gamma}\right] \tag{12}$$

The function H is equal to one in the absence of risk aversion ($\gamma = 0$) or if s goes to zero. It is increasing with respect to both arguments, and is convex in s ($H_s > 0$, $H_{ss} > 0$), going to infinity as s goes to one.

Using the shares (10)-(11) and the function (12), we define the following effective bargaining weight of b vis-à-vis x:

$$\tilde{\delta}_{xb} = \frac{\delta H(shbforx, \gamma_x)}{\delta H(shbforx, \gamma_x) + (1-\delta) H(shxforb, \gamma_b)} \tag{13}$$

If agents have a linear valuation of profits ($\gamma_x = \gamma_b = 0$) the effective bargaining weight corresponds to the formal one: $\tilde{\delta}_{xb} = \delta$. Otherwise, (13) is an increasing function of the share of the importer in the exporter's profits, $shbforx$, and a decreasing function of the share of the exporter in the importer's profits, $shxforb$. The importer has an effective bargaining weight that exceeds her formal weight δ when a) the importer is large, b) the exporter is small, c) the importer's valuation of profits is not strongly concave (γ_b is low), or d) c) the exporter's valuation of profits is strongly concave (γ_x is high). Intuitively, failing to reach an agreement with a large importer leaves the exporter with low profits, and thus a high marginal valuation of profits as (2) is concave. The exporter thus cares more about striking an agreement with a large importer than with a smaller one. Similarly, the importer has a low effective bargaining weight when dealing with a large exporter.

As $\bar{s} = 0$ both sides of (9) are zero in the steady state, and the equation is irrelevant. Intuitively, the invoicing share is not a meaningful dimension of the model in the absence of exchange rate fluctuations.[2] Evaluating (8) at the steady state we get:

$$\begin{aligned} &\tilde{\delta}_{xb}\left(\bar{P}_{xb} - \frac{\rho}{\rho-1}\bar{Z}_b\right)\left(\bar{P}_{xb} - \overline{AC}_{xb}\right) \\ = \;&\left(1-\tilde{\delta}_{xb}\right)\left(\bar{P}_{xb} - \frac{\rho}{\rho-1}\overline{MC}_{xb}\right)\left(\bar{Z}_b - \bar{P}_{xb}\right) \end{aligned} \tag{14}$$

[2] A parallel can be made with the allocation of a portfolio across various assets. If all assets yield the same return with certainty, investors are indifferent across portfolios.

(14) takes a simple form if demand is not price sensitive ($\rho = 0$). In that case the price is a simple average between the average production cost and the resale price:

$$\bar{P}_{xb} = \tilde{\delta}_{xb}\overline{AC}_{xb} + \left(1 - \tilde{\delta}_{xb}\right)\bar{Z}_b$$

A high effective bargaining weight of the importer brings the price close to the production cost, thus shifting the allocation of the margin between the final price and the cost towards the importer. The opposite is the case when the importer's effective weight is low. When demand is price sensitive ($\rho > 0$), (14) does not lead to a simple analytical solution. It is nonetheless the case that $\bar{P}_{xb}$ lies between the marginal production cost and the resale price, being closer to the later when the importer's bargaining weight is low (the reasoning is presented in the appendix).

Our analysis shows that the steady state is characterized by the shares of the exporter (importer) in their counterpart's payoff, (10)-(11), which are functions of the price between them, and by the price (14) which is a function of the shares 10)-(11) through the effective bargaining weight (13). The steady-state solution is the fixed point of these relations. While we cannot derive an analytical solution for this fixed point in general, we can compute it for specific cases presented below.

3.2 Approximation around the steady state

The next step of the solution method is to expand the first-order conditions (8) and (9) around the steady-state. We need to consider quadratic approximations to capture the various variances and covariances among the variables. This is extent of LCP β_{xb} determines who bear the exchange rate risk, and therefore cannot be solved using a linear approximation that by definition abstracts from second moments. Furthermore, the presence of risk implies that the preset component of the price p_{xb}^{fix} differs from the steady state value $\bar{P}_{xb}$ as forward-looking agents take account of the second moments of the model in their price setting.[3]

In the analysis that follows we focus on the optimality condition with respect to the extent of LCP (9) and abstract from the one with respect to the preset component of the price (8). The motivation is twofold. First, as

[3] This element is a standard feature in the analysis of optimal monetary policy in models where prices are set ex-ante by forward looking agents.

shown below, the optimal β_{xb} are not affected by the difference between the preset price p_{xb}^{fix} and its steady state value.

Second, the magnitude of the difference between the preset price and its steady state value is proportional to the variance of the shocks affecting the economy, and can therefore be set to be arbitrarily small by considering a small value of volatility. This is not an issue for the extent of LCP β_{xb} which is the same regardless whether the volatility of shocks is low or high, as long as it is nonzero.[4]

We expand (9) around the steady state with respect to the preset component of the price p_{xb}^{fix}, and the exogenous exchange rate s, the exogenous component of the input cost w_x, the exogenous final price z_b, and the exogenous component of demand q_{xb}^{set}.[5] These terms do not only enter for the xb pair but also involving other importers and exporters. We denote logs deviation from the steady state with hatted values: $\hat{z}_b = z_b - \ln\left[\bar{Z}_b\right]$.

The quadratic approximation of (9) leads to the following expression (the steps are presented in the appendix):

$$\begin{aligned} 0 \;=\;& \gamma_b \left(\sum_{i=1}^{X} shiforb \left(\begin{array}{c} \frac{\bar{Z}_b}{\bar{Z}_b - \bar{P}_{ib}} \frac{E\hat{z}_b\hat{s}}{E\hat{s}^2} + \frac{\bar{P}_{ib}}{\bar{Z}_b - \bar{P}_{ib}} \left(1 - \beta_{ib}\right) \\ + \frac{E\hat{q}_{ib}^{set}\hat{s}}{E\hat{s}^2} - \rho\left(\beta_{ib} - \eta_{ib}\right) \end{array} \right) \right) \\ & - \gamma_x \left(\sum_{j=1}^{B} shjforx \left(\begin{array}{c} \frac{\bar{P}_{xj}}{\bar{P}_{xj} - \overline{AC}_{xj}} \beta_{xj} - \frac{\overline{AC}_{xj}}{\bar{P}_{xj} - \overline{AC}_{xj}} \left(\frac{E\hat{w}_x\hat{s}}{E\hat{s}^2} + \zeta_x \right) \\ + \frac{\bar{P}_{xj} - \overline{MC}_{xj}}{\bar{P}_{xj} - \overline{AC}_{xj}} \left(\frac{E\hat{q}_{xj}^{set}\hat{s}}{E\hat{s}^2} - \rho\left(\beta_{xj} - \eta_{xj}\right) \right) \end{array} \right) \right) \\ & - \frac{\rho\bar{Z}_b}{\rho\bar{Z}_b + (1-\rho)\bar{P}_{xb}} \frac{E\hat{z}_b\hat{s}}{E\hat{s}^2} + \frac{\rho\overline{MC}_{xb}}{(1-\rho)\bar{P}_{xb} + \rho\overline{MC}_{xb}} \left(\frac{E\hat{w}_x\hat{s}}{E\hat{s}^2} + \zeta_x \right) \\ & + \frac{\rho\overline{MC}_{xb}}{(1-\rho)\bar{P}_{xb} + \rho\overline{MC}_{xb}} \frac{1-\lambda}{\lambda} \left(\frac{E\hat{q}_{xb}^{set}\hat{s}}{E\hat{s}^2} - \rho\left(\beta_{xb} - \eta_{xb}\right) \right) \\ & + \frac{(1-\rho)\bar{P}_{xb}}{\rho\bar{Z}_b + (1-\rho)\bar{P}_{xb}} \left(1 - \beta_{xb}\right) + \frac{(1-\rho)\bar{P}_{xb}}{(1-\rho)\bar{P}_{xb} + \rho\overline{MC}_{xb}} \beta_{xb} \end{aligned} \tag{15}$$

(15) shows that the extent of LCP is affected by the sensitivity of production cost with respect to the exchange rate (the terms $\left(E\hat{w}_x\hat{s}\right)\left(E\hat{s}^2\right)^{-1} + \zeta_x$),

[4] In tecnhical terms, β_{xb} is similar to a portfolio share in models of endogenous portolfio choice such as Tille and van Wincoop (2010). Such so-called "zero-order" shares depend not on the magnitude of volatility (as long as it is positive) but on the co-movements between asset returns and pricing kernels.

[5] The deviation of the price e p_{xb}^{fix} from the steady state being proportional to the variance of shocks (i.e. "second order") it ends up dropping out of the approximation.

the sensitivity of demand to the exchange rate (the terms $\left(E\hat{q}_{xb}^{set}\hat{s}\right)\left(E\hat{s}^2\right)^{-1} - \rho\left(\beta_{xb} - \eta_{xb}\right)$) to the extent that it affects costs through decreasing returns to scale ($\lambda < 1$), and the sensitivity of the final price to the exchange rate (the term $\left(E\hat{z}_b\hat{s}\right)\left(E\hat{s}^2\right)^{-1}$). The first two terms of (15) show that the determination of the exposure for the xb pair is also affected by the exposure between importer b and other exporters (the β_{ib} shares) and between exporter x and other importers (the β_{xj} shares) as long as exporters and importers have concave valuations of payoffs (γ_b and γ_x differ from zero).

The overall solution of the model is given by a system of (15) for each xb pairs. As each involves elements for all exporter-importer pairs in the first two terms, this makes for a complex system that has no analytical solution in general. We therefore focus on two particular cases, one highlighting the impact of exporter or importer fragmentation, and the other considering the impact of heterogeneity among exporters and among importers.

4 Importer and exporters fragmentation

We first focus on the number of exporters and importers, and we assume that all individual exporters (importers) are identical. This implies that the shares (10)-(11) are simply $shbforx = 1/B$ and $shxforb = 1/X$. The effective bargaining weight (13) is then:

$$\tilde{\delta} = \frac{\delta H\left(B^{-1}, \gamma_x\right)}{\delta H\left(B^{-1}, \gamma_x\right) + (1-\delta) H\left(X^{-1}, \gamma_b\right)}$$

As the price set between exporter x and importer b affects the quantity sold, we need to specify the reference price R_{xb} in (5). We consider that it consists of a weighted average of the price sets by other exporters, which is equilibrium is equal to P_{xb}^b, to and an external price R_{xb}^{ex}, with a weight ν on the former component: $R_{xb} = \left(R_{xb}^{ex}\right)^{1-\nu}\left(P_{xb}^b\right)^{\nu}$. The case $\nu = 1$ is the situation where only the X exporters are the reference price. We consider that the external component R_{xb}^{ex} has a preset component and moves with the exchange rate: $R_{xb}^{ex} = \exp\left[r_{xb}^{ex,,fix} - \left(1 - \eta_{xb}^{set}\right)s\right]$, where $1 - \eta_{xb}^{set}$ is the extent to which the price is affected by the exchange rate. As $P_{xb}^b = \exp\left[p_{xb}^{fix} - \left(1 - \beta_{xb}\right)s\right]$, our specification implies that:

$$\begin{aligned} R_{xb} &= \exp\left[r_{xb}^{fix} - (1-\eta_{xb})\, s\right] \\ &= \exp\left[(1-\nu)\, r_{xb}^{ex,,fix} + \nu p_{xb}^{fix} - \left[1-(1-\nu)\,\eta_{xb}^{set} - \nu\beta_{xb}\right] s\right] \end{aligned}$$

Using (5) the demand is:

$$Q_{xb} = Q_{xb}^{set} \exp\left[-\rho\,(1-\nu)\left(p_{xb}^{fix} - r_{xb}^{ex,,fix}\right) - \rho\,(1-\nu)\left(\beta_{xb} - \eta_{xb}^{set}\right) s\right]$$

We denote the exogenous component of overall quantity traded in the steady state by $\bar{Q}^{set}$, so that $\bar{Q}_{xb}^{set} = \bar{Q}^{set}/(XB)$.

We set the productivity term $\bar{A} = \left(\bar{Q}^{set}\right)^{1-\lambda}(XB)^{\lambda-1}$ so that in the steady state the marginal and average costs are not affected by the number of exporters and importers (as all exporter-importer pairs are identical, we drop the x and b subscripts):

$$\overline{MC} = \frac{1}{\lambda}\bar{W}\left(\bar{R}^{ex,,fix}\right)^{\rho(1-\nu)\frac{1-\lambda}{\lambda}}\left(\bar{P}\right)^{-\rho(1-\nu)\frac{1-\lambda}{\lambda}} \tag{16}$$

Using (14) the steady state price $\bar{P}$ solves:

$$\begin{aligned} 0 &= \tilde{\delta}_{xb}\left(1 - \frac{\rho}{\rho-1}\frac{\bar{Z}}{\bar{P}}\right)\left(\bar{P}\left(\frac{\bar{P}}{\bar{R}^{ex,,fix}}\right)^{\rho(1-\nu)\frac{1-\lambda}{\lambda}} - \bar{W}\right) \\ &\quad - \left(1-\tilde{\delta}_{xb}\right)\left(\bar{P}\left(\frac{\bar{P}}{\bar{R}^{ex,,fix}}\right)^{\rho(1-\nu)\frac{1-\lambda}{\lambda}} - \frac{\rho}{\rho-1}\frac{1}{\lambda}\bar{W}\right)\left(\frac{\bar{Z}}{\bar{P}} - 1\right) \end{aligned} \tag{17}$$

Turning to the optimal exposure to the exchange rate, our specification implies that η_{xb} in (15) for any xb pair is $(1-\nu)\,\eta_{xb}^{set} + \nu\beta_{xb}$. The first-order condition (15) is then written as:

$$\begin{aligned} 0 &= \left[Coef_1 + Coef_2 \cdot \zeta\right] \\ &\quad + \left[\gamma_b \frac{\bar{Z}}{\bar{Z}-\bar{P}} - \frac{\rho\bar{Z}}{\rho\bar{Z} + (1-\rho)\,\bar{P}}\right]\frac{E\hat{z}\hat{s}}{E\hat{s}^2} \\ &\quad + Coef_2 \frac{E\hat{w}\hat{s}}{E\hat{s}^2} + Coef_3 \cdot \frac{E\hat{q}^{set}\hat{s}}{E\hat{s}^2} \\ &\quad + Coef_3 \cdot \rho\,(1-\nu)\,\eta^{set} \\ &\quad - \left[Coef_1 - \frac{(1-\rho)\,\bar{P}}{(1-\rho)\,\bar{P} + \rho\overline{MC}} + \gamma_x \frac{\bar{P}}{\bar{P}-\overline{AC}} + Coef_3 \cdot \rho\,(1-\nu)\right]\beta \end{aligned} \tag{18}$$

where the various coefficients are:

$$
\begin{aligned}
Coef_1 &= \gamma_b \frac{\bar{P}}{\bar{Z}-\bar{P}} + \frac{(1-\rho)\,\bar{P}}{\rho\bar{Z}+(1-\rho)\,\bar{P}} \\
Coef_2 &= \gamma_x \frac{\overline{AC}}{\bar{P}-\overline{AC}} + \frac{\rho\overline{MC}}{(1-\rho)\,\bar{P}+\rho\overline{MC}} \\
Coef_3 &= \gamma_b - \gamma_x \frac{\bar{P}}{\bar{P}-\overline{AC}} + \gamma_x \frac{\overline{AC}}{\bar{P}-\overline{AC}}\frac{1}{\lambda} - \frac{\rho\overline{MC}}{(1-\rho)\,\bar{P}+\rho\overline{MC}}\frac{\lambda-1}{\lambda}
\end{aligned}
$$

We illustrate our results through a numerical example. In our baseline specification, we assume an even formal power, $\delta = 0.5$, and set both γ_x and γ_b to 2. We set $\rho = 2$, consider that production exhibits constant returns to scale ($\lambda = 1$), set $\eta_{xb}^{set} = 0$, and set the cost and price parameters at $\bar{W} = 1$, $\bar{R}^{ex,,fix} = 1$, $\nu = 1$, $\bar{Q}^{set} = 10$. We parametrize $\bar{Z} = 2\bar{W}/\lambda$ to ensure that it always exceeds the production cost. We consider that input costs are insulated from the exchange rate: $(E\hat{w}_x\hat{s})\,(E\hat{s}^2)^{-1} = \zeta_x = 0$, and that prices and quantities do not commove with the exchange rate: $(E\hat{q}_{xb}^{set}\hat{s})\,(E\hat{s}^2)^{-1} = (E\hat{z}_b\hat{s})\,(E\hat{s}^2)^{-1} = 0$.

The top-left panel of figure 1 shows the effective bargaining eight, $\tilde{\delta}_{xb}$, relative to the formal weight δ, as a function of the numbers of importers and exporters. Importers have a higher effective weight when they are more concentrated than exporters are, i.e. when B is low or X is high. Most of the impact takes place are relatively low values of X and B.

The effective bargaining weight of the importer is reflected in the steady state price shown in the top-right panel. Importers who dominate the bargaining are able to secure a lower price. The bottom-left panel displays the value of individual transactions in the steady state. The exogenous component $\bar{Q}_{xb}^{set} = \bar{Q}^{set}/\,(XB)$ is equally reduced by a high number of importers or a high number of exporters. However, when importers are fragmented (B is high and X is low) their bargaining weight is limited and they are charged a relatively high price. Conversely, they are charged a low price when fragmentation is on the exporters' side (B is low and X is high). Therefore, small transactions in real terms have a higher nominal value when the small size reflects importer fragmentation than when it reflects exporter fragmentation.

The extent of LCP β_{xb} is presented in the bottom-right panel. It follows a pattern similar to the steady-state price, with a higher exposure of importers to exchange rate movements (a lower β_{xb}) when importers have a high effective bargaining weight. This result can seem puzzling as it seems that importers take on more exposure to risk when they are more powerful. The reason is that they also benefit from low prices and thus get more of the joint surplus. The marginal utility of importers' is then small, relative to that of exporters, implying that they care relatively little about exchange rate fluctuations. Interestingly, a market structure where the extent of LCP β_{xb} is high (B is high and X is low) is also a market structure where the value of transactions (the price) is high. Therefore, there is a small (7.4%) positive correlation across market structures between transaction value and the extent of LCP.

We now assess the sensitivity of the results to the model parameters. For brevity we focus on the steady state price and the extent of LCP. Figure 2 shows that increasing the concavity of payoff valuation ($\gamma_x = \gamma_b = 4$) makes the pricing and invoicing pattern more sensitive to the market structure. Intuitively, agents are more sensitive towards failing to reach an agreement with a counterpart when this failure substantially affects their marginal utility. Increasing the sensitivity of demand to prices (figure 3 with $\rho = 5$) lowers the steady state price and the extent of LCP and makes them insensitive to the fragmentation among agents. This does not reflect the effective bargaining share, which is similar as in the baseline. Instead, this results can be seen from (14). Recall that the right-hand side increases with the price $\bar{P}_{xb}$ and is zero when the price is equal to $\bar{Z}_b$. If ρ is low, the right-hand side is substantially negative when $\bar{P}_{xb}$ is equal to $\overline{AC}_{xb}$ (for simplicity consider that $\overline{MC}_{xb} = \overline{AC}_{xb}$). If ρ is large however it is only slightly negative at that point. Setting ρ to a large value then ensures that at $\bar{P}_{xb} = \overline{AC}_{xb}$ the left-hand side of (14) is zero and the right-hand side is only slightly negative. The two will then cross at a value of $\bar{P}_{xb}$ close to $\overline{AC}_{xb}$. Our analysis thus shows that in industries with a high price-sensitivity of demand importers benefit from a low steady state price and are willing to tolerate a high exposure to exchange rate fluctuations as their marginal utility of payoff is low.

Figure 4 displays the case with decreasing returns to scale ($\lambda = 0.75$). We see that this has an impact on the steady state price, which is now higher, but not on the extent of LCP which is similar to figure 1. Finally, we consider that exchange rate movements directly impact the cost of inputs in figure 5 ($\zeta_x = 0.5$), so that a depreciation of the exporter's currency raises her costs. While this has little impact on the steady state price, it raises the extent of LCP substantially, and makes it insensitive to the market structure. Intuitively, stabilizing the price in the importer's currency provides a hedging benefit to exporters, as a depreciation of their currency then increases their unit revenue and thus offsets the increase in costs.

To sum up, our analysis shows that the extent of fragmentation among importers and exporters impacts the effective bargaining weights, the prices, and the extent of LCP. Interestingly, a higher bargaining power for importers benefits them through a lower steady state price. This gives them high payoffs and thus lowers their marginal utility. This in turns make them more tolerant towards volatility and leads them to accept a high exposure to exchange rate fluctuations.

5 Intra-group heterogeneity

We now focus on the impact of heterogeneity among exporter and importers. There are two exporters, denoted by $X1$ and $X2$, and two importers, denoted by $B1$ and $B2$. Without loss of generality we consider that exporter and importer 1 are relatively large. Specifically, the steady values of the Q_{xb}^{set} terms in (5) are:

$$\begin{aligned}\bar{Q}_{X1B1}^{set} &= \alpha\beta\bar{Q}^{set} \quad ; \quad \bar{Q}_{X1B2}^{set} = \alpha\left(1-\beta\right)\bar{Q}^{set} \\ \bar{Q}_{X2B1}^{set} &= \left(1-\alpha\right)\beta\bar{Q}^{set} \quad ; \quad \bar{Q}_{X2B2}^{set} = \left(1-\alpha\right)\left(1-\beta\right)\bar{Q}^{set}\end{aligned}$$

where $\bar{Q}^{set}$ is the total quantity exchange in the steady state. The coefficients $\alpha \in [0.5, 1]$ and $\beta \in [0.5, 1]$ denote the sizes of larger exporter and the larger importer, respectively. The case of homogeneity ($\alpha = \beta = 0.5$) corresponds to the fragmentation case with $X = B = 2$.

As in the previous example, we need to specify the reference price R_{xb} in (5). We again consider that it is a weighted average of an index of prices set by exporters to importer b, which we denote by P_b^{ind}, and an external price $R_{xb}^{ex} = \exp\left[r_{xb}^{ex,,fix} - (1-\eta_{xb}^{set})\, s\right]$, with a weight ν on the former component. The price index P_b^{ind} encompasses the prices of the two exporters selling to importer b:

$$P_b^{ind} = \left[\alpha\left[P_{X1b}\right]^{1-\rho} + \left(1-\alpha\right)\left[P_{X2b}\right]^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{19}$$

For simplicity we assume that the final price Z_b and the input cost w_x are the same for all importers and exporters. In the steady state, we set the productivity parameter $\bar{A}_{xb} = \left(\bar{Q}_{xb}^{set}\right)^{1-\lambda}$ so that the marginal cost is not affected by $\bar{Q}_{xb}^{set}$. In the steady state R_{xb}^{ex} is equal to $\bar{R}^{ex,,fix}$ for all xb pairs.

The steady state solution consists of 14 non-linear equations. The first two are the price indexes (19) for $B1$ and $B2$. The next four equations are the shares (10)-(11), then we have four effective bargaining weights (13), and finally four pricing equations (14). The specific equations are given in the appendix.

Turning to the determination of the optimal extent of LCP, we first $\hat{z}_b$, $\hat{q}^{set}_{xb}$, ζ_x and $\hat{w}_x$ to be the same for all xb pairs for simplicity. A log linear approximation of the reference price R_{xb} around the steady state implies that:

$$\hat{r}_{xb} = -\hat{s} + \left[(1-\nu)\, \eta^{set} + \nu \left[\alpha \beta_{X1b} + (1-\alpha)\, \beta_{X2b} \right] \right] \hat{s} = -\left(1-\eta_{xb}\right) \hat{s}$$

Using this result, we obtain four variants of (15), one per importer-exporter pair. The relation for the $X1B2$ pair is presented in the appendix.

We illustrate our results with a numerical example, taking the same baseline calibration as in section 4. Figure 6 shows the effective bargaining weights relative to the formal weight, $\tilde{\delta}_{xb} - \delta$, as a function of the heterogeneity among exporters (α) and importers (β) for all exporter-importer pairs. The top left panel considers the large importer's weight vis-a-vis the large exporter ($\tilde{\delta}_{X1B1}$), and shows that it increases with importer heterogeneity (higher β) and decreases with the exporter heterogeneity (higher α). Interestingly, the marginal effect of either heterogeneity is increasing with heterogeneity itself.

The bottom left panel shows that the large importer's weight vis-a-vis the small exporter ($\tilde{\delta}_{X2B1}$) is high and increases with importer heterogeneity, especially at high levels of heterogeneity. While it also increases with exporter heterogeneity, the effect is smaller. A mirror pattern is seen for the effective weight of the small importer vis-a-vis the large exporter ($\tilde{\delta}_{X1B2}$, top right panel), which is relatively insensitive to the importer heterogeneity but falls rapidly as the exporter heterogeneity increases. Finally, the small importer's weight vis-a-vis the small exporter ($\tilde{\delta}_{X2B2}$, bottom right panel) is close to the formal weight and relatively insensitive to heterogeneity.

The pattern for the effective bargaining weights is mirrored in the steady state price (figure 7). The price is lower for sales to the larger importer (left panels) than for sales to the small importer (right panels). The gap is more pronounced when importer fragmentation is high, and for sales from the large exporter (top panels).

The extent of LCP, β_{xb}, is displayed in figure 8 for the four importer-exporter pairs. Starting from the point of full homogeneity ($\alpha = \beta = 0.5$), the extent of LCP between the large importer and the large exporter (top left panel) falls with importer heterogeneity, but increases with exporter heterogeneity. This is a similar pattern to the one of the steady state price in figure

7. When the importer can shift the surplus her way through a low steady state price, her marginal value of profits is low. She is then little affected by exchange rate volatility and willing to be more exposed to fluctuations with a low extent of LCP. The similarity between the steady state price and the extent of LCP is also seen for sales from the large exporter to the small importer (top right panel). As the small importer carries less weight than the large one, she is faced with a higher price but a more limited exposure to exchange rate movements. The large importer also faces a smaller extent of LCP on sales from the small exporter (bottom left panel) than on sales from the large exporter (top left panel), reflecting the fact that she obtains a lower price on the former. The extent of LCP between the small importer and small exporter (bottom right panel) increases with heterogeneity, although that particular pair becomes marginal in that case.

To obtain summary measures of the pricing and invoicing, we compute the average and standard deviation of the steady state price and extent of LCP across the four exporter-importer pairs, weighting each pair by its share in total steady state transaction value. The results are presented in figure 9. Exporter heterogeneity raises the average price (top left panel), while the cross-sectional dispersion of prices (top right panel) is raised by either heterogeneity, but somewhat more so by exporter heterogeneity. Turning to the extent of LCP, the extent of heterogeneity has a substantial impact on the average extent (bottom left panel) which increases with exporter heterogeneity. Heterogeneity on either side of the market raises the dispersion of LCP shares.

As the market structure impacts the steady state price, and hence the steady state value of transactions, as well as the extent of LCP, we consider the linkage between the two by computing the coefficient of correlation across the four exporter-importer pairs between the steady state value of transactions and the extents of LCP (figure 10). This correlation is negative when importer heterogeneity dominates, but turns positive as exporter heterogeneity raises.

Our numerical example shows that the market structure has a sizable impact on the effective bargaining weight, price, and extent of LCP across the various importer-exporter pairs. This impact is also observed in aggregate terms, as the average value and dispersion of prices and extents of LCP, as well as the correlation between invoicing and transaction size, vary depending on the degrees of heterogeneity among importers and exporters.

We now consider the impact of varying the model parameters along the same lines as in section 4. For brevity, we focus on the averages and standard deviations of steady state prices and the extent of LCP, as well as the correlation between transaction value and invoicing. Increasing the concavity of payoff valuation ($\gamma_x = \gamma_b = 4$) raises the average extent of LCP somewhat (figure 11 bottom left panel) and increases the dispersion of prices and extent of LCP (right panels). The average value of prices and invoicing remains sensitive to the amount of heterogeneity on both sides of the market. The correlation between transaction value and invoicing remains close to the baseline case (figure 12). Increasing the sensitivity of demand to prices ($\rho = 5$) substantially lower the average price and the average extent of LCP, as importers' marginal utility is then less sensitive to prices (figure 13 left panels). The average price and invoicing is also much less sensitive to the market structure. In addition, the cross sectional dispersion of the two measures is reduced (right panels), and shifts the correlation between transaction value and extent of LCP towards positive values (figure 14).

Introducing decreasing returns to scale ($\lambda = 0.75$) raises the average price and reduces the extent of LCP (figure 15 left panels) and leads to more dispersion in prices (top right panel). The dispersion in invoicing is now mostly driven by importer heterogeneity. The market structure has a sizable impact on the average price, and a more moderate one on the average extent of LCP. The correlation between transaction value and extent of LCP shifts towards negative values (figure 16). We finally consider a direct impact of the exchange rate on input costs $\zeta_x = 0.5$). This has little impact on prices (figure 17 top panel) and raises the average extent of LCP (bottom left panel) while lowering its dispersion somewhat. The sensitivity of the average price and invoicing to the market structure remains similar to the baseline case. The correlation between transaction value and invoicing remains close to the baseline case (figure 17).

6 Conclusion

This paper analyzes the determination of prices and exposure to exchange rate fluctuations among exporter and importers through a simple model of bargaining. This setting expands the theoretical analysis beyond the standard assumption of unilateral choice by the exporter. We show that the market structure, reflecting in the share of specific exporters and importers in each other's total profits, has a substantial impact on the effective bargaining weights, prices, and exchange rate exposure. This impact is not limited to specific exporter-importer pairs but also affects the aggregate values of prices and exposure. A striking result of our analysis is that powerful agents end up being more exposed to exchange rate fluctuations. This reflects the fact that their power allows them to shift the steady state price in their favor, which lowers their marginal utility and makes exchange rate fluctuations less of a concern.

Our analysis is the first step towards building a bargaining view in the theory of international trade pricing under uncertainty. Under this view understanding aggregate prices requires one to take account of the microeconomic structure of the market, such as the degrees of fragmentation and heterogeneity among exporters and importers. A promising area of future research is to go beyond our Nash bargaining solution and get more detailed evidence on the specific process of interaction between importers and exporters in specific industries.

Another avenue for further work is to allow for prices to respond ex-post to cost movements. It is reasonable to expect that this response will be substantially affected by the market structure. Recall that the importer's currency is used for the invoicing when the importer's weight is low, as she is then faced with a high preset price, hence low profits and a high marginal valuation of profits. A reasonable conjecture is that in such a situation the high marginal utility would also leads to limited movements of prices when they can be adjusted. This would be in line with the finding of Gopinath, Itskhoki, and Rigobon (2010) that price adjustment is low for US imports invoiced in dollars.

7 Appendix

This technical appendix presents specific analytical points on the key aspects of the model and its solution. A complete presentation of the technical aspects of the paper is in an exhaustive technical appendix available on request.

7.1 Derivatives of the joint surplus

The derivatives with respect to the fixed component of the price that enter (8) are:

$$\frac{\partial SB^{bx}}{\partial p_{xb}^{fix}} = -E\left[\begin{array}{c} \left(\sum_{i=1}^{X}\left(\begin{array}{c} \exp\left[\begin{array}{c} z_b + q_{ib}^{set} \\ -\rho\left(p_{ib}^{fix} - r_{ib}^{fix} + \left(\beta_{ib} - \eta_{ib}\right)s\right)\end{array}\right] \\ -\exp\left[\begin{array}{c} p_{ib}^{fix} - \left(1-\beta_{ib}\right)s + q_{ib}^{set} \\ -\rho\left(p_{ib}^{fix} - r_{ib}^{fix} + \left(\beta_{ib} - \eta_{ib}\right)s\right)\end{array}\right]\end{array}\right)\right)^{-\gamma_b} \\ \times\left(\begin{array}{c} \rho\exp\left[\begin{array}{c} z_b + q_{xb}^{set} \\ -\rho\left(p_{xb}^{fix} - r_{xb}^{fix} + \left(\beta_{xb} - \eta_{xb}\right)s\right)\end{array}\right] \\ +\left(1-\rho\right)\exp\left[\begin{array}{c} p_{xb}^{fix} - \left(1-\beta_{xb}\right)s + q_{xb}^{set} \\ -\rho\left(p_{xb}^{fix} - r_{xb}^{fix} + \left(\beta_{xb} - \eta_{xb}\right)s\right)\end{array}\right]\end{array}\right)\end{array}\right]$$

and:

$$\frac{\partial SX^{bx}}{\partial p_{xb}^{fix}} = E\left[\begin{array}{c} \left(\sum_{j=1}^{B}\left(\begin{array}{c} \exp\left[\begin{array}{c} p_{xj}^{fix} + \beta_{xj}s + q_{xb}^{set} \\ -\rho\left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right)s\right)\end{array}\right] \\ -\exp\left[\begin{array}{c} w_x + \zeta_x s + \frac{1}{\lambda}q_{xj}^{set} \\ -\frac{\rho}{\lambda}\left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right)s\right) - \frac{1}{\lambda}a_{xj}\end{array}\right]\end{array}\right)\right)^{-\gamma_x} \\ \times\left(\begin{array}{c} \left(1-\rho\right)\exp\left[\begin{array}{c} p_{xj}^{fix} + \beta_{xj}s + q_{xb}^{set} \\ -\rho\left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right)s\right)\end{array}\right] \\ +\frac{\rho}{\lambda}\exp\left[\begin{array}{c} w_x + \zeta_x s + \frac{1}{\lambda}q_{xj}^{set} \\ -\frac{\rho}{\lambda}\left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right)s\right) - \frac{1}{\lambda}a_{xj}\end{array}\right]\end{array}\right)\end{array}\right]$$

The derivatives with respect to the exchange rate exposure in (9) are:

$$\frac{\partial SB^{bx}}{\partial \beta_{xb}} = -E\left[\begin{array}{c}\left(\sum_{i=1}^{X}\left(\begin{array}{c}\exp\left[\begin{array}{c}z_b + q_{ib}^{set}\\ -\rho\left(p_{ib}^{fix} - r_{ib}^{fix} + \left(\beta_{ib} - \eta_{ib}\right)s\right)\end{array}\right]\\ -\exp\left[\begin{array}{c}p_{ib}^{fix} - \left(1-\beta_{ib}\right)s + q_{ib}^{set}\\ -\rho\left(p_{ib}^{fix} - r_{ib}^{fix} + \left(\beta_{ib} - \eta_{ib}\right)s\right)\end{array}\right]\end{array}\right)\right)^{-\gamma_b}\\ \times\left(\begin{array}{c}\rho\exp\left[\begin{array}{c}z_b + q_{xb}^{set}\\ -\rho\left(p_{xb}^{fix} - r_{xb}^{fix} + \left(\beta_{xb} - \eta_{xb}\right)s\right)\end{array}\right]\\ +\left(1-\rho\right)\exp\left[\begin{array}{c}p_{xb}^{fix} - \left(1-\beta_{xb}\right)s + q_{xb}^{set}\\ -\rho\left(p_{xb}^{fix} - r_{xb}^{fix} + \left(\beta_{xb} - \eta_{xb}\right)s\right)\end{array}\right]\end{array}\right)s\end{array}\right]$$

and:

$$\frac{\partial SX^{bx}}{\partial \beta_{xb}} = E\left[\begin{array}{c}\left(\sum_{j=1}^{B}\left(\begin{array}{c}\exp\left[\begin{array}{c}p_{xj}^{fix} + \beta_{xj}s + q_{xb}^{set}\\ -\rho\left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right)s\right)\end{array}\right]\\ -\exp\left[\begin{array}{c}w_x + \zeta_x s + \frac{1}{\lambda}q_{xj}^{set}\\ -\frac{\rho}{\lambda}\left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right)s\right) - \frac{1}{\lambda}a_{xj}\end{array}\right]\end{array}\right)\right)^{-\gamma_x}\\ \times\left(\begin{array}{c}\left(1-\rho\right)\exp\left[\begin{array}{c}p_{xj}^{fix} + \beta_{xj}s + q_{xb}^{set}\\ -\rho\left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right)s\right)\end{array}\right]\\ +\frac{\rho}{\lambda}\exp\left[\begin{array}{c}w_x + \zeta_x s + \frac{1}{\lambda}q_{xj}^{set}\\ -\frac{\rho}{\lambda}\left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right)s\right) - \frac{1}{\lambda}a_{xj}\end{array}\right]\end{array}\right)s\end{array}\right]$$

7.2 Steady state price

The price charged by the exporter x to the importer b in the steady state is given by 14) and lies between the average cost $\overline{AC}_{xb}$ and the final price $\bar{Z}_b$. To see this, consider how the various elements of 14) evolve as we increase $\bar{P}_{xb}$ from low to high values. Recall that the final price $\bar{Z}_b$ exceeds the marginal cost $\overline{MC}_{xb}$, which itself exceeds the average cost $\overline{AC}_{xb}$.

Consider first the left-hand side of (14). When $\bar{P}_{xb} < \overline{AC}_{xb}$ it is clearly positive as it is the product of two negative components. Increasing $\bar{P}_{xb}$ to $\overline{AC}_{xb}$, the left-hand side reaches zero from above. As we increase $\bar{P}_{xb}$ above $\overline{AC}_{xb}$, but remain below $\bar{Z}_b$, the left-hand side becomes more and more negative as it is the product of a negative terms and an increasing positive one. This is still the case when $\bar{P}_{xb} = \bar{Z}_b$. Therefore, the left-hand side of (14) is a decreasing function that crosses zero at $\bar{P}_{xb} = \overline{AC}_{xb}$.

We now turn to the right-hand side of (14). First, notice that when $\bar{P}_{xb} \leq \overline{AC}_{xb}$ it is negative as it is the product between a negative first term and a positive second one. Second, when $\bar{P}_{xb} = \bar{Z}_b$ the right-hand side is clearly zero. As $\bar{P}_{xb}$ increases from $\overline{AC}_{xb}$ to $\bar{Z}_b$, the second term remains positive. If $\rho(\rho-1)^{-1}\overline{MC}_{xb}$ is large enough, the first term remains negative as $\bar{P}_{xb}$ increases to $\bar{Z}_b$, so the right-hand side reaches zero from below. If $\rho(\rho-1)^{-1}\overline{MC}_{xb}$ is small enough, the first term turns positive for some value of $\bar{P}_{xb}$ between $\overline{AC}_{xb}$ and $\bar{Z}_b$, so the right-hand side reaches zero from above as $\bar{P}_{xb}$ increases to $\bar{Z}_b$. Therefore the right-hand side of (14) increases from a negative value to zero as $\bar{P}_{xb}$ goes from $\overline{AC}_{xb}$ to $\bar{Z}_b$.

Combining our analyses of the left- and right-hand sides of (14) we see that there is a unique value of $\bar{P}_{xb}$ between $\overline{AC}_{xb}$ and $\bar{Z}_b$ that equalizes them. Thus, the price between exporter x and importer b is such that both make profits as the price lies between the average production cost and the final price.

7.3 Quadratic approximation

To write a quadratic approximation of (9), we first notice that left- and right-hand sides are expressions of the form (the detailed expression for (9) is given in a long technical appendix):

$$\begin{aligned}\Phi &= \frac{\delta_1}{1-\gamma_1} E\left[\left(\sum_{s1}(\exp[a]-\exp[b])\right)^{-\gamma_2}(c_1\exp[c]+h_1\exp[h])\,s\right] \\ &\quad\times\left[E\left(\sum_{s2}(\exp[d]-\exp[e])\right)^{1-\gamma_1} - E\left(\begin{array}{c}\sum_{s2}(\exp[d]-\exp[e]) \\ -(\exp[f]-\exp[g])\end{array}\right)^{1-\gamma_1}\right]\end{aligned}$$

Bearing in mind that we expand around $\bar{s}=0$ and that $E\hat{s}=0$, we write:

$$\begin{aligned}\Phi &= \frac{\delta_1}{1-\gamma_1}\left(\sum_{s2}\left(\bar{D}-\bar{E}\right)\right)^{1-\gamma_1}\left(c_1\bar{C}+h_1\bar{H}\right)\left(\sum_{s1}\left(\bar{A}-\bar{B}\right)\right)^{-\gamma_2} \\ &\quad\times\left[1-\left(1-\frac{\bar{F}-\bar{G}}{\sum_{s2}\left(\bar{D}-\bar{E}\right)}\right)^{1-\gamma_1}\right] E\left[\begin{array}{c}-\gamma_2\left(\sum_{s1}\left(\bar{A}-\bar{B}\right)\right)^{-1}\left(\sum_{s1}\left(\bar{A}\hat{a}-\bar{B}\hat{b}\right)\right)\hat{s} \\ +\left(c_1\bar{C}+h_1\bar{H}\right)^{-1}\left(c_1\bar{C}\hat{c}+h_1\bar{H}\hat{h}\right)\hat{s}\end{array}\right]\end{aligned}$$

We now apply this to the left-hand side of (9). The various elements are:

$$\begin{aligned}
s1 &= i = 1...X \quad ; \quad s2 = j = 1...B \\
\delta_1 &= \delta \quad ; \quad \gamma_1 = \gamma_x \quad ; \quad \gamma_2 = \gamma_b \\
c_1 &= \rho \quad ; \quad h_1 = 1 - \rho \\
a &= z_b + q_{ib}^{set} - \rho \left(p_{ib}^{fix} - r_{ib}^{fix} + (\beta_{ib} - \eta_{ib})\, s \right) \\
b &= p_{ib}^{fix} - (1 - \beta_{ib})\, s + q_{ib}^{set} - \rho \left(p_{ib}^{fix} - r_{ib}^{fix} + (\beta_{ib} - \eta_{ib})\, s \right) \\
c &= z_b + q_{xb}^{set} - \rho \left(p_{xb}^{fix} - r_{xb}^{fix} + (\beta_{xb} - \eta_{xb})\, s \right) \\
d &= p_{xj}^{fix} + \beta_{xj} s + q_{xj}^{set} - \rho \left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right) s \right) \\
e &= w_x + \zeta_x s + \frac{1}{\lambda} q_{xj}^{set} - \frac{\rho}{\lambda} \left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right) s \right) - \frac{1}{\lambda} a_{xj} \\
f &= p_{xb}^{fix} + \beta_{xb} s + q_{xb}^{set} - \rho \left(p_{xb}^{fix} - r_{xb}^{fix} + (\beta_{xb} - \eta_{xb})\, s \right) \\
g &= w_x + \zeta_x s + \frac{1}{\lambda} q_{xb}^{set} - \frac{\rho}{\lambda} \left(p_{xb}^{fix} - r_{xb}^{fix} + (\beta_{xb} - \eta_{xb})\, s \right) - \frac{1}{\lambda} a_{xb} \\
h &= p_{xb}^{fix} - (1 - \beta_{xb})\, s + q_{xb}^{set} - \rho \left(p_{xb}^{fix} - r_{xb}^{fix} + (\beta_{xb} - \eta_{xb})\, s \right)
\end{aligned}$$

As all the pre-set component of all prices deviate from the steady-state allocation only because of second moments, we can omit the various p^{fix} and r^{fix} from the quadratic elements. We also recall that $\hat{a}_{xb} = 0$ as A_{xb} is constant. The quadratic approximation of the left-hand side is then:

$$\begin{aligned}
&\frac{\delta}{1-\gamma_x} \left(\sum_{i=1}^{X} \left(\bar{Z}_b - \bar{P}_{ib} \right) \bar{Q}_{ib}^{set} \left(\bar{R}_{ib} \right)^{\rho} \left(\bar{P}_{ib} \right)^{-\rho} \right)^{-\gamma_b} \\
&\times \left(\sum_{j=1}^{B} \left(\bar{P}_{xj} - \bar{W}_x \left(\bar{A}_{xj} \right)^{-\frac{1}{\lambda}} \left(\bar{Q}_{xj}^{set} \right)^{\frac{1-\lambda}{\lambda}} \left(\bar{R}_{xj} \right)^{\rho \frac{1-\lambda}{\lambda}} \left(\bar{P}_{xj} \right)^{-\rho \frac{1-\lambda}{\lambda}} \right) \bar{Q}_{xj}^{set} \left(\bar{R}_{xj} \right)^{\rho} \left(\bar{P}_{xj} \right)^{-\rho} \right)^{1-\gamma_x} \\
&\times \left(\rho \bar{Z}_b + (1-\rho)\, \bar{P}_{xb} \right) \bar{Q}_{xb}^{set} \left(\bar{R}_{xb} \right)^{\rho} \left(\bar{P}_{xb} \right)^{-\rho} \left[1 - (1 - shbforx)^{1-\gamma_x} \right] \\
&\times E \left[\begin{array}{c} -\gamma_b \left(\sum_{i=1}^{X} shiforb \left(\frac{\bar{Z}_b}{\bar{Z}_b - \bar{P}_{ib}} \frac{E \hat{z}_b \hat{s}}{E \hat{s}^2} + \frac{\bar{P}_{ib}}{\bar{Z}_b - \bar{P}_{ib}} \left(1 - \beta_{ib}\right) + \frac{E \hat{q}_{ib}^{set} \hat{s}}{E \hat{s}^2} - \rho \left(\beta_{ib} - \eta_{ib} \right) \right) \right) E \hat{s}^2 \\ + \left(\frac{\rho \bar{Z}_b}{\rho \bar{Z}_b + (1-\rho) \bar{P}_{xb}} \frac{E \hat{z}_b \hat{s}}{E \hat{s}^2} - \frac{(1-\rho) \bar{P}_{xb}}{\rho \bar{Z}_b + (1-\rho) \bar{P}_{xb}} \left(1 - \beta_{xb}\right) + \frac{E \hat{q}_{xb}^{set} \hat{s}}{E \hat{s}^2} - \rho \left(\beta_{xb} - \eta_{xb} \right) \right) E \hat{s}^2 \end{array} \right]
\end{aligned}$$

We now turn to the right-hand side of (9). The various elements are:

$$
\begin{aligned}
s1 &= j = 1...B \quad ; \quad s2 = i = 1...X \\
\delta_1 &= 1-\delta \quad ; \quad \gamma_1 = \gamma_b \quad ; \quad \gamma_2 = \gamma_x \\
c_1 &= 1-\rho \quad ; \quad h_1 = \frac{\rho}{\lambda} \\
a &= p_{xj}^{fix} + \beta_{xj} s + q_{xj}^{set} - \rho\left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right) s\right) \\
b &= w_x + \zeta_x s + \frac{1}{\lambda} q_{xj}^{set} - \frac{\rho}{\lambda}\left(p_{xj}^{fix} - r_{xj}^{fix} + \left(\beta_{xj} - \eta_{xj}\right) s\right) - \frac{1}{\lambda} a_{xj} \\
c &= p_{xb}^{fix} + \beta_{xb} s + q_{xb}^{set} - \rho\left(p_{xb}^{fix} - r_{xb}^{fix} + \left(\beta_{xb} - \eta_{xb}\right) s\right) \\
d &= z_b + q_{ib}^{set} - \rho\left(p_{ib}^{fix} - r_{ib}^{fix} + \left(\beta_{ib}^{d} - \eta_{ib}^{d}\right) s^{d}\right) \\
e &= p_{ib}^{fix} - \left(1-\beta_{ib}\right) s + q_{ib}^{set} - \rho\left(p_{ib}^{fix} - r_{ib}^{fix} + \left(\beta_{ib} - \eta_{ib}\right) s\right) \\
f &= z_b + q_{xb}^{set} - \rho\left(p_{xb}^{fix} - r_{xb}^{fix} + \left(\beta_{xb} - \eta_{xb}\right) s\right) \\
g &= p_{xb}^{fix} - \left(1-\beta_{xb}\right) s + q_{xb}^{set} - \rho\left(p_{xb}^{fix} - r_{xb}^{fix} + \left(\beta_{xb} - \eta_{xb}\right) s\right) \\
h &= w_x + \zeta_x s + \frac{1}{\lambda} q_{xb}^{set} - \frac{\rho}{\lambda}\left(p_{xb}^{fix} - r_{xb}^{fix} + \left(\beta_{xb} - \eta_{xb}\right) s\right) - \frac{1}{\lambda} a_{xb}
\end{aligned}
$$

The quadratic approximation of the right-hand side is then:

$$
\begin{aligned}
&\frac{1-\delta}{1-\gamma_b}\left(\sum_{i=1}^{X}\left(\bar{Z}_b - \bar{P}_{ib}\right)\bar{Q}_{ib}^{set}\left(\bar{R}_{ib}\right)^{\rho}\left(\bar{P}_{ib}\right)^{-\rho}\right)^{1-\gamma_b} \\
&\times\left(\sum_{j=1}^{B}\left(\bar{P}_{xj} - \bar{W}_x\left(\bar{A}_{xj}\right)^{-\frac{1}{\lambda}}\left(\bar{Q}_{xj}^{set}\right)^{\frac{1-\lambda}{\lambda}}\left(\bar{R}_{xj}\right)^{\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{xj}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)\bar{Q}_{xj}^{set}\left(\bar{R}_{xj}\right)^{\rho}\left(\bar{P}_{xj}\right)^{-\rho}\right)^{-\gamma_x} \\
&\times\left(\left(1-\rho\right)\bar{P}_{xb} + \rho\overline{MC}_{xb}\right)\bar{Q}_{xb}^{set}\left(\bar{R}_{xb}\right)^{\rho}\left(\bar{P}_{xb}\right)^{-\rho}\left[1-\left(1-shxforb\right)^{1-\gamma_b}\right] \\
&\times E\left[\begin{array}{c}
-\gamma_x\left(\sum_{j=1}^{B} shjforx\left(\begin{array}{c}
\frac{\bar{P}_{xj}}{\bar{P}_{xj}-\overline{AC}_{xj}}\left[\beta_{xj} + \frac{E\hat{q}_{xj}^{set}\hat{s}}{E\hat{s}^2} - \rho\left(\beta_{xj}-\eta_{xj}\right)\right] \\
-\frac{\overline{AC}_{xb}}{\bar{P}_{xj}-\overline{AC}_{xj}}\left[\frac{E\hat{w}_x\hat{s}}{E\hat{s}^2} + \zeta_x + \frac{1}{\lambda}\frac{E\hat{q}_{xj}^{set}\hat{s}}{E\hat{s}^2} - \frac{\rho}{\lambda}\left(\beta_{xj}-\eta_{xj}\right)\right]
\end{array}\right)\right) E\hat{s}^2 \\
+\left(\begin{array}{c}
\frac{(1-\rho)\bar{P}_{xb}}{(1-\rho)\bar{P}_{xb}+\rho\overline{MC}_{xb}}\left[\beta_{xb} + \frac{E\hat{q}_{xb}^{set}\hat{s}}{E\hat{s}^2} - \rho\left(\beta_{xb}-\eta_{xb}\right)\right] \\
+\frac{\rho\overline{MC}_{xb}}{(1-\rho)\bar{P}_{xb}+\rho\overline{MC}_{xb}}\left[\frac{E\hat{w}_x\hat{s}}{E\hat{s}^2} + \zeta_x + \frac{1}{\lambda}\frac{E\hat{q}_{xb}^{set}\hat{s}}{E\hat{s}^2} - \frac{\rho}{\lambda}\left(\beta_{xb}-\eta_{xb}\right)\right]
\end{array}\right) E\hat{s}^2
\end{array}\right]
\end{aligned}
$$

Combining our results, the approximation of (9) is written as:

$$
\begin{aligned}
0 &= \gamma_b \left(\sum_{i=1}^{X} shiforb \left(\frac{\bar{Z}_b}{\bar{Z}_b - \bar{P}_{ib}} \frac{E\hat{z}_b\hat{s}}{E\hat{s}^2} + \frac{\bar{P}_{ib}}{\bar{Z}_b - \bar{P}_{ib}} (1-\beta_{ib}) + \frac{E\hat{q}_{ib}^{set}\hat{s}}{E\hat{s}^2} - \rho(\beta_{ib} - \eta_{ib}) \right) \right) \\
&\quad - \gamma_x \left(\sum_{j=1}^{B} shjforx \left(\begin{array}{c} \frac{\bar{P}_{xj}}{\bar{P}_{xj} - \overline{AC}_{xj}} \left[\beta_{xj} + \frac{E\hat{q}_{xj}^{set}\hat{s}}{E\hat{s}^2} - \rho(\beta_{xj} - \eta_{xj}) \right] \\ - \frac{\overline{AC}_{xj}}{\bar{P}_{xj} - \overline{AC}_{xj}} \left[\frac{E\hat{w}_x\hat{s}}{E\hat{s}^2} + \zeta_x + \frac{1}{\lambda} \frac{E\hat{q}_{xj}^{set}\hat{s}}{E\hat{s}^2} - \frac{\rho}{\lambda}(\beta_{xj} - \eta_{xj}) \right] \end{array} \right) \right) \\
&\quad - \frac{\rho\bar{Z}_b}{\rho\bar{Z}_b + (1-\rho)\bar{P}_{xb}} \frac{E\hat{z}_b\hat{s}}{E\hat{s}^2} + \frac{(1-\rho)\bar{P}_{xb}}{\rho\bar{Z}_b + (1-\rho)\bar{P}_{xb}} (1-\beta_{xb}) - \frac{E\hat{q}_{xb}^{set}\hat{s}}{E\hat{s}^2} + \rho(\beta_{xb} - \eta_{xb}) \\
&\quad + \frac{(1-\rho)\bar{P}_{xb}}{(1-\rho)\bar{P}_{xb} + \rho\overline{MC}_{xb}} \left[\beta_{xb} + \frac{E\hat{q}_{xb}^{set}\hat{s}}{E\hat{s}^2} - \rho(\beta_{xb} - \eta_{xb}) \right] \\
&\quad + \frac{\rho\overline{MC}_{xb}}{(1-\rho)\bar{P}_{xb} + \rho\overline{MC}_{xb}} \left[\frac{E\hat{w}_x\hat{s}}{E\hat{s}^2} + \zeta_x + \frac{1}{\lambda} \frac{E\hat{q}_{xb}^{set}\hat{s}}{E\hat{s}^2} - \frac{\rho}{\lambda}(\beta_{xb} - \eta_{xb}) \right]
\end{aligned}
$$

which is (15) in the text after re-arranging terms.

7.4 Intra-group heterogeneity

The first two equations are the price indexes (19):

$$
\begin{aligned}
\bar{P}_{B1}^{ind} &= \left[\alpha \left[\bar{P}_{X1B1} \right]^{1-\rho} + (1-\alpha) \left[\bar{P}_{X2B1} \right]^{1-\rho} \right]^{\frac{1}{1-\rho}} \\
\bar{P}_{B2}^{ind} &= \left[\alpha \left[\bar{P}_{X1B2} \right]^{1-\rho} + (1-\alpha) \left[\bar{P}_{X2B2} \right]^{1-\rho} \right]^{\frac{1}{1-\rho}}
\end{aligned}
$$

The four shares between importers and exporters (10)-(11) are (recall that $shB2forXi = 1 - shB1forXi$, and $shX2forBi = 1 - shX1forBi$ for $i =$

1, 2):

$$shB1forX1 = \frac{1}{1+\frac{\left(\bar{P}_{X1B2}-\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B2}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X1B2}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)(1-\beta)\left(\bar{P}_{B2}^{ind}\right)^{\rho\nu}\left(\bar{P}_{X1B2}\right)^{-\rho}}{\left(\bar{P}_{X1B1}-\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B1}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X1B1}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)\beta\left(\bar{P}_{B1}^{ind}\right)^{\rho\nu}\left(\bar{P}_{X1B1}\right)^{-\rho}}}$$

$$shB1forX2 = \frac{1}{1+\frac{\left(\bar{P}_{X2B2}-\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B2}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X2B2}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)(1-\beta)\left(\bar{P}_{B2}^{ind}\right)^{\rho\nu}\left(\bar{P}_{X2B2}\right)^{-\rho}}{\left(\bar{P}_{X2B1}-\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B1}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X2B1}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)\beta\left(\bar{P}_{B1}^{ind}\right)^{\rho\nu}\left(\bar{P}_{X2B1}\right)^{-\rho}}}$$

$$shX1forB1 = \frac{1}{1+\frac{\left(\bar{Z}-\bar{P}_{X2B1}\right)(1-\alpha)\left(\bar{P}_{X2B1}\right)^{-\rho}}{\left(\bar{Z}-\bar{P}_{X1B1}\right)\alpha\left(\bar{P}_{X1B1}\right)^{-\rho}}}$$

$$shX1forB2 = \frac{1}{1+\frac{\left(\bar{Z}-\bar{P}_{X2B2}\right)(1-\alpha)\left(\bar{P}_{X2B2}\right)^{-\rho}}{\left(\bar{Z}-\bar{P}_{X1B2}\right)\alpha\left(\bar{P}_{X1B2}\right)^{-\rho}}}$$

The four effective bargaining weights (13) are:

$$\tilde{\delta}_{X1B1} = \frac{1}{1+\frac{(1-\delta)H(shX1forB1,\gamma_b)}{\delta H(shB1forX1,\gamma_x)}}$$

$$\tilde{\delta}_{X1B2} = \frac{1}{1+\frac{(1-\delta)H(shX1forB2,\gamma_b)}{\delta H((1-shB1forX1),\gamma_x)}}$$

$$\tilde{\delta}_{X2B1} = \frac{1}{1+\frac{(1-\delta)H((1-shX1forB1),\gamma_b)}{\delta H(shB1forX2,\gamma_x)}}$$

$$\tilde{\delta}_{X2B2} = \frac{1}{1+\frac{(1-\delta)H((1-shX1forB2),\gamma_b)}{\delta H((1-shB1forX2),\gamma_x)}}$$

Finally, we have four pricing equations (14). The specific equations are given in the appendix. For the $X1B1$ pair we write:

$$\tilde{\delta}_{X1B1}\left(\bar{P}_{X1B1}-\frac{\rho}{\rho-1}\bar{Z}\right)\left(\bar{P}_{X1B1}-\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B1}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X1B1}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)$$
$$= \left(1-\tilde{\delta}_{X1B1}\right)\left(\bar{P}_{X1B1}-\frac{\rho}{\rho-1}\frac{1}{\lambda}\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B1}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X1B1}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)\left(\bar{Z}-\bar{P}_{X1B1}\right)$$

For the $X1B2$ pair we write:

$$\tilde{\delta}_{X1B2}\left(\bar{P}_{X1B2}-\frac{\rho}{\rho-1}\bar{Z}\right)\left(\bar{P}_{X1B2}-\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B2}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X1B2}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)$$
$$=\left(1-\tilde{\delta}_{X1B2}\right)\left(\bar{P}_{X1B2}-\frac{\rho}{\rho-1}\frac{1}{\lambda}\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B2}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X1B2}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)\left(\bar{Z}-\bar{P}_{X1B2}\right)$$

For the $X2B1$ pair we write:

$$\tilde{\delta}_{X2B1}\left(\bar{P}_{X2B1}-\frac{\rho}{\rho-1}\bar{Z}\right)\left(\bar{P}_{X2B1}-\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B1}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X2B1}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)$$
$$=\left(1-\tilde{\delta}_{X2B1}\right)\left(\bar{P}_{X2B1}-\frac{\rho}{\rho-1}\frac{1}{\lambda}\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B1}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X2B1}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)\left(\bar{Z}-\bar{P}_{X2B1}\right)$$

For the $X2B2$ pair we write:

$$\tilde{\delta}_{X2B2}\left(\bar{P}_{X2B2}-\frac{\rho}{\rho-1}\bar{Z}\right)\left(\bar{P}_{X2B2}-\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B2}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X2B2}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)$$
$$=\left(1-\tilde{\delta}_{X2B2}\right)\left(\bar{P}_{X2B2}-\frac{\rho}{\rho-1}\frac{1}{\lambda}\bar{W}\left(\bar{R}^{ex,,fix}\right)^{(1-\nu)\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{B2}^{ind}\right)^{\nu\rho\frac{1-\lambda}{\lambda}}\left(\bar{P}_{X2B2}\right)^{-\rho\frac{1-\lambda}{\lambda}}\right)\left(\bar{Z}-\bar{P}_{X2B2}\right)$$

There are four variants of (15) for the four importer-exporter pairs. For instance, the one for the extent of LCP between importer $B1$ and exporter

$X2$ is:

$$
\begin{aligned}
0 \;=\; & -\gamma_x shB1forX1 \left[\begin{array}{c} \left(\frac{\bar{P}_{X1B1}}{\bar{P}_{X1B1}-\overline{AC}_{X1B1}} - \frac{1}{\lambda} \frac{\overline{AC}_{X1B1}}{\bar{P}_{X1B1}-\overline{AC}_{X1B1}} \right) \rho\nu\alpha \\ +\frac{\bar{P}_{X1B1}}{\bar{P}_{X1B1}-\overline{AC}_{X1B1}} (1-\rho) + \frac{\rho}{\lambda} \frac{\overline{AC}_{X1B1}}{\bar{P}_{X1B1}-\overline{AC}_{X1B1}} \end{array} \right] \beta_{X1B1} \\
& - \left[\begin{array}{c} \frac{(1-\rho)\bar{P}_{X1B2}}{\rho\bar{Z}+(1-\rho)\bar{P}_{X1B2}} - \frac{(1-\rho)\bar{P}_{X1B2}}{(1-\rho)\bar{P}_{X1B2}+\rho\overline{MC}_{X1B2}} \\ -\frac{\rho\overline{MC}_{X1B2}}{(1-\rho)\bar{P}_{X1B2}+\rho\overline{MC}_{X1B2}} \frac{\lambda-1}{\lambda}\rho \\ +\gamma_b shX1forB2 \left(+\frac{\bar{P}_{X1B2}}{\bar{Z}-\bar{P}_{X1B2}} + \rho \right) \\ -\left(\gamma_b - \frac{\rho\overline{MC}_{X1B2}}{(1-\rho)\bar{P}_{X1B2}+\rho\overline{MC}_{X1B2}} \frac{\lambda-1}{\lambda} \right) \rho\nu\alpha \\ +\gamma_x shB2forX1 \left[\begin{array}{c} \left(\frac{\bar{P}_{X1B2}}{\bar{P}_{X1B2}-\overline{AC}_{X1B2}} - \frac{1}{\lambda} \frac{\overline{AC}_{X1B2}}{\bar{P}_{X1B2}-\overline{AC}_{X1B2}} \right) \rho\nu\alpha \\ +\frac{\bar{P}_{X1B2}}{\bar{P}_{X1B2}-\overline{AC}_{X1B2}} (1-\rho) + \frac{\rho}{\lambda} \frac{\overline{AC}_{X1B2}}{\bar{P}_{X1B2}-\overline{AC}_{X1B2}} \end{array} \right] \end{array} \right] \beta_{X1B2} \\
& -\gamma_x shB1forX1 \left(\frac{\bar{P}_{X1B1}}{\bar{P}_{X1B1}-\overline{AC}_{X1B1}} - \frac{1}{\lambda} \frac{\overline{AC}_{X1B1}}{\bar{P}_{X1B1}-\overline{AC}_{X1B1}} \right) \rho\nu (1-\alpha) \beta_{X2B1} \\
& - \left[\begin{array}{c} \gamma_b shX2forB2 \left(\frac{\bar{P}_{X2B2}}{\bar{Z}-\bar{P}_{X2B2}} + \rho \right) - \left(\gamma_b - \frac{\rho\overline{MC}_{X1B2}}{(1-\rho)\bar{P}_{X1B2}+\rho\overline{MC}_{X1B2}} \frac{\lambda-1}{\lambda} \right) \rho\nu(1-\alpha) \\ +\gamma_x shB2forX1 \left(\frac{\bar{P}_{X1B2}}{\bar{P}_{X1B2}-\overline{AC}_{X1B2}} - \frac{1}{\lambda} \frac{\overline{AC}_{X1B2}}{\bar{P}_{X1B2}-\overline{AC}_{X1B2}} \right) \rho\nu(1-\alpha) \end{array} \right] \beta_{X2B2} \\
& + \left[\frac{(1-\rho)\bar{P}_{X1B2}}{\rho\bar{Z}+(1-\rho)\bar{P}_{X1B2}} + \gamma_b \left(shX1forB2 \frac{\bar{P}_{X1B2}}{\bar{Z}-\bar{P}_{X1B2}} + shX2forB2 \frac{\bar{P}_{X2B2}}{\bar{Z}-\bar{P}_{X2B2}} \right) \right] \\
& + \left[\gamma_b \left(shX1forB2 \frac{\bar{Z}}{\bar{Z}-\bar{P}_{X1B2}} + shX2forB2 \frac{\bar{Z}}{\bar{Z}-\bar{P}_{X2B2}} \right) - \frac{\rho\bar{Z}}{\rho\bar{Z}+(1-\rho)\bar{P}_{X1B2}} \right] \frac{E\hat{z}\hat{s}}{E\hat{s}^2} \\
& + \left[\begin{array}{c} \gamma_b - \frac{\rho\overline{MC}_{X1B2}}{(1-\rho)\bar{P}_{X1B2}+\rho\overline{MC}_{X1B2}} \frac{\lambda-1}{\lambda} \\ -\gamma_x shB1forX1 \left(\frac{\bar{P}_{X1B1}}{\bar{P}_{X1B1}-\overline{AC}_{X1B1}} - \frac{1}{\lambda} \frac{\overline{AC}_{X1B1}}{\bar{P}_{X1B1}-\overline{AC}_{X1B1}} \right) \\ -\gamma_x shB2forX1 \left(\frac{\bar{P}_{X1B2}}{\bar{P}_{X1B2}-\overline{AC}_{X1B2}} - \frac{1}{\lambda} \frac{\overline{AC}_{X1B2}}{\bar{P}_{X1B2}-\overline{AC}_{X1B2}} \right) \end{array} \right] \left(\frac{E\hat{q}^{set}\hat{s}}{E\hat{s}^2} + \rho(1-\nu)\eta^{set} \right) \\
& + \left[\begin{array}{c} \frac{\rho\overline{MC}_{X1B2}}{(1-\rho)\bar{P}_{X1B2}+\rho\overline{MC}_{X1B2}} + \gamma_x shB1forX1 \frac{\overline{AC}_{X1B1}}{\bar{P}_{X1B1}-\overline{AC}_{X1B1}} \\ +\gamma_x shB2forX1 \frac{\overline{AC}_{X1B2}}{\bar{P}_{X1B2}-\overline{AC}_{X1B2}} \end{array} \right] \left(\frac{E\hat{w}\hat{s}}{E\hat{s}^2} + \zeta \right)
\end{aligned}
$$

The other three relations are given in a separate detailed technical appendix.

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Figure 2: Role of risk aversion, fragmentation case

Change from baseline: $\gamma_x = 4$, $\gamma_b = 4$

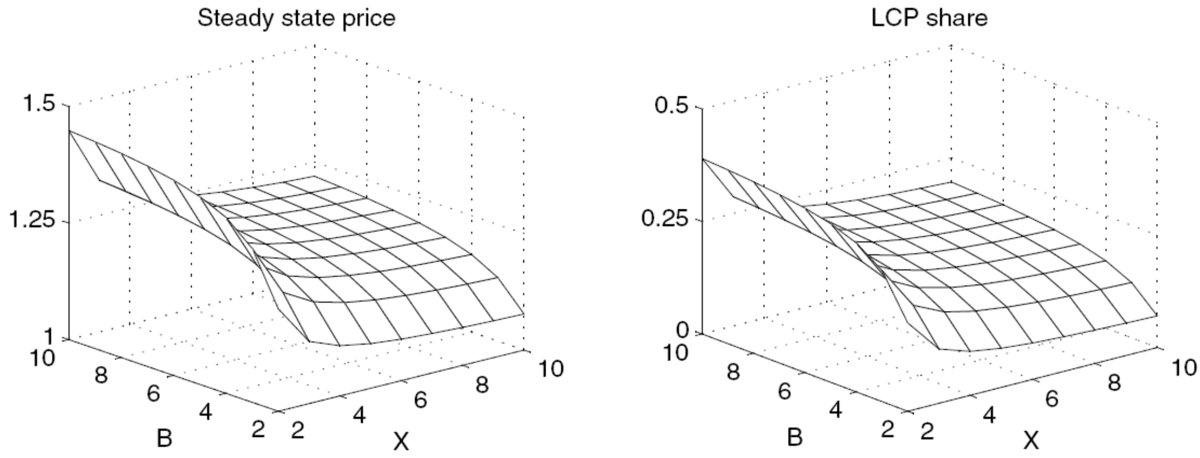


Figure 3: Role of price sensitivity, fragmentation case

Change from baseline: $\rho = 5$.

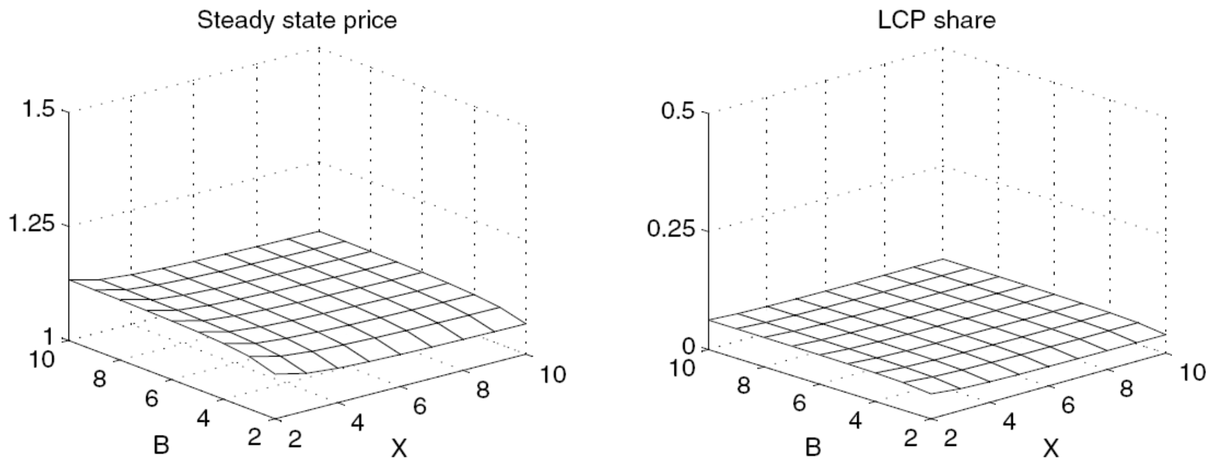


Figure 4: Role of returns to scale, fragmentation case

Change from baseline: $\lambda = 0.75$

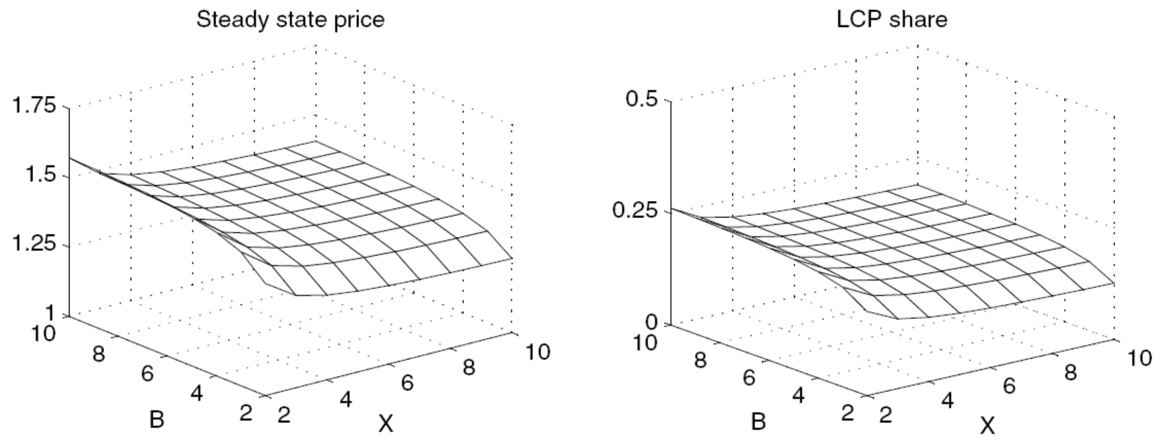


Figure 5: Role of link between cost and exchange rate, fragmentation case

Change from baseline: $\zeta_x = 0.5$

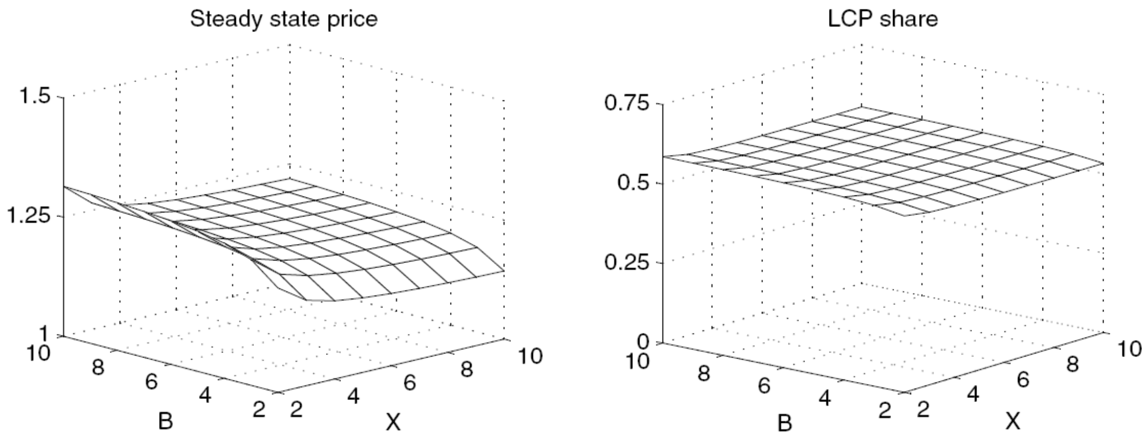
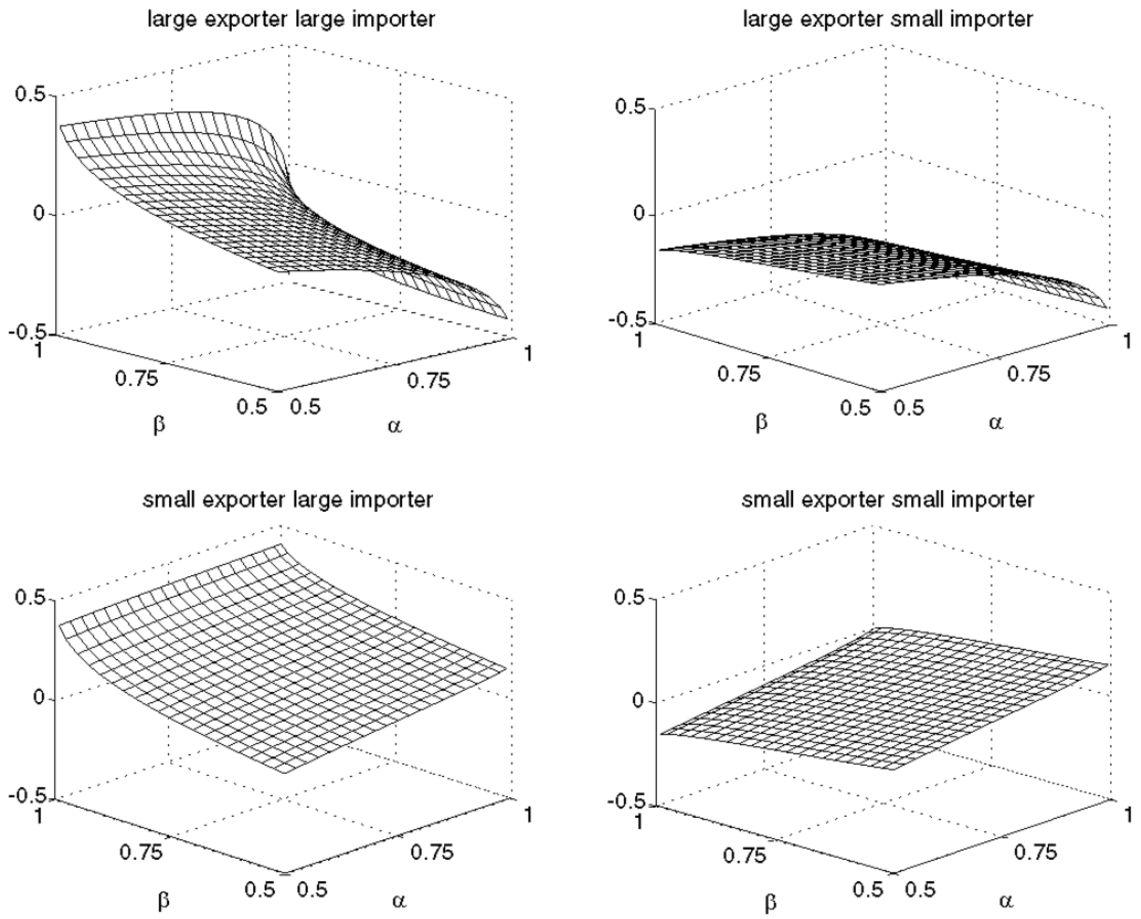


Figure 6: Effective bargaining weight (relative to formal weight), heterogeneity case

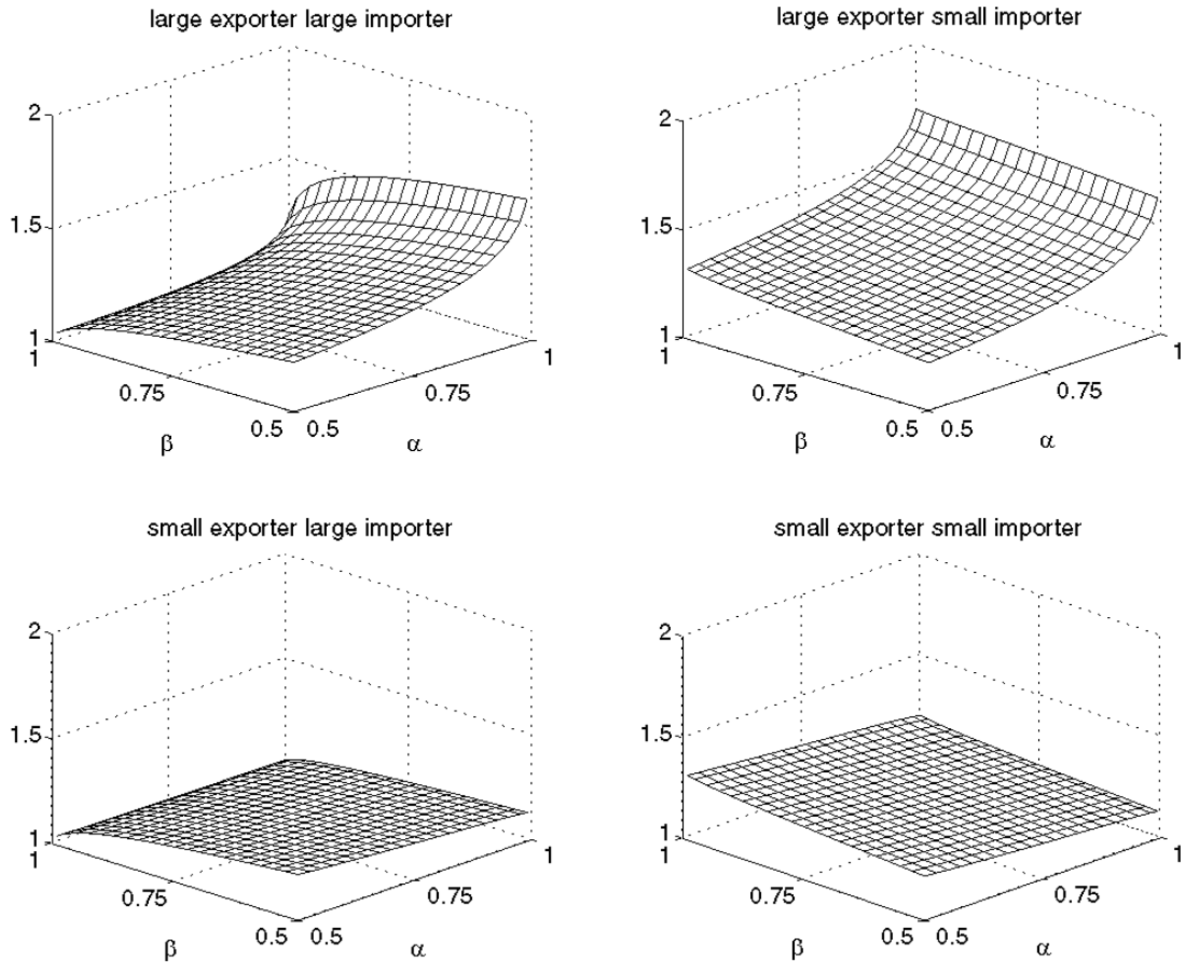
Baseline: δ = 0.5, γ_x = 2, γ_b = 2, ρ = 2, λ = 1, ν = 1, W_x = 1, Q^{set} = 10, $R^{ex,fix}$ = 1, η^{set} =0, ζ_x = 0, $E(q^{set} s)/(Es^2)$ =0, $E(w_x s)/(Es^2)$ =0, $E(z s)/(Es^2)$ =0.



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 7: Steady state price, heterogeneity case

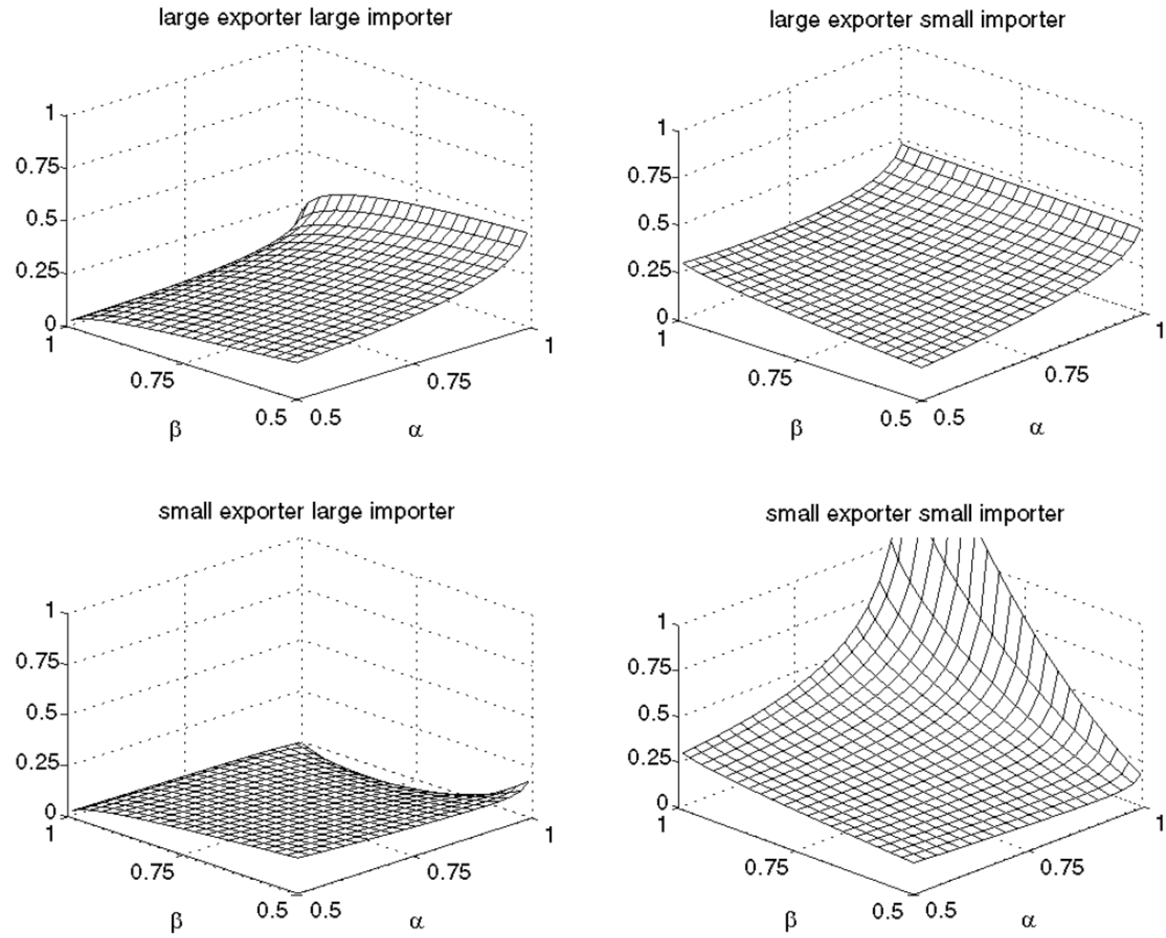
Baseline: $\delta = 0.5$, $\gamma_x = 2$, $\gamma_b = 2$, $\rho = 2$, $\lambda = 1$, $\nu = 1$, $W_x = 1$, $Q^{set} = 10$, $R^{ex,fix} = 1$, $\eta^{set} = 0$, $\zeta_x = 0$, $E(q^{set}\, s)/(Es^2) = 0$, $E(w_x\, s)/(Es^2) = 0$, $E(z\, s)/(Es^2) = 0$.



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 8: LCP share, heterogeneity case

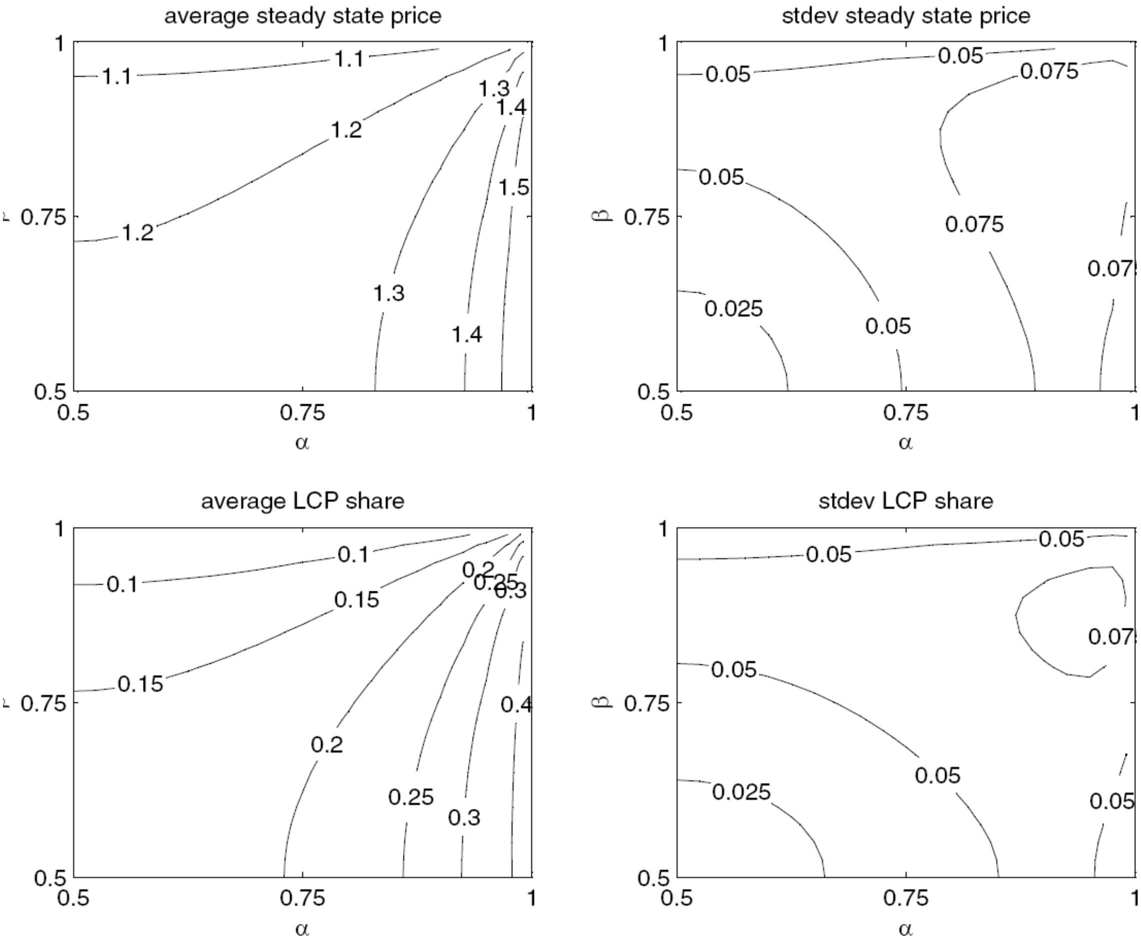
Baseline: $\delta = 0.5$, $\gamma_x = 2$, $\gamma_b = 2$, $\rho = 2$, $\lambda = 1$, $\nu = 1$, $W_x = 1$, $Q^{set} = 10$, $R^{ex,fix} = 1$, $\eta^{set} = 0$, $\zeta_x = 0$, $E(q^{set} s)/(Es^2) = 0$, $E(w_x s)/(Es^2) = 0$, $E(z s)/(Es^2) = 0$.



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 9: Average and standard deviation of steady state price and LCP share, heterogeneity case

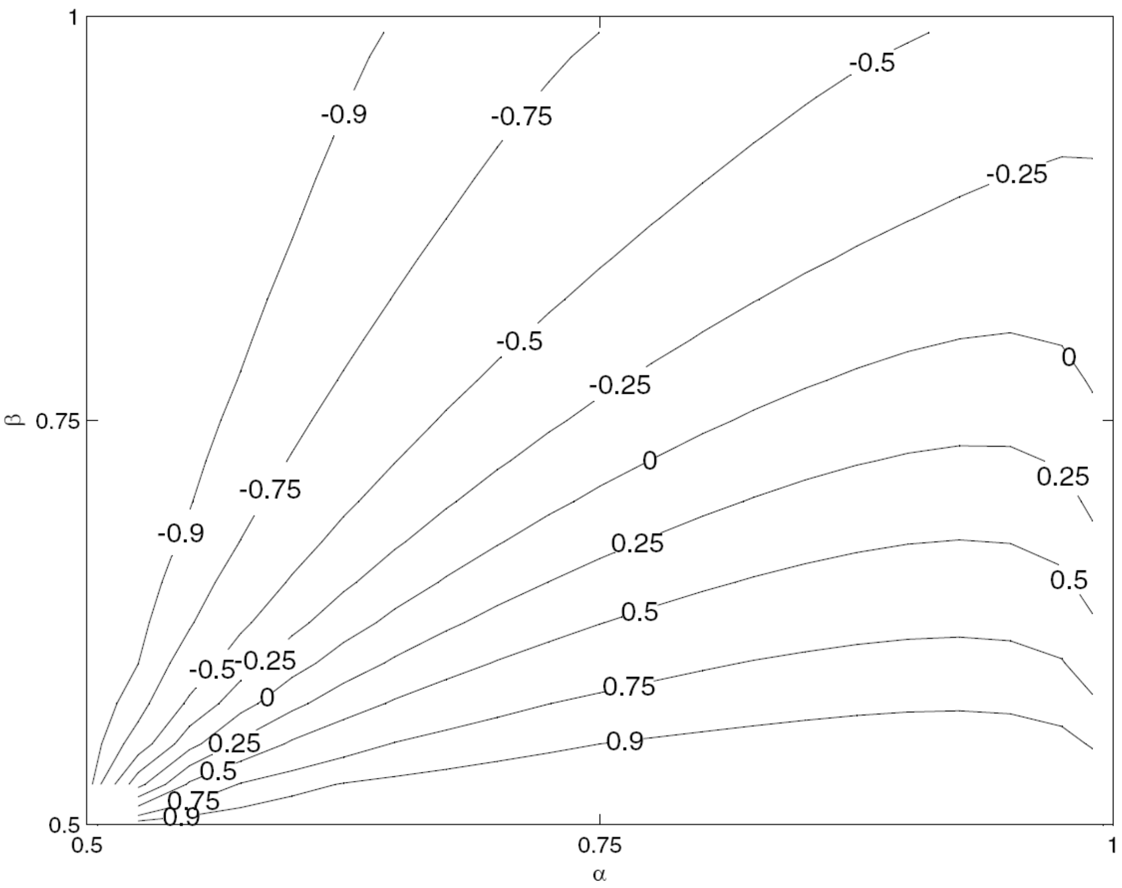
Baseline: $\delta = 0.5$, $\gamma_x = 2$, $\gamma_b = 2$, $\rho = 2$, $\lambda = 1$, $\nu = 1$, $W_x = 1$, $Q^{set} = 10$, $R^{ex,fix} = 1$, $\eta^{set} = 0$, $\zeta_x = 0$, $E(q^{set} s)/(Es^2) = 0$, $E(w_x s)/(Es^2) = 0$, $E(z s)/(Es^2) = 0$.



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 10: Correlation between transaction value and LCP share, heterogeneity case

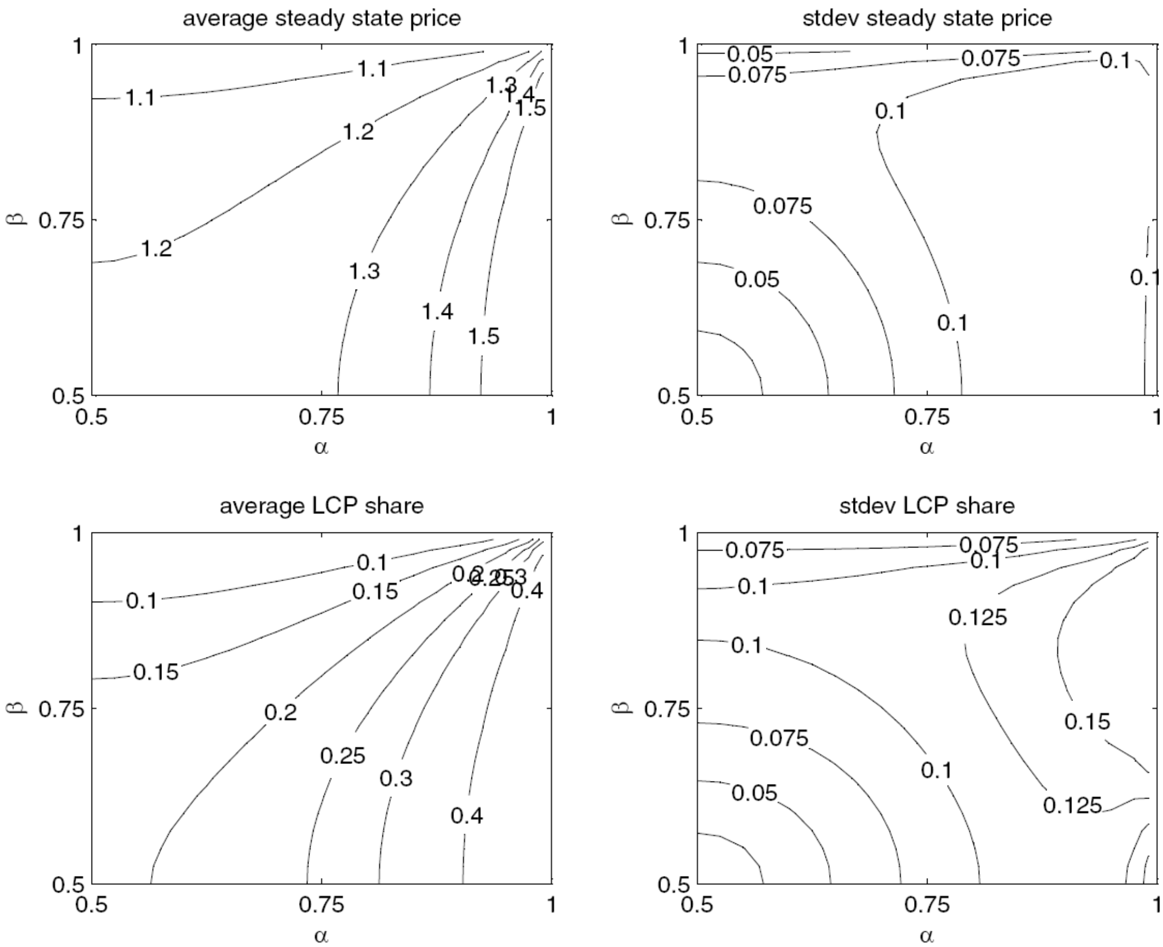
Baseline: $\delta = 0.5$, $\gamma_x = 2$, $\gamma_b = 2$, $\rho = 2$, $\lambda = 1$, $\nu = 1$, $W_x = 1$, $Q^{set} = 10$, $R^{ex,fix} = 1$, $\eta^{set} = 0$, $\zeta_x = 0$, $E(q^{set} s)/(Es^2) = 0$, $E(w_x s)/(Es^2) = 0$, $E(z s)/(Es^2) = 0$.



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 11: Role of risk aversion, Average and standard deviation of steady state price and LCP share, heterogeneity case

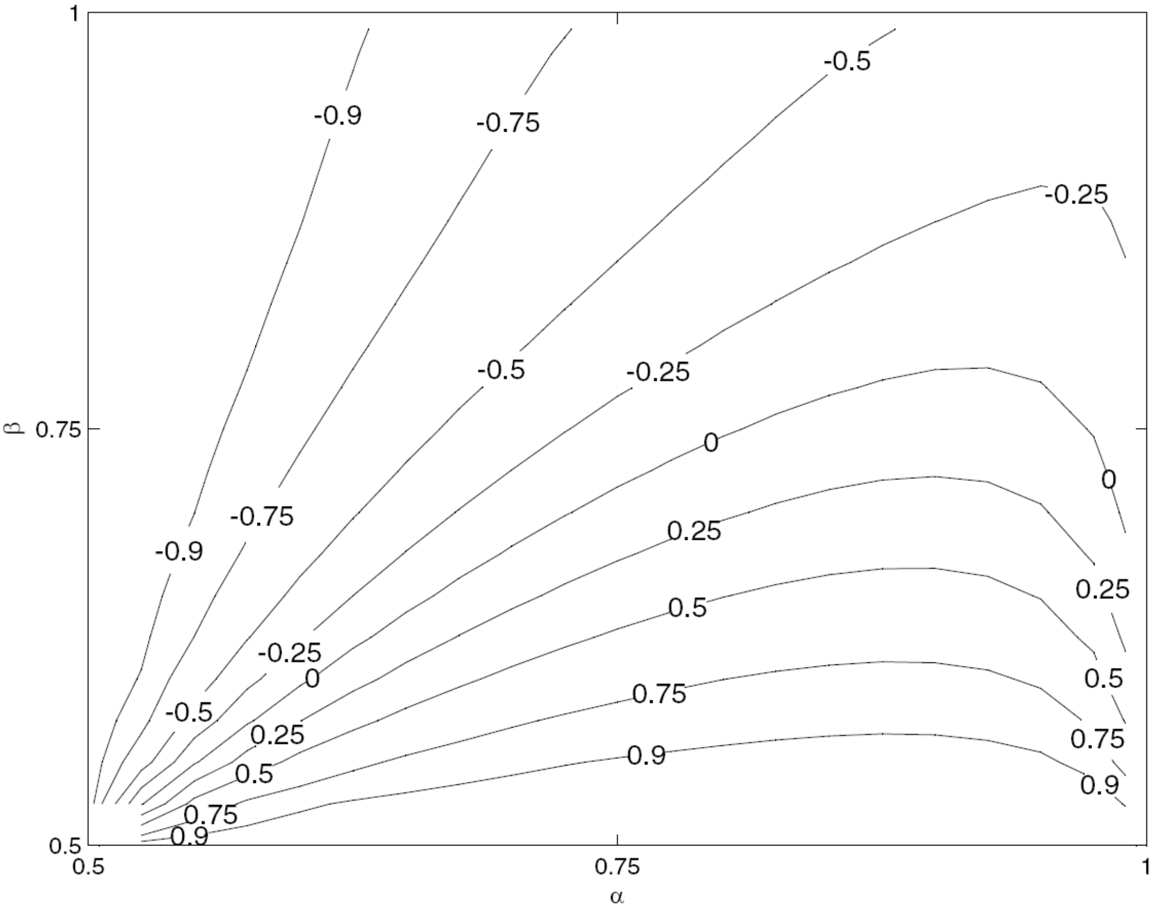
Change from baseline: $\gamma_x = 4$, $\gamma_b = 4$



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 12: Role of risk aversion, Correlation between transaction value and LCP share, heterogeneity case

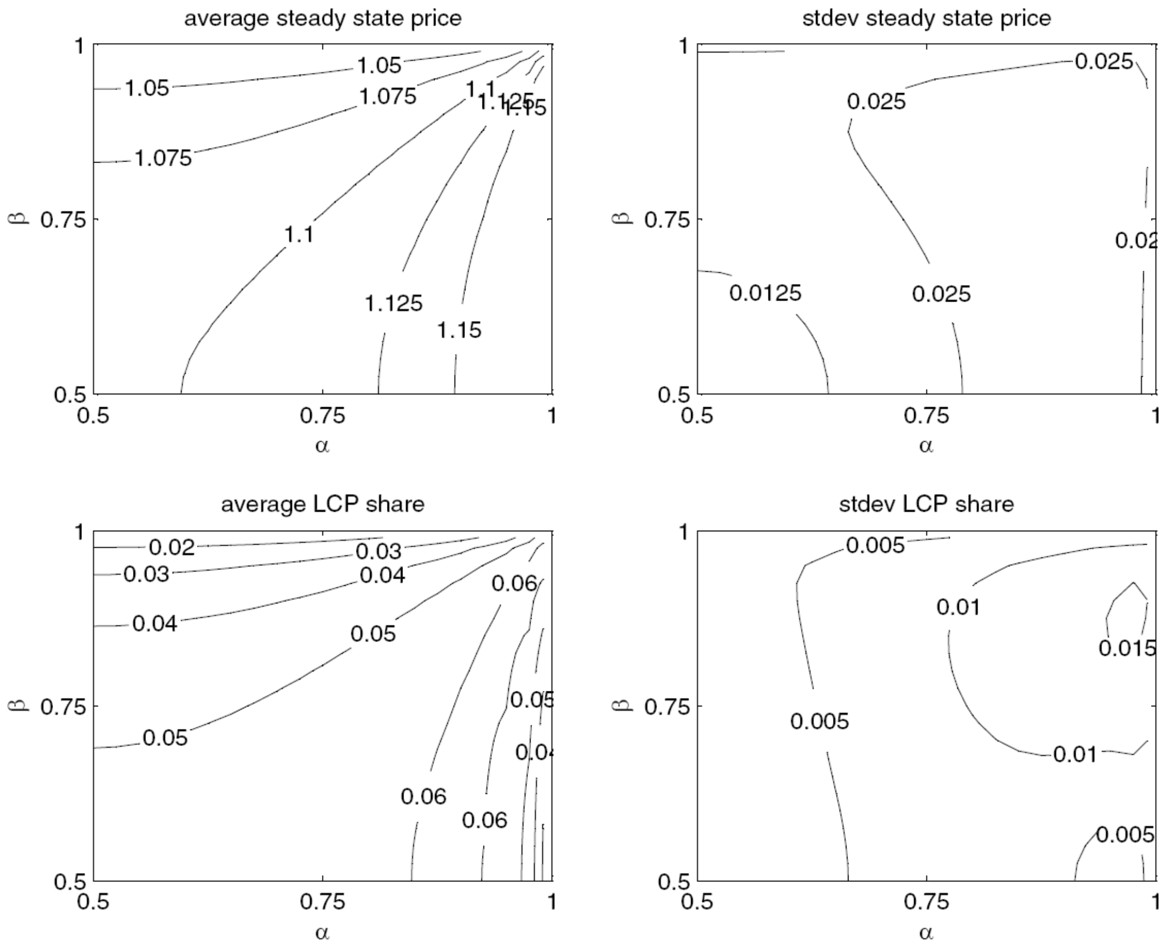
Change from baseline: $\gamma_x = 4$, $\gamma_b = 4$



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 13: Role of price sensitivity, Average and standard deviation of steady state price and LCP share, heterogeneity case

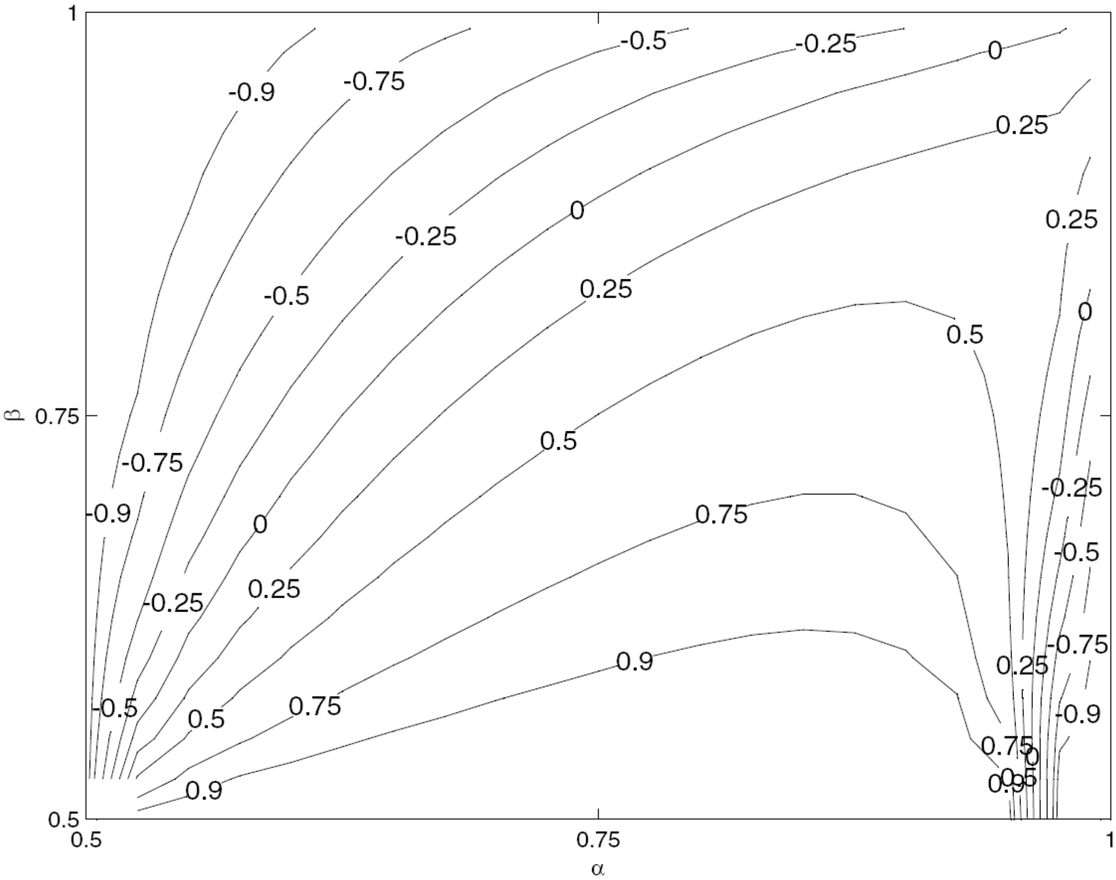
Change from baseline: ρ = 5.



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 14: Role of price sensitivity, Correlation between transaction value and LCP share, heterogeneity case

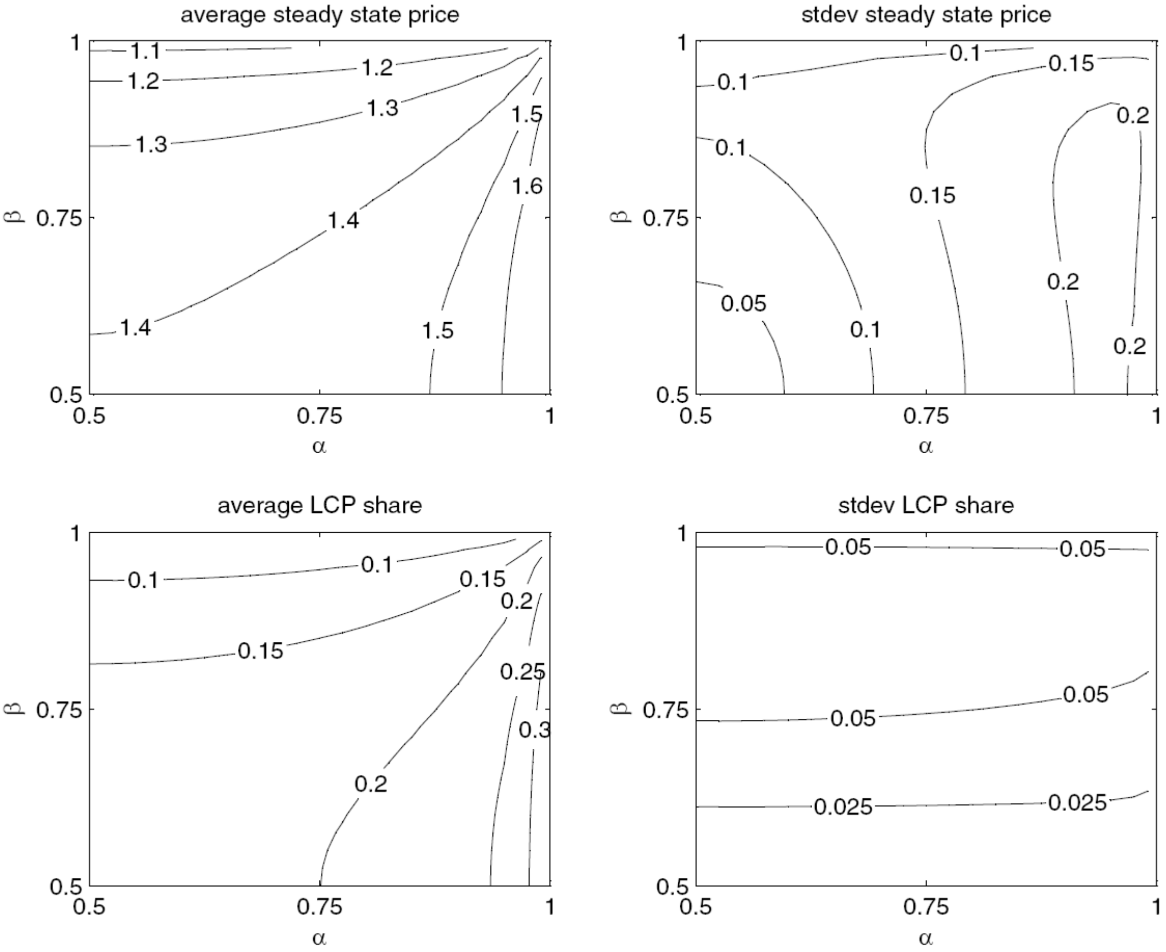
Change from baseline: ρ = 5.



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 15: Role of returns to scale, Average and standard deviation of steady state price and LCP share, heterogeneity case

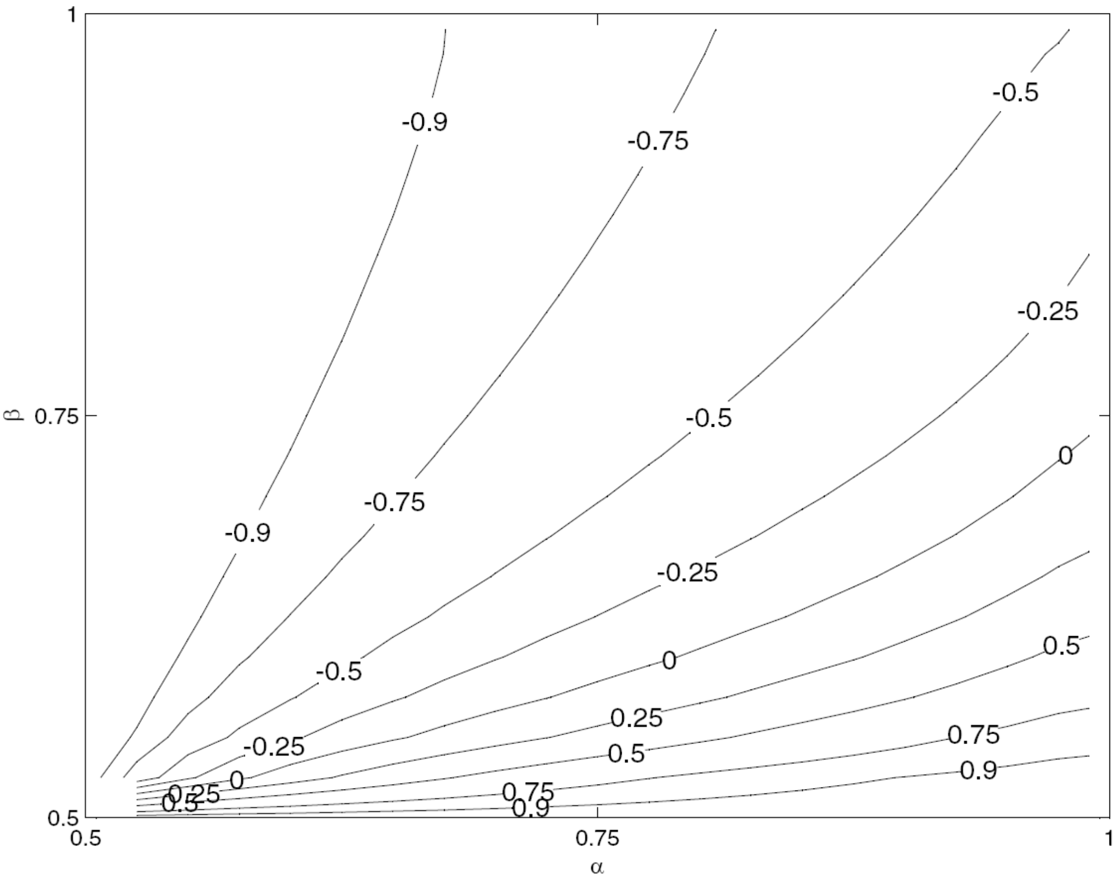
Change from baseline: λ = 0.75



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 16: Role of returns to scale, Correlation between transaction value and LCP share, heterogeneity case

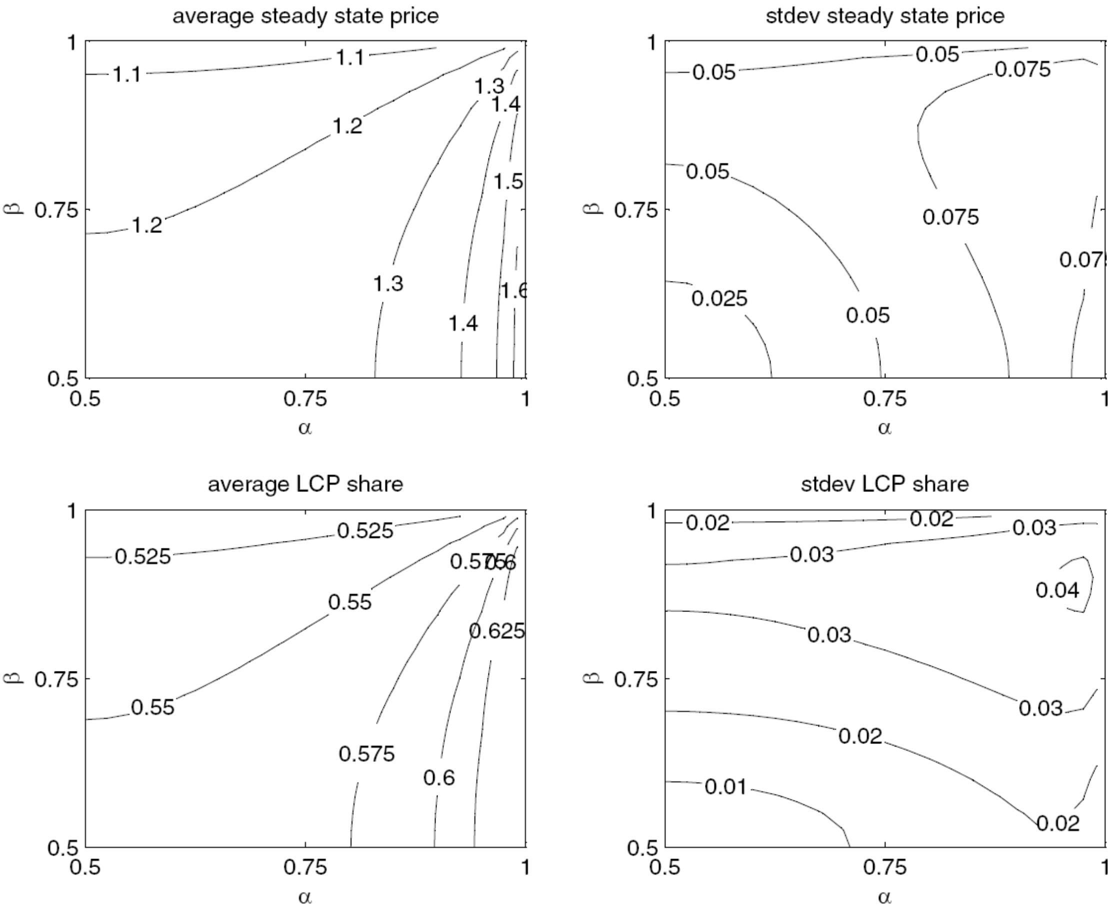
Change from baseline: λ = 0.75



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 17: Role of link between cost and exchange rate, Average and standard deviation of steady state price and LCP share, heterogeneity case

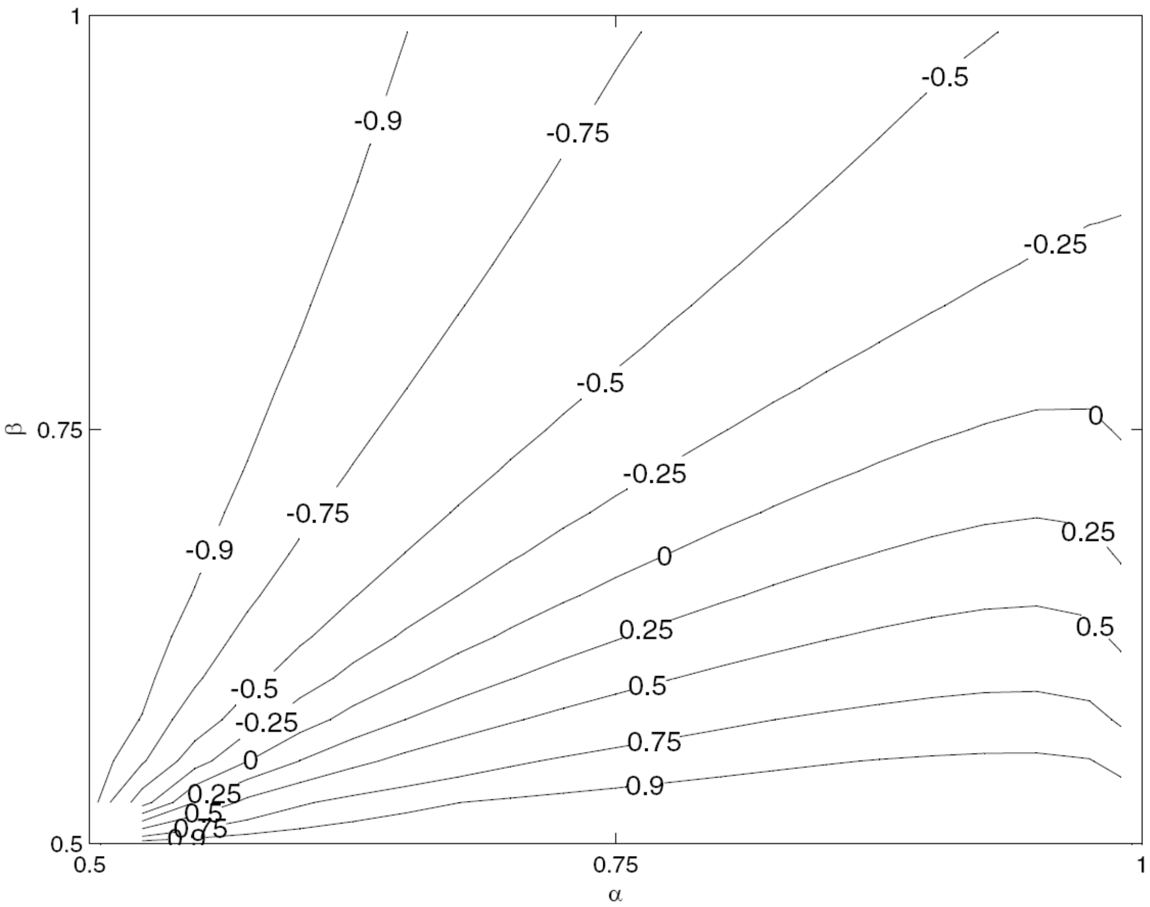
Change from baseline: $\zeta_x = 0.5$



Note : α and β denote the transaction share of the largest exporter and importer, respectively

Figure 18: Role of link between cost and exchange rate, Correlation between transaction value and LCP share, heterogeneity case

Change from baseline: $\zeta_x = 0.5$



Note : α and β denote the transaction share of the largest exporter and importer, respectively