Commitment Through Renegotiation-Proof Contracts under Asymmetric Information

Emanuele Gerratana[1] Levent Koçkesen[2]

[1]SIPA, Columbia University

[2]Koç University

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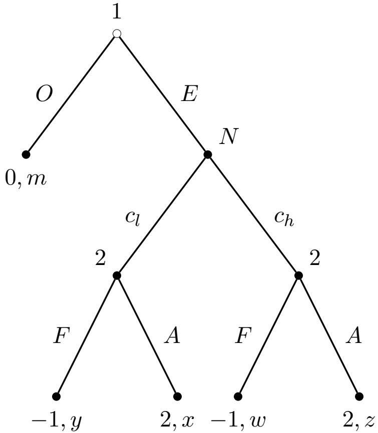
- We analyze this question in dynamic games with asymmetric information
- Contracts can be

	Non-renegotiable	Renegotiable
Observable	Known	?
Unobservable	Known	?

Example: Entry Game



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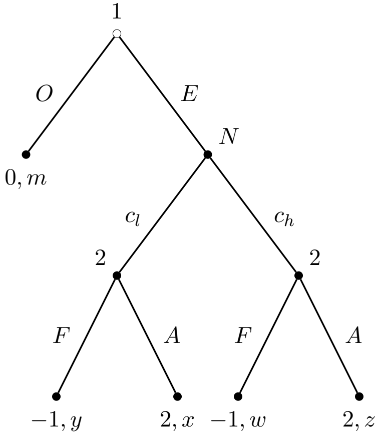

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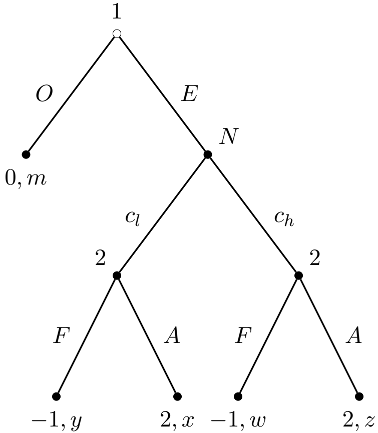

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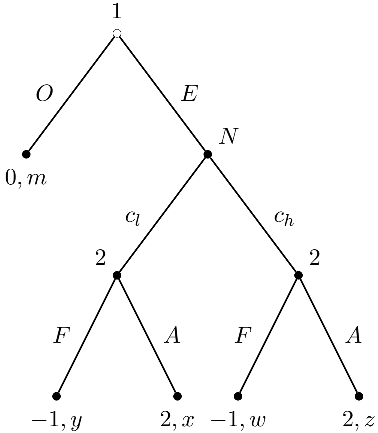

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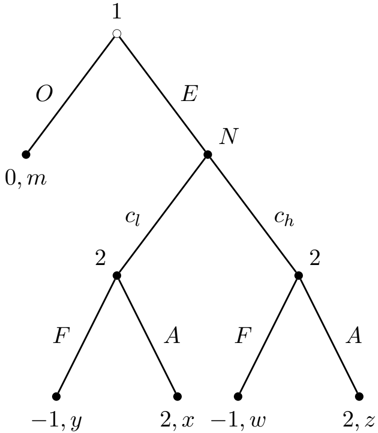

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- How about renegotiable contracts?

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- Contracts with third parties matter in strategic interactions

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Observable and Non-renegotiable Contracts

- Folk theorems

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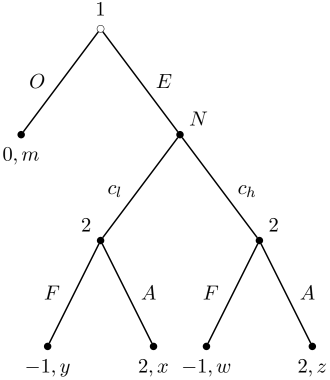
- Fershtman, Judd, and Kalai (1991), Polo and Tedeschi (2000), Katz (2006)

Entry Game: Observable and Non-renegotiable Contracts



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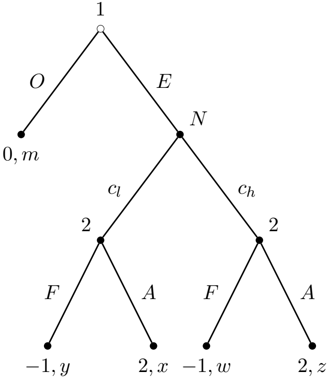

- FF deters entry
- A contract that supports FF:

$$f(F) = \delta, \quad f(A) = \delta + (z - w)$$

Unobservable and Non-renegotiable Contracts

Katz (1991)

- NE outcomes of game with contracts = NE outcomes of original game

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Koçkesen and Ok (2004) and Koçkesen (2007)

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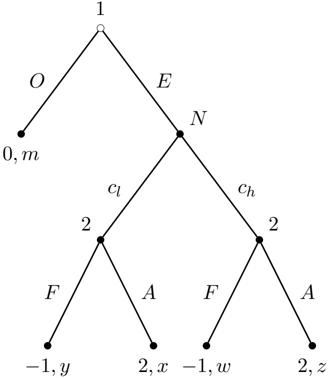

- (O, FF) is a (Bayesian) Nash equilibrium

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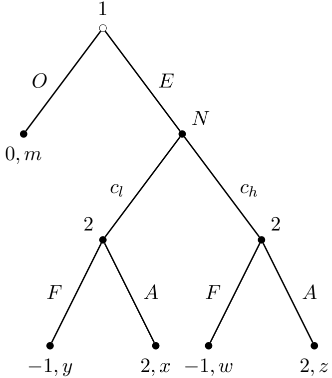

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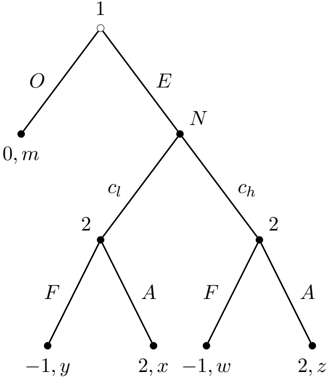
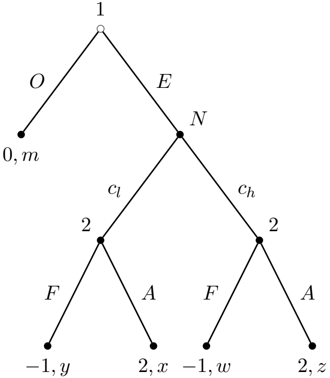


- (O, FF) is a (Bayesian) Nash equilibrium
- It can be supported with unobservable contracts
- Can use the same contract

$$f(F) = \delta, \quad f(A) = \delta + (z - w)$$

Entry Game: Renegotiable Contracts



- Can we support FF with renegotiable contracts?

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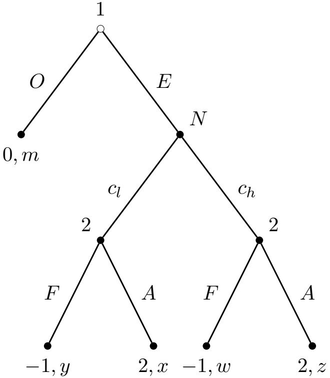

- Can we support FF with renegotiable contracts?
- Not if renegotiation is frictionless

Renegotiable Contracts

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    - Also we look at unobservable contracts and let informed player initiate renegotiation

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    - Only player 2 can write contracts

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- If player 1 observes f before choosing $a_1 \to$ Observable contracts

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Stage III. Nature chooses θ

Stage IV. Player 1 chooses a_1 (without observing θ)

Stage V. Player 2 observes (θ, a_1)

Stage VI. Player 2 chooses a_2 or a new contract g

- $a_2 \to$ game ends
- $g \to$

  Stage VII(i). Third party (without observing θ) accepts or rejects g

  Stage VII(ii). Player 2 chooses a_2

- If player 1 observes f before choosing $a_1 \to$ Observable contracts

Extensions

- Arbitrary extensive form games with incomplete information

Extensions

- Arbitrary extensive form games with incomplete information
- Interested Third-Party

Extensions

- Arbitrary extensive form games with incomplete information
- Interested Third-Party
- Strong renegotiation-proofness

Incentive Compatibility

θ non-contractible $\rightarrow$ need more structure

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Strictly Increasing Differences

u_2 has strictly increasing differences in $(\succsim_\theta, \succsim_2)$:
$\theta \succ_\theta \theta', a_2 \succ_2 a_2' \Rightarrow$

$$u_2(a_1, a_2, \theta) - u_2(a_1, a_2, \theta') > u_2(a_1, a_2', \theta) - u_2(a_1, a_2', \theta')$$

Incentive Compatibility

Increasing Strategies

$b_2 : A_1 \times \Theta \to A_2$ is increasing in $(\succsim_\theta, \succsim_2)$ if for all a_1

$$\theta \succsim_\theta \theta' \Rightarrow b_2(a_1, \theta) \succsim_2 b_2(a_1, \theta')$$

B_2^+: Set of all increasing b_2.

Incentive Compatibility

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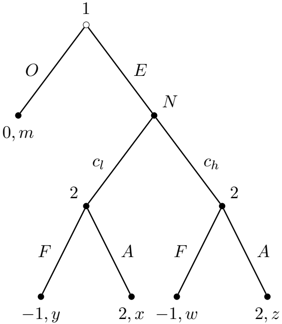
Incentive Compatibility

u_2 has strictly increasing differences $\Rightarrow$

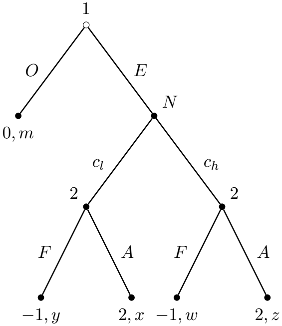
incentive compability $\Leftrightarrow$ b_2 increasing

Entry Game: IC Strategies



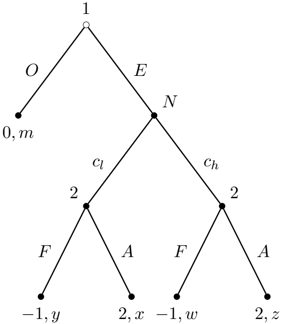
- $c_h \succ c_l$ and $A \succ F$

Entry Game: IC Strategies



- $c_h \succ c_l$ and $A \succ F$
- Increasing differences: $z - w > x - y$

Entry Game: IC Strategies



- $c_h \succ c_l$ and $A \succ F$
- Increasing differences: $z - w > x - y$
- Incentive compatible strategies: FF, FA, AA

Renegotiation-Proofness

Definition (Renegotiation-Proof Equilibria)

A PBE of $\Gamma_R(G)$ is renegotiation-proof if the equilibrium contract is not renegotiated after any a_1 and θ.

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Definition (Renegotiation-Proofness)

We say that (f, b_2^*) is renegotiation-proof if for all a_1 and θ for which there exists an incentive compatible (g, b_2) such that

$$u_2(a_1, b_2(a_1,\theta),\theta) - g(a_1, b_2(a_1,\theta)) > u_2(a_1, b_2^*(a_1,\theta),\theta) - f(b_2^*(a_1,\theta))$$

there exists a θ' such that

$$f(a_1, b_2^*(a_1,\theta')) \geq g(a_1, b_2(a_1,\theta'))$$

Renegotiation-Proofness

Renegotiation-Proof Strategies

A strategy b_2 is renegotiation-proof if there exists a contract f such that (f, b_2) is IC and RP

B_2^R: Set of all RP strategies

Renegotiation-Proof Contracts

Theorem 1

(f, b_2^*) is RP iff for any a_1, i, and $b_2 \in \mathfrak{B}(a_1, i, b_2^*)$ there exists k:

$$u_2(a_1, b_2(a_1, \theta^i), \theta^i) - u_2(a_1, b_2^*(a_1, \theta^i), \theta^i) + \sum_{j=k}^{i-1} \vec{U}_2(a_1, b_2)_{2j-1} \leq f_k - f_i$$

or there exists l:

$$u_2(a_1, b_2(a_1, \theta^i), \theta^i) - u_2(a_1, b_2^*(a_1, \theta^i), \theta^i) + \sum_{j=i+1}^{l} \vec{U}_2(a_1, b_2)_{2(j-1)} \leq f_l - f_i$$

▸ Proof

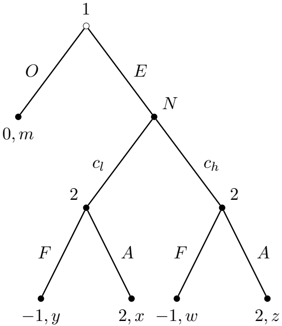
Renegotiation-Proof Strategies

- Necessary and sufficient conditions for renegotiation-proof strategies

Renegotiation-Proof Strategies

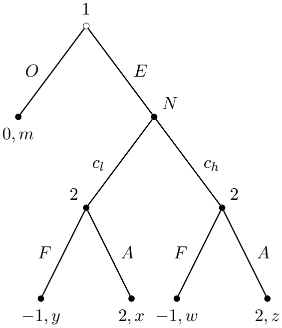
- Necessary and sufficient conditions for renegotiation-proof strategies
- Characterization when there are only two types

Entry Game: RP Strategies



- FF is not RP [Why is FF not RP?]

Entry Game: RP Strategies



- FF is not RP ▸ Why is FF not RP?
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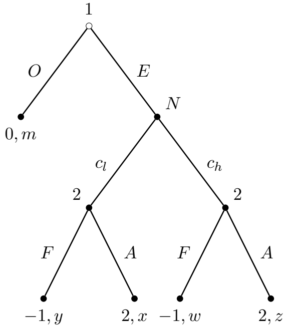
Entry Game: RP Strategies



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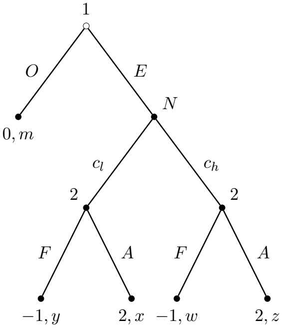
- RP strategies: FA, AA ‣ Why is FA RP?

Entry Game: RP Strategies



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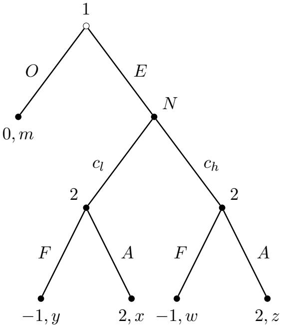

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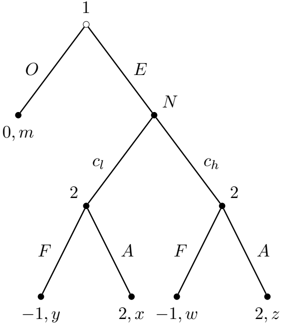

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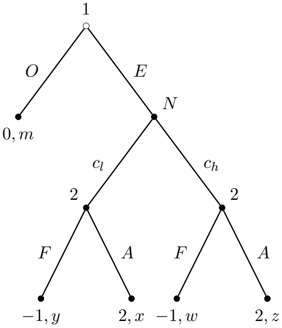

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- RP strategy: Allow best response for the type who benefits most
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- Credibility requires tolerance for the worst case scenarios

Observable and Non-renegotiable Contracts

Stackelberg Payoffs

$$\bar{U}_2^B = \max_{b_2 \in B_2^+} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$$

$$\bar{U}_2^W = \max_{b_2 \in B_2^+} \min_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$$

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Proposition 1

$\bar{U}_2^B - \delta$ *can be supported with non-renegotiable contracts.*

Observable and Non-renegotiable Contracts

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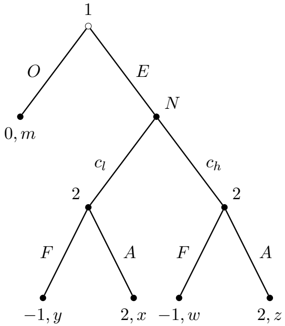
Proposition 1

$\bar{U}_2^B - \delta$ *can be supported with non-renegotiable contracts.*

Proposition 2

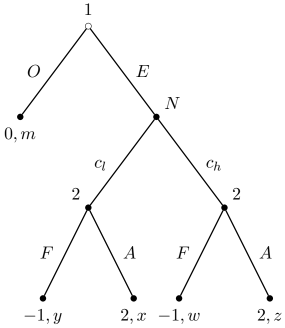
$\bar{U}_2^W - \delta$ *is the smallest payoff that can be supported with non-renegotiable contracts.*

Entry Game: Observable Non-renegotiable Contracts



- Unique outcome that can be supported is no-entry

Entry Game: Observable Non-renegotiable Contracts



- Unique outcome that can be supported is no-entry
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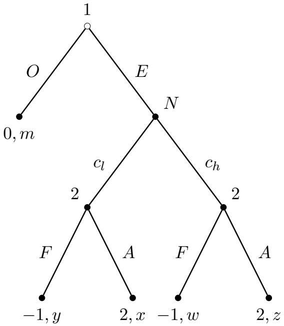
Entry Game: Observable Non-renegotiable Contracts



- Unique outcome that can be supported is no-entry
- Supported with strategy FF
- A contract that supports FF

$$f(F) = \delta, \quad f(A) = \delta + (z - w)$$

Observable and Renegotiable Contracts

Stackelberg Payoffs

$$\bar{U}_2^{BR} = \max_{b_2 \in B_2^R} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$$

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Observable and Renegotiable Contracts

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Proposition 3

$\bar{U}_2^{BR} - \delta$ *can be supported with renegotiation-proof contracts.*

Observable and Renegotiable Contracts

Stackelberg Payoffs

$$\bar{U}_2^{BR} = \max_{b_2 \in B_2^R} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$$
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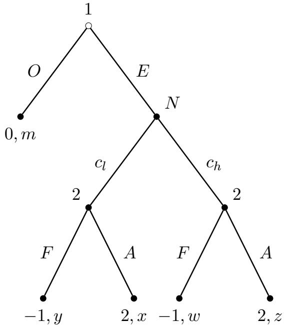
Proposition 4

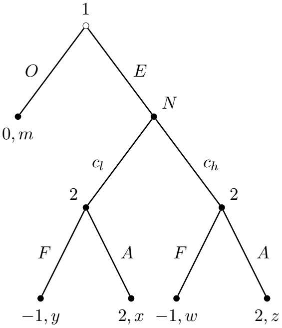
$\bar{U}_2^{WR} - \delta$ *is the smallest payoff that can be supported with renegotiation-proof contracts.*

Entry Game: Observable and RP Contracts



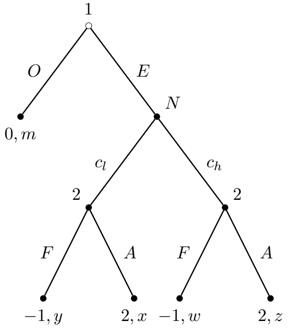
- RP strategies: FA, AA

Entry Game: Observable and RP Contracts



- RP strategies: FA, AA
  - RP contract that supports FA: $f(F) = \delta, \quad f(A) = \delta + (x - y)$

Entry Game: Observable and RP Contracts



- RP strategies: FA, AA
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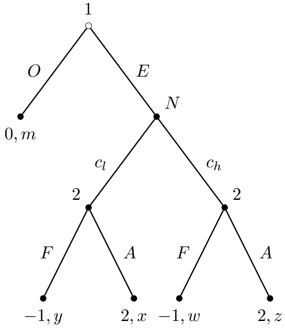
- Pl. 1's best response

$$br_1(AA) = E$$

$$br_1(FA) = \begin{cases} O, & p(c_l) > 2/3 \\ E, & p(c_l) < 2/3 \end{cases}$$

Entry Game: Observable and RP Contracts



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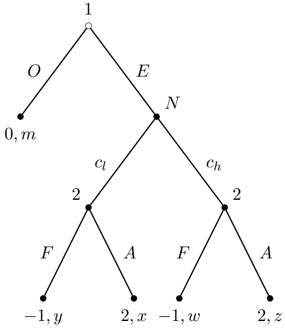

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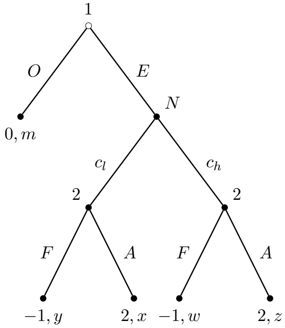

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- If $p(c_l) > 2/3$ unique outcome that can be supported is no-entry
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- If $p(c_l) < 2/3$ unique outcome that can be supported is entry and accommodate

Unobservable and Non-renegotiable Contracts

Proposition 5

(b_1^*, b_2^*) can be supported iff

1. *(b_1^*, b_2^*) is a Bayesian Nash equilibrium of G*
2. *b_2^* is increasing*

Unobservable and Non-renegotiable Contracts

Individually rational payoff of player 1:

$$\underline{U}_1^+ = \max_{a_1} \min_{b_2 \in B_2^+} U_1(a_1, b_2)$$

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Corollary 1

Outcome (a_1^*, a_2^*) *can be supported iff*

1. $a_2^*(\theta) \in BR_2(a_1^*, \theta)$ *for all* θ *and*
2. $U_1(a_1^*, a_2^*) \geq \underline{U}_1^+$

Unobservable and Renegotiation-Proof Contracts

Proposition 6

(b_1^*, b_2^*) can be supported iff

1. *(b_1^*, b_2^*) is a Bayesian Nash equilibrium of G*
2. *b_2^* is increasing and renegotiation-proof*

Unobservable and Renegotiation-Proof Contracts

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Unobservable and Renegotiation-Proof Contracts

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Outcome (a_1^*, a_2^*) *can be supported iff*

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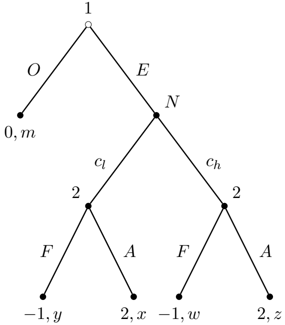
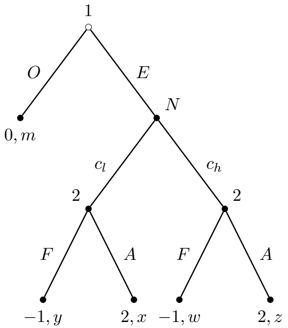
Entry Game: Unobservable Contracts



- In addition to no-entry, entry and accommodate also supported

Entry Game: Unobservable Contracts



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contradicting (1)

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$$f_j - f_{j+1} \leq u_2(a_1, b_2(a_1, \theta^j), \theta^j) - u_2(a_1, b_2(a_1, \theta^{j+1}), \theta^j)$$
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Proof of Theorem 1

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- Can write these as $Df \leq \vec{U}_2(b_2)$

Proof of Theorem 1

(f, b_2^*) not RP iff there exist a_1, i, and IC (g, b_2):

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