TFP Persistence and Monetary Policy

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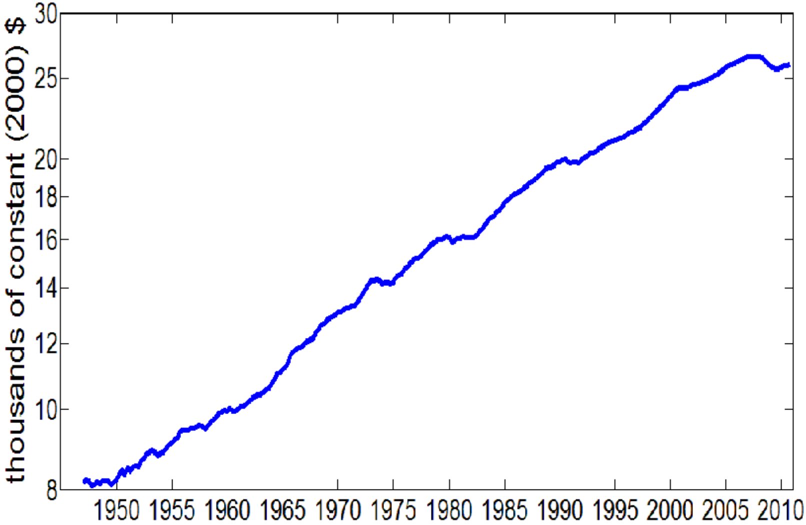
Motivation

1. Well Known Facts about the Evolution of Macroeconomic Volatility:
   - Evidence 1: Large reduction in the last 30 years *[Great Moderation: Kim and Nelson (1999), Stock and Watson (2003)]*
2. Our Findings about the Evolution of Macroeconomic Volatility
   - Evidence 2: Uneven decline of the volatility across frequencies
   - Evidence 3: Increased persistence

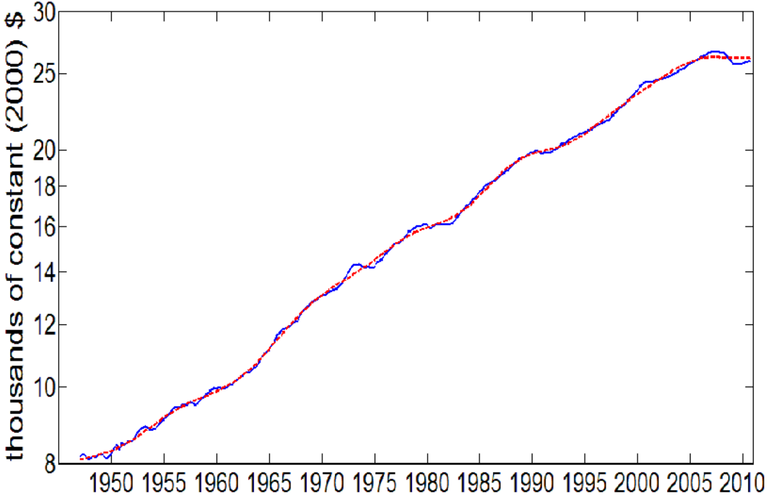
Macroeconomic Variables are Trending

Real Per Capita Consumption (US)



We Need to Isolate the Trend

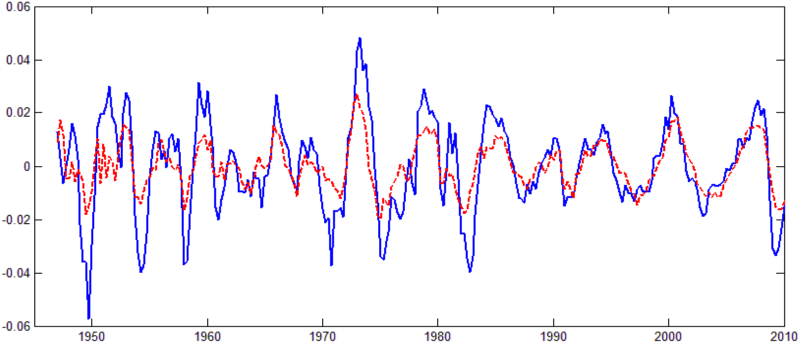
Real Per Capita Consumption (US) and its Trend



Motivation

Business Cycle Frequencies (2q-32q) component

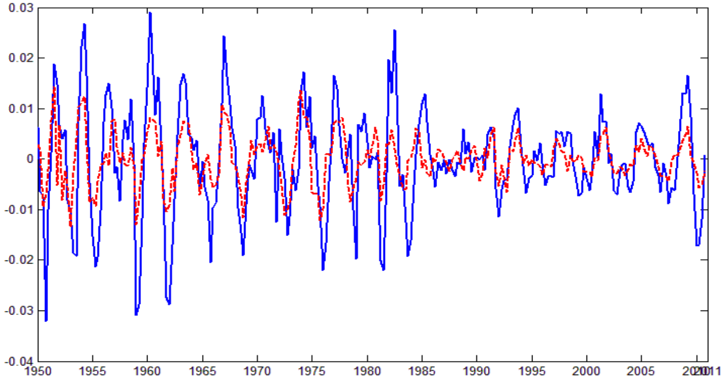
- US real per capita Consumption (red-dashed) and Output (blue-solid): 1950:1 - 2010:4



Motivation

High-Frequencies (2q-16q) components

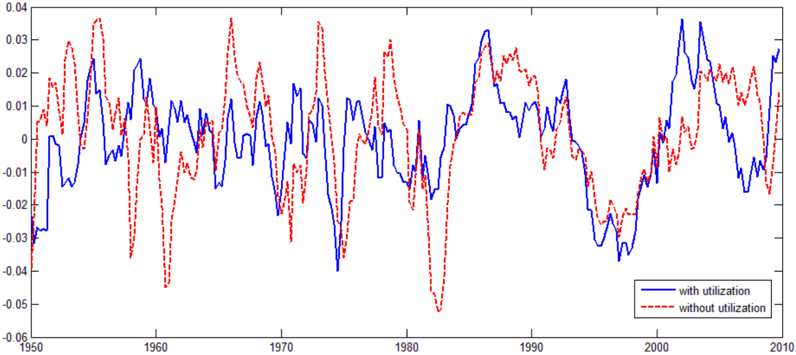
- US real per capita Consumption (red-dashed) and Output (blue-solid): 1950:1 - 2010:4
- Band-Pass filters: Christiano and Fitzgerald 2003.



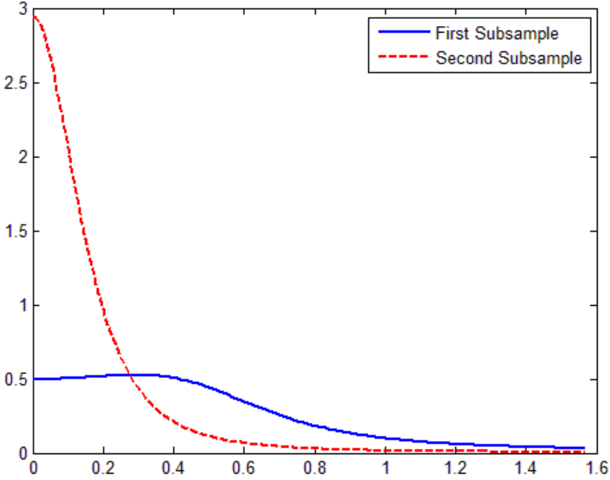
Motivation

Total Factor Productivity

- What drives the real variables dynamics? A possible candidate is TFP.
- Stationary component of U.S. TFP with varying capacity utilization (blue-solid) and constant utilization (red-dashed): 1950:1 - 2010:4



Normalized Spectrum of the TFP



Motivation

Equilibrium dynamics of a model:

$$y_t = g(x_t; \Theta, \Phi) \qquad x_{t+1} = h(x_t; \Theta, \Phi)$$

$$\Theta = \text{structural parameters}$$

$$\Phi = \begin{bmatrix} \varrho & \Sigma \end{bmatrix} \text{ laws of motion parameters}$$

- A change in the autocorrelations structure of an exogenous process has first-order effect in the equilibrium:

$$y_t \simeq \tilde{g}\left(\Theta, \varrho\right) x_t$$

$$x_{t+1} \simeq \tilde{h}\left(\Theta, \varrho\right) x_t$$

- Policy-makers maximize some objective functions to determine their policy, taking into account the equilibrium dynamics.
- A changed autocorrelation structure modifies the policy functions, thus altering the optimal policy

In this paper:

1. Provide evidence of a change in the autocorrelation structure of TFP
   - Split Sample, Rolling Windows, Recursive Regressions, TVP-SV Estimates
2. Analyze the analytical relationship between TFP persistence and monetary policy
   - Classical Monetary Model (provide intuition)
   - New-Keynesian Model (workhorse for monetary economics)
   - Medium Scale DSGE Model (numerical methods)
3. Derive the optimal monetary policy as a function of the TFP persistence

Road-map

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Total Factor Productivity

Constructing the data

- Y_t and L_t = non-farm business hours and output (BLS). K_t = Capital (BLS). U_t =capacity utilization manufacturing (FED Board)
- $$TFP_t = \frac{Y_t}{L_t^{1-\alpha}\left(U_t K_t\right)^{\alpha}}$$
- Stationary component $\overline{TFP}_t$ follows an autoregressive process:

$$\left(1 - B\left(L\right)\right)\overline{TFP}_t = \sigma_\varepsilon \varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} N\left(0, 1\right).$$

Split Sample Statistics

- Split Sample at the early eighties (GM break)

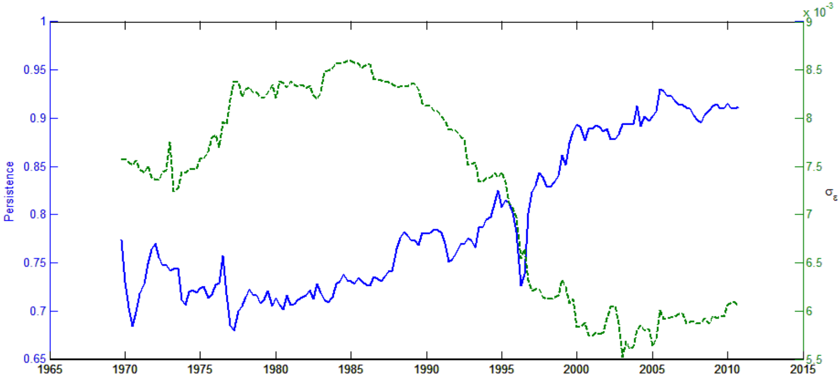
	Sample 1: 1950:1- 1982:4		Sample 2: 1983:1-2009:4	
	Largest Root	Std. Dev. Innovations	Largest Root	Std. Dev. Innovations
		AR(1)		
Varying utilization	0.74 [0.06]	0.78 [0.05]	0.95 [0.04]	0.61 [0.04]
Constant utilization	0.83 [0.05]	1.03 [0.06]	0.91 [0.04]	0.63 [0.04]
		AR(4)		
Varying utilization	0.60	0.77	0.84	0.60
Constant utilization	0.63	1.00	0.92	0.59

spectrum

Rolling Windows Statistics

- Assume an $AR(1)$ process for $\overline{TFP}_t$: compute $\hat{\rho}_t$ (blue-solid) and $\hat{\sigma}_{\varepsilon,t}$ (green-dashed)

$$\hat{\rho}_t = \hat{\rho}\left(\left\{\overline{TFP}\right\}_{j=t-k}^{t}\right) \quad \hat{\sigma}_{\varepsilon,t} = \hat{\sigma}_{\varepsilon}\left(\left\{\overline{TFP}\right\}_{j=t-k}^{t}\right) \quad for\ t = k+1, \ldots, T$$

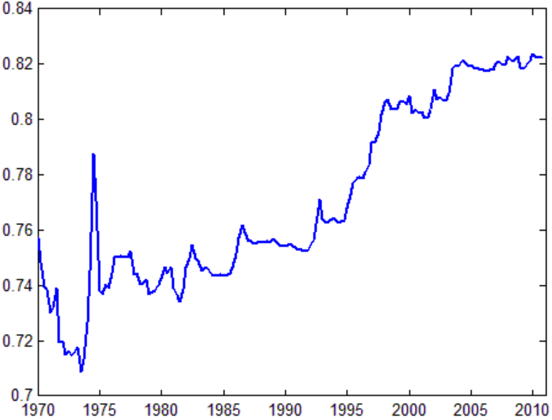


- $k = 80$

Recursive Regression Estimates

- Assume an $AR(1)$ process for $\overline{TFP}_t$: compute $\hat{\rho}_t^{RE}$

$$\hat{\rho}_t^{RE} = \hat{\rho}\left(\left\{\overline{TFP}\right\}_{\bar{k}}^{t}\right) \qquad for \ \ t = \bar{k}+1, ..., T$$



TVP-SV Model

- Evidence suggests that $\hat{\rho}$ and $\hat{\sigma}_\varepsilon$ are time-varying:

$$\overline{TFP}_t = \rho \overline{TFP}_{t-1} + \sigma_\varepsilon \varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} N(0,1)$$

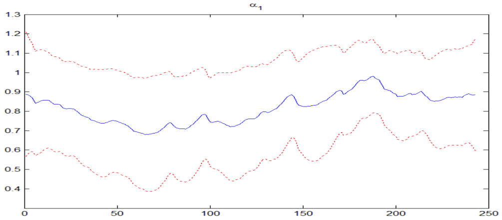
- We formally estimate a Time-Varying-Parameter with Stochastic Volatility:

$$\begin{aligned}
\overline{TFP}_t &= \rho_t \overline{TFP}_{t-1} + \varepsilon_t \qquad \varepsilon_t \sim N\left(0, \sigma_t^2\right) \\
\rho_{t+1} &= \rho_t + u_t \qquad u_t \sim N\left(0, \sigma_u^2\right) \\
\sigma_t^2 &= \gamma \exp\left(h_t\right) \\
h_{t+1} &= \phi h_t + \eta_t \qquad \eta_t \sim N\left(0, \sigma_\eta^2\right).
\end{aligned}$$

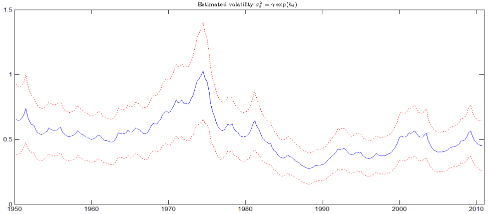
- Use 1 million repetitions with MCMC methods.

TVP-SW Model

- Posterior mean estimate for α_t :



- Posterior mean estimate for σ_t^2 :



Road-map

1. Provide evidence of a change in the autocorrelation structure of TFP
    - Split sample, Rolling Windows, Recursive Regressions, TVP-SV Estimates
2. **Analyze the analytical relationship between TFP persistence and monetary policy**
    - **Classical Monetary Model (provide intuition)**
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Classical Monetary Model (Cooley-Hansen 1989)

- Money is neutral. Example to illustrate inflation dynamics.
- Perfect competition, flexible prices.
- Agents trade one-period nominally risk-less bonds
- Monetary authority sets the nominal interest rate:

$$i_t = \rho + \phi_\pi \pi_t$$

- Fisherian equation:

$$i_t = E_t \pi_{t+1} + r_t$$

Simple Monetary Model

- Real interest rate dynamics:

$$r_t = \rho + \sigma\psi E_t \left\{\Delta a_{t+1}\right\}$$

- Exogenous TFP:

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t \qquad \varepsilon_t \overset{iid}{\sim} N(0,1)$$

- Equilibrium inflation dynamics:

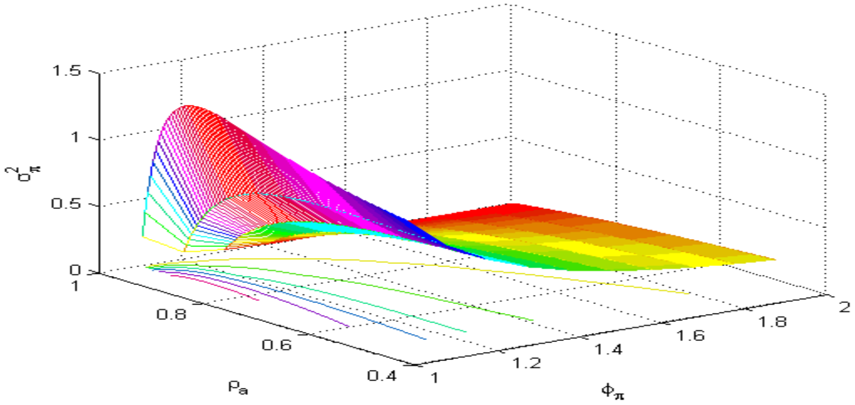
$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t \left(r_{t+k} - \rho\right),$$

$$\pi_t = \delta_a a_t \qquad \delta_a = -\frac{\sigma\psi\left(1 - \rho_a\right)}{\phi_\pi - \rho_a}$$

$$\sigma_\pi^2 = \delta_a^2 \frac{\sigma_a^2}{(1-\rho_a^2)}$$

Inflation Variance and TFP Persistence

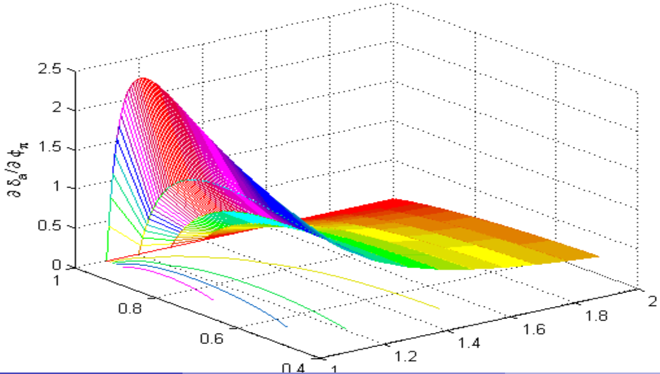
- Inflation variance as a function of TFP persistence and monetary policy parameter:



Effectiveness of Monetary Policy and TFP Persistence

- Effectiveness of Monetary Policy on the instantaneous response of inflation to a technology shock:

$$\frac{\partial \delta_a}{\partial \phi_\pi} = \frac{\sigma\psi\left(1 - \rho_a\right)}{\left(\phi_\pi - \rho_a\right)^2}.$$



Effectiveness of Monetary Policy and TFP Persistence

Proposition: *If we consider a simple monetary model just described, then we can show that the variance of inflation is non-monotone in* ρ_a *and the value of the monetary policy response to inflation* ϕ_π *that maximizes the variance of inflation in is given by:*

$$\phi_\pi^* = 1 + \rho_a - \rho_a^2$$

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Standard New-Keynesian Model

- Introducing features to obtain non-neutrality of money
- Staggered price setting and imperfect competition
- Agents trade one-period nominally risk-less bonds
- Monetary authority sets the nominal interest rate:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

- Exogenous TFP and monetary shocks

$$\begin{aligned} a_t &= \rho_a a_{t-1} + \sigma_a \varepsilon_t^a, \text{ where } \varepsilon_t^a \sim N(0,1) \\ v_t &= \rho_v v_{t-1} + \sigma_v \varepsilon_t^v, \text{ where } \varepsilon_t^v \sim N(0,1) \end{aligned}$$

Equilibrium: first order approximation
Output Gap

$$\tilde{y}_t = \Lambda_v \left(\phi_\pi, \phi_y, \rho_v, \Theta\right) v_t + \Lambda_a \left(\phi_\pi, \phi_y, \boxed{\rho_a}, \Theta\right) a_t$$

$$\Lambda_v \left(\phi_\pi, \phi_y, \rho_v, \Theta\right) = -\frac{(1-\beta\rho_v)}{(1-\beta\rho_v)\left(\sigma(1-\rho_v)+\phi_y\right)+\kappa(\phi_\pi-\rho_v)}$$

$$\Lambda_a \left(\phi_\pi, \phi_y, \rho_a, \Theta\right) = \frac{-\psi\sigma\left(1-\boxed{\rho_a}\right)\left(1-\beta\boxed{\rho_a}\right)}{\left(1-\beta\boxed{\rho_a}\right)\left(\sigma\left(1-\boxed{\rho_a}\right)+\phi_y\right)+\kappa\left(\phi_\pi-\boxed{\rho_a}\right)}$$

Equilibrium: first order approximation

Inflation

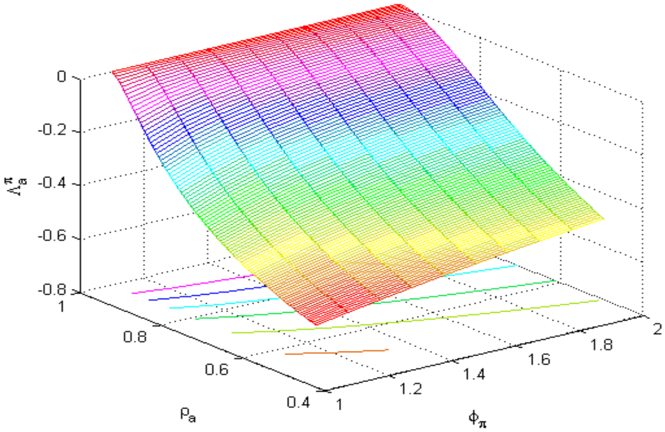
$$
\begin{aligned}
\pi_t &= \Lambda_v^\pi\left(\phi_\pi, \phi_y, \rho_v, \Theta\right) v_t + \Lambda_a^\pi\left(\phi_\pi, \phi_y, \boxed{\rho_a}, \Theta\right) a_t \\
\Lambda_v^\pi\left(\phi_\pi, \phi_y, \rho_v, \Theta\right) &= -\frac{\kappa}{(1-\beta\rho_v)\left(\sigma(1-\rho_v)+\phi_y\right)+\kappa(\phi_\pi-\rho_v)} \\
\Lambda_a^\pi\left(\phi_\pi, \phi_y, \rho_a, \Theta\right) &= \frac{-\psi\sigma\left(1-\boxed{\rho_a}\right)\kappa}{\left(1-\beta\boxed{\rho_a}\right)\left(\sigma\left(1-\boxed{\rho_a}\right)+\phi_y\right)+\kappa\left(\phi_\pi-\boxed{\rho_a}\right)}
\end{aligned}
$$

Standard Calibration:

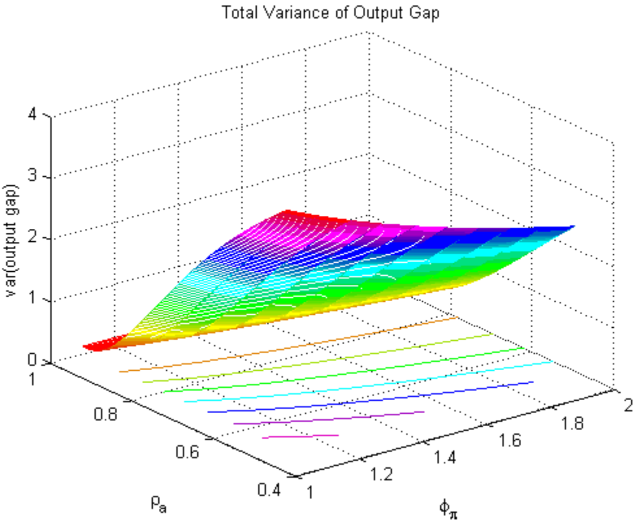
- Discount factor:$\beta = 0.99$
- Inverse of intertemporal elast. of substitution: $\sigma = 1$
- Labor share in the production function: $1 - \alpha = \frac{2}{3}$
- Elasticity of subs among differentiated goods: $\varepsilon = 6$
- Price stickiness parameter: $\theta = \frac{2}{3}$
- Inverse of the Frish elast. of labor supply: $\varphi = 1$
- Monetary response to output gap: $\phi_y = 0.125$

Equilibrium: Response of Output Gap to a Technology Shock

- Monotone relationship between TFP persistence and output gap response to technology (Λ_a)



Equilibrium: Variance of Output Gap



Intuition:

- $$r_t^n = \rho + \sigma\psi E_t\{\Delta a_{t+1}\} = \rho + \sigma\psi(1-\rho_a)\,a_t$$
- When $\rho_a \to 1$, then r_t^n constant, as well as $r_{t.}$
- The output gap in this model results from the current and anticipated deviations of the real interest rate from its natural level:

$$\tilde{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\left(r_{t+k} - r_{t+k}^n\right)$$

- Marginal cost:

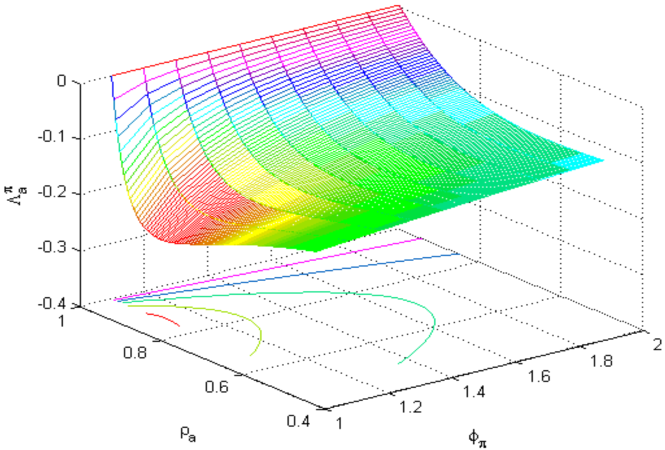
$$\widetilde{mc}_t \propto \tilde{y}_t$$

- Inflation results from the price-setting decisions by firms that adjust their price considering present and current cost conditions:

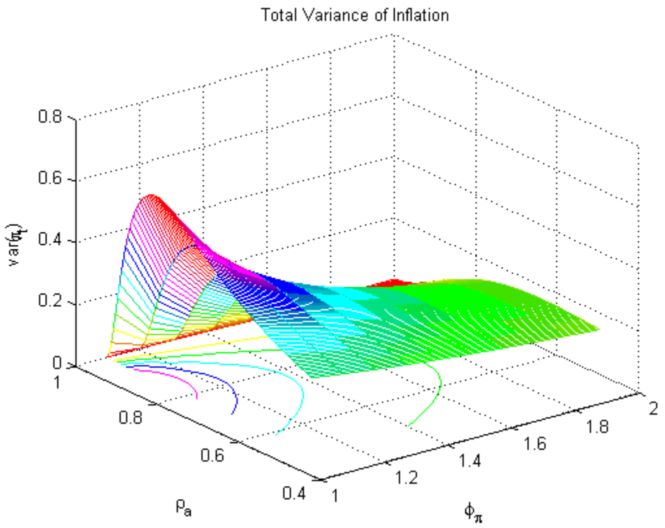
$$\pi_t = \lambda\sum_{k=0}^{\infty}\beta^k E_t\left\{\widetilde{mc}_{t+k}\right\}$$

Equilibrium: Response of Inflation to a technology shock

- Non-Monotone relationship between TFP persistence and inflation response to technology (Λ_a^π)



Equilibrium: Variance of Inflation



Proposition: *Assume that* $\rho_a \in (-1, 1)$, $\beta < 1$, $\phi_y > 0$, $\theta < 1$, $\alpha < 1$, $\sigma > 0$, $\varepsilon > 0$, $\zeta > 0$, *and* $\phi_\pi > 1$. *Then*

$$\frac{\partial \ \Lambda_a \left(\phi_\pi, \phi_y, \rho_a, \Theta\right)}{\partial \ \phi_\pi} > 0$$

and

$$\frac{\partial \ \Lambda_a \left(\phi_\pi, \phi_y, \rho_a, \Theta\right)}{\partial \ \rho_a} > 0$$

for any structural parameter vector Θ.*Moreover, there exists a value* $\phi_\pi^{\pi*}$ *that maximizes instantaneous response* $\left|\Lambda_a^\pi \left(\phi_\pi, \phi_y, \rho_a, \Theta\right)\right|$. *This value is:*

$$\phi_\pi^{\pi*} = \frac{\kappa + \beta\sigma \left(1 - \rho_a\right)^2 - \left(1 - \beta\right)\phi_y}{\kappa}$$

for any structural parameter vector Θ

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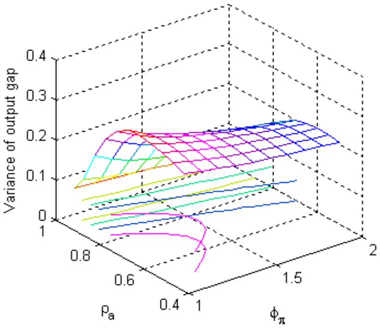
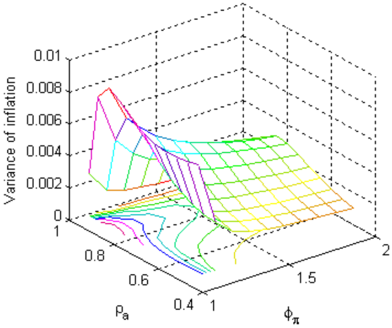
DSGE Model

- More features are considered (add capital, real rigidities - habit persistence, investment adjustment cost, variable capacity utilization)
- Smets and Wouters (2007)
- Nonlinear relationship among technology persistence, monetary policy response to inflation and variances of output gap and inflation

DSGE Model

Variance of Output Gap and Inflation





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Welfare

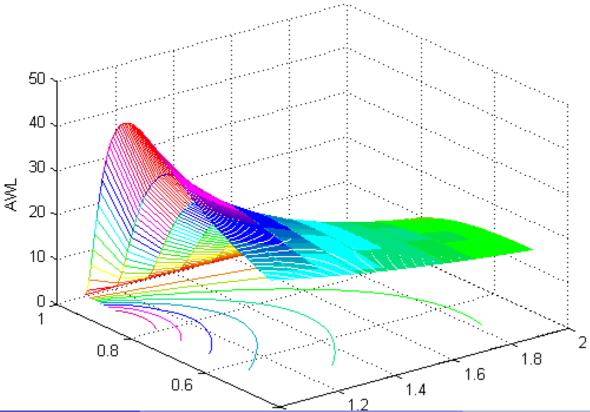
- Average Welfare Loss: Rotemberg and Woodford (1999)
- Second order approximation to the consumer utility loss due to deviations from efficient allocation:

$$AWL = \frac{1}{2}\left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) var\left(\tilde{y}_t\right) + \frac{\varepsilon}{\lambda} var\left(\pi_t\right)\right].$$

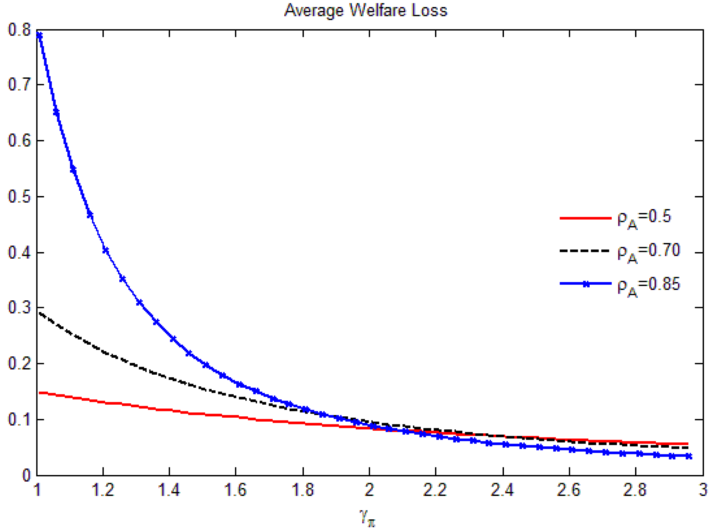
- Calibrated parameters: $\sigma_a = 0.45\%$, $\sigma_v = 0.24\%$, $\rho_v = 0.15$.
- The ratio of variance of v_t shock and α_t shock is kept constant at 3% as estimated by Smets and Wouters (2007)
- Isolate the effect of the increasing persistence by keeping constant the unconditional variance of a_t when varying ρ_a

Welfare

- Average Welfare Loss
- No Trade-off: respond to inflation arbitrarily strong (*devine coincidence*)
- Interesting dynamics driven by ρ_a



Welfare



Average Welfare Loss
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3
γ_π
ρ_A=0.5
ρ_A=0.70
ρ_A=0.85

Optimal Monetary Policy with Trade-off

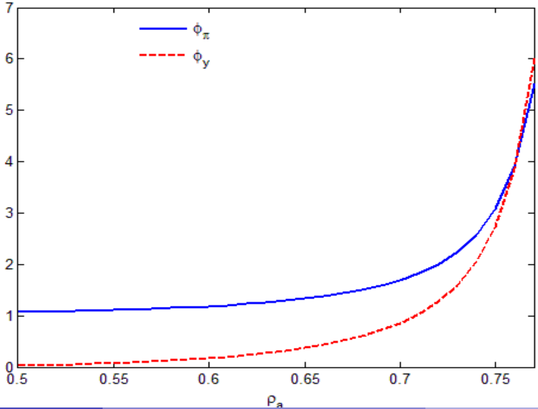
- Without cost-push shocks, the monetary authority **does not face trade off** between stabilizing output variance and inflation variance and it is optimal to respond to inflation as strongly as possible
- Add cost-push shocks and then characterize optimal Taylor rule under commitment (ϕ_π^*, ϕ_y^*) by minimizing expected welfare loss:

$$E\left(WL\right) = E\left\{(1-\beta)\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2 + \lambda_y\left(\tilde{y}_t - y^*\right)^2 + \lambda_i\left(i_t - i^*\right)^2\right]\right\}$$

- Optimal ϕ_π and ϕ_y depend on the persistence of productivity: optimal response is to increase both ϕ_π and ϕ_y as a response to higher persistence of TFP

Optimal Monetary Policy Parameters as a Function of the Persistence of Technology

- Higher TFP response calls for a higher response of monetary policy, implying a lower ability of monetary policy to smooth the volatility of macroeconomic variables



Conclusions

- Statistical evidences for increasing TFP persistence
- Analyze the relationship between TFP persistence and monetary policy in the equilibrium dynamics of monetary models
- Study the effects of increased TFP persistence on the optimal monetary policy
- Investigation of the productivity in the different sectors of the economy [*Ongoing Research*]