

Sovereign Debt Sustainability in Advanced Economies

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April 25, 2014

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§We would like to thank Barry Adler, Marco Becht, Bruno Biais, Olivier Blanchard, Henning Bohn, François Bourguignon, Gabrielle Demange, Bastien Drut, Ester Faia, Hans Gersbach, Gérard Hertig, Howell Jackson, Luc Laeven, Luisa Lambertini, Leonardo Martinez, Colin Mayer, Manuel Mayer, Geoffrey Miller, Dirk Niepelt, Marco Pagano, Bruno Parigi, Enrico Perotti, Paul Pichler, Lev Ratnovsky, Mark Roe, Roberta Romano, Stefano Rossi, Chikahisa Sumi, Elu von Thadden, Jean Tirole, Alain Trannoy, and seminar participants at the IMF, the Universities of Aix-Marseille, Konstanz, and Sydney, the Paris and Toulouse Schools of Economics, the Third ETH-NYU Law and Banking/Finance Conference at the ETH Zurich, the CEPR-Michigan-Wien Transatlantic Conference on Banks and Governments in Globalised Financial Markets at the Oesterreichische Nationalbank in Vienna, the Banca d'Italia-CEPR Macroeconomic Policy Research Workshop on Growth, Rebalancing and Macroeconomic Adjustment after Large Shocks at the Magyar Nemzeti Bank in Budapest, the AEA Meetings in Philadelphia, and the EGCI Law and Finance Seminar on Sovereign Debt in Brussels for very helpful comments and suggestions, and Ugo Panizza for having encouraged the third author to write an earlier paper exploring similar ideas. Claire Myriam Célérier and especially Ethem Ibrahim Güney provided superb research assistance. Rochet acknowledges financial support from ERC (Project RMAC) and NCCR-FINRISK (Project Banking Regulation).

Abstract

We develop a measure of maximum sustainable government debt for advanced economies. How much investors are willing to lend to a country's government depends on how high a primary surplus they expect that government to generate, how fast they expect the country to grow, how volatile they expect that growth to be, and how much debt they expect the government will be able to raise in the future for the purpose of servicing the debt it seeks to raise today. This last observation points to the presence of a borrowing multiplier, which raises a country's borrowing well above what it would be, absent the ability to service maturing debt out of new debt's proceeds. Present debt is sustainable when implied future debt remains bounded. A country's probability of default displays a marked asymmetry around that country's maximum sustainable debt (MSD): it increases slowly below and rapidly above. We calibrate our measure for 23 OECD countries: Korea has the highest MSD at 281% of GDP and Greece the lowest at 89%. The probabilities of default at MSD are very low, from Norway's 0.27% to Korea's 0.81%. Most countries' actual debt-to-GDP ratios in 2010 are below MSD; some countries are above; these are generally the countries that have received some form of financial support in the wake of the financial crisis.

1 Introduction

What has come to be called the ‘Great Recession’ has seen an explosion of sovereign debt, to levels which perhaps would not have been thought possible just a few years ago. Most advanced economies saw sizeable increases in their debt-to-GDP ratios in the years since 2009. This increase in debt, and the ensuing debt crisis that has engulfed many advanced economies, naturally have prompted much thought about the nature and level of a country’s maximum sustainable debt, a concept not unlike Reinhart, Rogoff, and Savastano’s (2003) concept of ‘debt intolerance’ for developing countries.

The present paper constitutes an attempt at estimating advanced economies’ maximum sustainable debt ratios, MSD for short. It follows in the footsteps of Bohn (1998, 2008), Ghosh, Kim, Mendoza, Ostry, and Qureshi (2011), and Tanner (2013). As do Ghosh et al. (2011) and Tanner (2013), the present paper computes a country’s MSD on the basis of that country’s maximum primary surplus, MPS. The paper does, however, use the MPS in a slightly different manner. Unlike Tanner (2013) who equates maximum borrowing to the present value of future MPS to obtain a measure that is not so much debt- as equity-like, the present paper accounts for debt’s fixed payments and the possibility of default. Unlike Gosh et al. (2011) whose maximum debt is unstable, the present paper seeks maximum debt that is stable and thus more properly described as sustainable.

The starting point of our analysis is the recognition that maximum debt is determined by lenders: a country can only borrow as much as lenders are willing to provide. That amount depends on the country’s MPS, which we express as a fraction of the country’s GDP, and on the mean and volatility of the country’s growth in GDP. It further depends on the country’s attitude to repayment, as lenders naturally lend more to a country that defaults only when unable to service its debt than to a country that defaults strategically when it deems the payoff from default to be higher than that from debt service. Last but not least, it depends on lenders’ expectation of the amount of new debt that can be raised to service existing debt.¹ There is a ‘borrowing multiplier’ that raises the country’s borrowing well above what it would be, absent the ability to service maturing debt out of new debt’s proceeds. Where that multiplier is finite, sustainable borrowing has a unique

¹We abstract from possible coordination problems among lenders. See for example Calvo (1988), Cole and Kehoe (2000), Lorenzoni and Werning (2013), and Morris and Shin (2004) for the analysis of various aspects of such problems.

maximum that defines the country’s maximum sustainable borrowing, MSB. Borrowing beyond MSB is not sustainable, for the high interest rate would make future borrowing for debt service requirement unbounded.

We characterize the properties of MSB and its associated MSD. We find these to be increasing in the mean growth in GDP and the MPS, and to be decreasing in the risk-free rate. MSB is further decreasing in the volatility of the growth in GDP. Together with a country’s actual debt, MSB determines that country’s probability of default, PD. We find PD to be decreasing in the mean growth in GDP and the MPS and to be increasing in the volatility of the growth in GDP and the risk-free rate. In a nutshell, countries with higher MPS and higher and less volatile growth can afford to borrow more, yet present a lower probability of default.

Consistently with our focus on advanced economies, we limit our analysis to OECD countries. We use OECD data for growth rates and primary surpluses and Reinhart and Rogoff data for debt-to-GDP ratios. We exclude 11 countries that have missing or insufficient data. Our final sample consists of 23 countries, which we study over the period 1980-2010 for the most part.²

Our benchmark model assumes that countries grow at a rate that is iid lognormally distributed. We obtain a wide range of MSD, reflecting both the assumptions we make about MPS and country-specific characteristics such as mean growth rate and, to a much lesser extent, volatility. Where, in line with IMF (2011) analyses, we assume MPS of 5%, MSD ranges from Greece’s 89% to Korea’s 282%, largely a reflection of these countries’ mean growth rates (1.56% for Greece, 5.75% for Korea). Where we use instead a country’s historical MPS over the period of study, we find MSD ranging from Portugal’s 5% to Norway’s 720%, reflecting these countries’ historical MPS (0.23% for Portugal, 20.25% for Norway). The wide range of country MSD suggests that the wide variation in debt intolerance documented for developing countries by Reinhart, Rogoff, and Savastano (2003) extends to advanced economies as well. Should a country’s historical MPS be considered a measure of the quality of that country’s fiscal structure, the strong relation between historical MPS and MSD would be consistent with Reinhart, Rogoff, and Savastano’s (2003, p. 1) observation that “debt-intolerant countries tend to have weak fiscal structures.”³

²Notably absent from our sample is Japan, excluded because of the lack of OECD government income and spending data prior to 2005: government income and spending are essential to the computation of the primary surplus.

³Unlike Bohn (1998, 2008), we do not allow a country’s primary surplus to respond to its level of debt. Lukkezen and Rojas-Romagosa (2012) use such ‘fiscal response’ to derive a measure of debt sustainability.

The PD associated with MSD are very low. They range from 0.27% (Norway) to 0.81% (Korea). PD are *a fortiori* lower still where actual debt is lower than MSD. Where in contrast actual debt is above MSD, PD increases rather quickly towards one. Consider for example Italy in 2010: there is a relatively modest 27% difference between debt of 118% of GDP and MSD of 91% with 4% MPS, yet PD is 0.46% in the latter case and 89% in the former. There is a marked asymmetry in the relation between debt and PD on the two sides of MSD, with PD increasing slowly in debt when debt is smaller than MSD, and rapidly when debt is larger than MSD. This asymmetry stems from the very nature of MSD, which we show to be the level of debt at which the average interest rate is minimized. The average interest rate therefore decreases in debt below MSD and increases above. This necessarily implies that the interest rate increases slowly in debt below MSD and rapidly above. What is true of the interest rate naturally is also true of PD, for the interest rate's premium over the risk-free rate is but a compensation for default.

The same asymmetry between slow increases in PD below MSD and rapid increases above characterizes the variation over time of countries' PD. Thus, Greece's PD barely budged as it increased its debt-to-GDP ratio from 53% in 1987 to 89% in 1990, precisely its MSD with 5% MPS. Yet, the subsequent increase in Greece's debt-to-GDP ratio to 127% in 2009 increased its PD from below 1% in 1990 to 85.6% in 2009. The 36% debt-to-GDP ratio increase to the MSD was accompanied by an increase in Greece's PD of little more than 1%, the very slightly higher 38% increase away from the MSD was accompanied by an increase of nearly 86%.

The borrowing multiplier plays a central role in our analysis. We show that the expectation of repeated reliance on future borrowing for the purpose of servicing present debt increases sustainable borrowing by a factor of magnitude. For example, with 5% MPS, Switzerland's MSB increases from 16% to 88%, Germany's from 17% to 111%. As is to be expected, our debt measure is well below Tanner's (2013) equity-like measure of maximum government liabilities. For example, with 5% MPS, the United Kingdom's MSB is 108%, its Tanner equity-like measure is 303%; the corresponding numbers for the United States are 103% and 272%, respectively. More can be raised in the form of equity than of debt, for default provides borrowers with a means to escape debt liabilities that has no equivalent for equity-like liabilities.

Our benchmark model assumes zero recovery in default, a constant risk-free interest rate, and no growth disasters. Assuming maximum recovery in default results in only a very slight increase in

maximum sustainable debt, a reflection of the very low probability of default at MSD. Assuming a time-varying risk-free interest rate generally decreases MSD, as lenders recognize that there will be fewer funds for debt service when high realized interest rates decrease future borrowing proceeds. Allowing for the possibility of rare disasters (Rietz, 1988; Barro, 2006) results in a marked decline in MSD, as lenders fear a collapse in output that will leave governments with little revenue for debt service.

Our analysis eschews strategic default—default despite having income sufficient for debt service—for what Grossman and Van Huyck (1988) call ‘excusable default:’ default that occurs when the sum of government income and debt issuance proceeds falls short of debt service requirements. This is consistent with Levy Yeyati and Panizza’s (2011) evidence of governments’ reluctance to default, which they attribute to (i) governments’ desire not to be seen as engaging in strategic default and (ii) governments’ fear of losing office upon default. On the first point, Tomz (2007) presents strong evidence that governments engaging in strategic default later incur very high costs of borrowing. On the second point, Borensztein and Panizza (2008) and Malone (2011) find that governments that default see a marked decline in their prospects for reelection: domestic bondholders are also voters; even the bonds sold initially to foreign bondholders may ultimately accrue to domestic bondholders through trading in secondary markets (Broner, Martin, and Ventura, 2010).⁴ Gümbel and Sussman (2009) show that, where voters differ in wealth and in their holdings of government bonds, the median voter may favor debt repayment even when much of the benefit accrues to foreign bondholders, because much of the cost is borne by low-income voters who hold few bonds and receive little of the benefit of repayment. Bolton and Jeanne (2011) provide another, important reason for governments’ reluctance to default: government bonds provide the collateral for interbank loans; government default jeopardizes the value of that collateral, thereby impeding the functioning of the banking system and its ability to finance investment.⁵

The paper proceeds as follows. Section 2 reviews the related literature. Section 3 presents the basic setting. Section 4 obtains a country’s maximum sustainable debt ratio. Section 5 presents the

⁴However Foley-Fischer (2012) defends the opposite view that an incoming government may signal its competence through default.

⁵Mengus (2012) shows that the attempt to preserve investment through the bailout of domestic entrepreneurs may have the undesirable effect of inducing excessive investment. Where entrepreneurs differ in their portfolios but receive identical bailout payments from the government, some entrepreneurs may ultimately dispose of so many funds as to invest in projects which, albeit privately profitable, are socially unprofitable in the sense of not justifying the deadweight cost of the bailout.

data. Section 6 discusses the results of the calibration. Section 7 extends the analysis to the cases of recovery in default, a time-varying risk-free interest rate, and the possibility of growth disasters. Finally, Section 8 concludes.

2 Literature review

As noted in the Introduction, our work seeks to estimate advanced economies' maximum sustainable debt ratios. This focus on MSD distinguishes our work from the bulk of the very rich literature on sovereign debt, which has been concerned primarily with calibrating a country's *optimal* debt ratio for purposes of intertemporal consumption smoothing (Eaton and Gersovitz, 1981), rather than the *maximal* debt ratio that is our concern.⁶ Consistently with this difference in focus, our work assumes excusable default where the bulk of existing work has assumed strategic default: as more debt is forthcoming under excusable than under strategic default, the former form of default is clearly more suited to our attempt at estimating maximum debt.

It is probably fair to say much of the literature on sovereign debt can be viewed as constituting a very rich tapestry weaved on the loom of Eaton and Gersovitz's (EG, 1981) seminal work.⁷ Later work has quantified, refined, and extended EG's predictions, and endogenized some of what had been exogenous in EG. Thus Aguiar and Gopinath (2006) and Arellano (2008) have embedded the basic EG framework into the setting of a small open economy to study the interactions of default risk with output, consumption, the trade balance, interest rates, and foreign debt. Arellano (2008) ascribes the countercyclicality of interest rates and the current account to incomplete financial contracts. As interest and principal payments cannot be made to depend on output, the incentive to default is higher in recessions than in expansions. Interest rates consequently are lower in expansions, thereby inducing countries to borrow more when output is high. Borrowing finances imports, which deteriorate the current account. Aguiar and Gopinath (2006) incorporate a trend into the output process. They distinguish between the two cases of stable and volatile trends and show that only in the latter case can observed default frequencies be replicated in calibration. Where the trend in output is stable, there is little value to the insurance provided by access to

⁶Optimal and maximal debt ratio need not always differ: Bohn (2013) argues that 'low altruism' will tend to make these coincide.

⁷The present literature review is by necessity rather limited in scope. For extensive literature reviews, see Panizza, Sturzenegger, and Zettelmeyer (2009), Reinhart and Rogoff (2009), and Sturzenegger and Zettelmeyer (2006).

international debt markets. A borrower in recession therefore has a strong incentive to default. The interest rate schedule consequently is extremely steep and borrowing does not extend to the range where default occurs. Where in contrast the trend in output is volatile, insurance is valuable and the incentive to default is weakened. The interest rate schedule is less steep and borrowing extends to the range where default occurs.

Mendoza and Yue (2012) have endogenized the collapse in output that accompanies default. There is a general equilibrium model in which domestic firms borrow internationally to finance their purchase of foreign inputs. A sovereign default jeopardizes firms' access to foreign working capital loans, thereby forcing the firms to substitute domestic inputs for the previously purchased foreign inputs. As the former are imperfect substitutes for the latter, TFP declines and the effects of the negative output shock that triggered default are amplified. Cuadra and Sapriza (2008) have considered the role of political risk.⁸ They show that political instability (one party may lose power to another party) and political polarization (different parties represent different constituents with differing interests) combine to increase borrowing by decreasing the importance a party in power attaches to the future. The negative consequences of borrowing are lessened when shared with another party that has other constituents. A positive consequence of borrowing is to 'tie the other party's hands,' thereby preventing that party from lavishing its constituents with debt-financed favors should the party come to power.

Yue (2009) and Benjamin and Wright (2009) have considered the role of renegotiation in default. Yue (2009) considers Nash bargaining under symmetric information. Disagreement payoffs are zero for creditors and the autarkic payoff for the defaulting country. Yue (2009) shows that the parties bargain to a reduced level of debt that does not depend on the defaulting country's original debt: the parties 'let bygones be bygones.' Haircuts therefore are increasing in the defaulting country's debt. They are decreasing in the country's output: countercyclical interest rates increase the payoff for the country to rejoining international debt markets; they increase the bargaining surplus that is shared by the parties. Benjamin and Wright (2009) note that the period to the resolution of default extends over many years. They attribute the delay in default resolution to the requirement that the defaulting country's commitment to servicing post-resolution debt be credible. As the incentive to default generally decreases in output, credibility requires that the defaulting country's

⁸See also Hatchondo, Martinez, and Sapriza (2009).

output recovers from the low level that likely prompted default in the first place. This is often a protracted process. That the country emerges from default only after output has recovered provides an explanation for the otherwise puzzling observation that default resolution often results in post-resolution debt that is no lower than the original, pre-default debt.

Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012) have considered the role of debt maturity: when not all debt is retired every period, the issuance of new debt serves to dilute the value of existing debt; lack of commitment creates a ‘prisoner’s dilemma’ that results in increased government borrowing at higher interest rates. While short-term debt therefore should dominate long-term debt, this need not be true where self-fulfilling rollover crises may occur (Chatterjee and Eyigungor, 2012).⁹ Fink and Scholl (2011) have considered the role of conditionality. They show that international financial institution (IFI) involvement may increase rather than decrease interest rates, by inducing additional borrowing on the part of a government that expects to benefit from IFI support.

Cohen and Villemot (2013) have noted the difficulty of existing models simultaneously to match the first moments of debt and default probabilities: high default costs that make possible the matching of debt ratios preclude that of default probabilities; low default costs have the opposite effect.¹⁰ Building on Levy Yeyati and Panizza’s (2011) finding that output contractions generally precede rather than follow default, Cohen and Villemot (2013) have developed a model in which the cost of default is borne ‘in advance.’ Governments in such case do not have the incentive to stave off a default whose cost they have already borne.

Catão and Kapur (2004) and Acharya and Rajan (2012) have replaced the assumption of infinite government horizon made by the preceding papers by that of short horizon. Catão and Kapur attribute the debt intolerance documented by Reinhart, Rogoff, and Savastano (2003) to country macroeconomic volatility. Acharya and Rajan show that myopic governments intent on maximizing present borrowing proceeds may purposely exacerbate their countries’ financial fragility in order to commit successor governments to debt repayment.¹¹ Bi and Leeper (2012) maintain the assumption

⁹Niepelt (2013) argues that optimal maturity structure trades off the consumption smoothing benefits from the marginal unit of debt against the revenue costs from the inframarginal units: debt issuance affects the default and rollover choices of subsequent governments, thereby affecting the prices of the maturities currently issued; such effect varies with the maturity structure.

¹⁰See in particular Table 1 in Cohen and Villemot (2013).

¹¹See Rochet (2006) for an early attempt at analyzing the implications for government borrowing of short government horizon.

of infinite horizon but dispense with that of strategic default. They characterize the fiscal limit that arises from the dynamic Laffer curve.

We conclude the present section by noting that, unlike the assumption of strategic default, the assumption of excusable default neither is subject to the well-known Bulow-Rogoff critique (Bulow and Rogoff, 1989a, 1989b) nor requires that there be a bubble (Hellwig and Lorenzoni, 2009). Bulow and Rogoff (1989a, 1989b) argue that exclusion from debt markets alone fails to deter default, because a defaulting government can use the amount otherwise to be reimbursed effectively to purchase an insurance contract that provides the same risk sharing as does government borrowing. Hellwig and Lorenzoni (2009) argue that the low interest rates that are necessary to preserve a government's incentive to repay result in the formation of a bubble. A government that has excusably rather than strategically defaulted has no income with which to purchase the insurance contract hypothesized by Bulow and Rogoff (1989a, 1989b).¹² That government allocates the entirety of its MPS to servicing its debt regardless of the level of interest rates.

3 The benchmark model

Consider a government that is in office at date t . Let y_t denote the country's GDP at that date; α denote the government's maximum primary surplus, MPS, expressed as a fraction of GDP y_t ; b_t denote the proceeds from issuing debt at date t , again expressed as a fraction of GDP y_t ; d_t denote the face value of that debt, expressed as a fraction of GDP y_t but payable at date $t+1$; $g_t \equiv y_{t+1}/y_t$ denote the gross rate of growth in GDP from t to $t+1$, independently and identically distributed with cdf $F(\cdot)$ and pdf $f(\cdot)$; and r denote the risk-free interest rate. We assume competitive, risk-neutral investors, neither recovery nor bail-out in default, no renegotiation, and an independent central bank that resists possible government demand to decrease through inflation the real value of the debt.¹³ Maximum proceeds $b_t y_t$ for given face value $d_t y_t$ are

$$b_t y_t = \frac{\Pr[(\alpha + b_{t+1}) y_{t+1} > d_t y_t] d_t y_t}{1 + r}. \quad (1)$$

¹²Amador (2003) presents a solution to the Bulow-Rogoff critique that does not require the abandonment of the strategic default assumption. Where political parties alternate in power, Bulow-Rogoff-type insurance contracts are not feasible because a party in power would rather (i) rescind any such contract to spend its proceeds whilst in power than (ii) leave the benefits of the contract to be enjoyed by the other party when that party in turn will be in power.

¹³We relax the assumption of zero recovery in default in Section 7; see Tirole (2012) for an analysis of the incentives for sovereign bailout.

Default occurs at date $t + 1$ when the sum of the MPS at date $t + 1$ (αy_{t+1}) and the amount the government can borrow at that date ($b_{t+1}y_{t+1}$) is not sufficient to repay the debt raised at date t ($d_t y_t$). We assume no new borrowing is possible during default: default allows the government to escape its debt obligations; lenders naturally do not ‘throw good money after bad;’ there is a ‘sudden stop.’¹⁴ This is unlike the case of no-default, in which proceeds from new borrowing can be used to service existing debt. The amount the government can borrow at date t , $b_t y_t$, consequently depends on the amount the government can borrow at date $t + 1$, $b_{t+1} y_{t+1}$.

Rearranging (1), we have

$$\begin{aligned}
 b_t &= \Pr \left[\frac{y_{t+1}}{y_t} > \frac{d_t}{\alpha + b_{t+1}} \right] \frac{d_t}{1+r} \\
 &= \Pr \left[g_t > \frac{d_t}{\alpha + b_{t+1}} \right] \frac{d_t}{1+r} \\
 &= \left[1 - F \left(\frac{d_t}{\alpha + b_{t+1}} \right) \right] \frac{d_t}{1+r}.
 \end{aligned} \tag{2}$$

4 Maximum sustainable debt

We wish to determine the maximum amount the government can borrow at date t , expressed as a fraction of GDP. We thus seek

$$b_t = \max_{d_t} \left[1 - F \left(\frac{d_t}{\alpha + b_{t+1}} \right) \right] \frac{d_t}{1+r}. \tag{3}$$

The maximization of present borrowing b_t depends on lenders’ expectation of future borrowing b_{t+1} .¹⁵ We define

$$x_t \equiv \frac{d_t}{\alpha + b_{t+1}} \tag{4}$$

to be the minimum growth rate necessary to avoid default. Problem (3) therefore can be rewritten

$$b_t = \max_{x_t} \frac{1}{1+r} [1 - F(x_t)] (\alpha + b_{t+1}) x_t, \tag{5}$$

¹⁴See for example Calvo and Reinhart (2000), Calvo, Izquierdo, and Loo-Kung (2006), and Mendoza (2010) for an analysis of sudden stops.

¹⁵For simplicity, we refer to b_t as ‘borrowing’ rather than ‘borrowing as a fraction of GDP,’ more exact but also longer. We likewise refer to d_t as debt.

where we have used $d_t = (\alpha + b_{t+1}) x_t$. That $(\alpha + b_{t+1})$ enters the objective function in (5) multiplicatively implies that the optimal x_t is independent of t .

Definition 1 *The borrowing factor γ is*

$$\gamma \equiv \max_x [1 - F(x)] x. \quad (6)$$

Remark 1 *The borrowing factor γ is smaller than the mean growth rate \bar{g} :*

$$\gamma = [1 - F(x_M)] x_M < \int_{x_M}^{\infty} g f(g) dg \leq \int_{-\infty}^{\infty} g f(g) dg = E(g) \equiv \bar{g},$$

where $x_M \equiv \arg \max_x [1 - F(x)] x$.

In order to understand the concept of borrowing factor, substitute (6) into (5) to obtain

$$b_t = \frac{\gamma}{1+r} (\alpha + b_{t+1}) \equiv \tau (b_{t+1}). \quad (7)$$

Consider the case where the government cannot rely on future borrowing to repay present borrowing: $b_{t+1} = 0$. Maximum borrowing in such case equals maximum static borrowing $b_t = \alpha\gamma/(1+r) \equiv b_S$ from (7). Were the government to raise funds in the form of equity, that is, credibly to commit to paying out to investors the entirety of next period's maximum primary surplus, maximum proceeds would be $\alpha\bar{g}/(1+r)$; this is the expected present value of next period's maximum primary surplus. Remark 1 implies that $\alpha\gamma/(1+r) < \alpha\bar{g}/(1+r)$. The borrowing factor γ therefore measures the impact on government proceeds of raising funds in the form of debt, accounting for debt's fixed payments and possibility of bankruptcy. These decrease debt's proceeds below the present value of next period's expected primary surplus, that is, below the proceeds that would be had were funds to be raised in the form of equity. We shall show this result to extend to the case where the government can rely on future borrowing to repay present borrowing.

Definition 2 *The borrowing multiplier Γ is*

$$\Gamma \equiv 1 + \frac{\gamma}{1+r} + \left(\frac{\gamma}{1+r}\right)^2 + \dots \quad (8)$$

Remark 2 *The borrowing multiplier Γ is finite when $\gamma < 1 + r$ and infinite when $\gamma \geq 1 + r$.*

In order to understand the concept of borrowing multiplier, iterate (7) forward to obtain

$$b_t = \frac{\alpha\gamma}{1+r} \left[1 + \frac{\gamma}{1+r} + \left(\frac{\gamma}{1+r} \right)^2 + \dots \right] = b_S \Gamma. \quad (9)$$

The term $1 + \gamma/(1+r)$ measures the gross, fractional increase in present borrowing made possible by the ability to rely on future borrowing on one single occasion. The borrowing multiplier Γ measures the increase made possible by infinitely repeated reliance on future borrowing. When $\gamma < 1 + r$, the series converges, the borrowing multiplier is finite; when $\gamma \geq 1 + r$, the series diverges, the borrowing multiplier is infinite,

Definition 3 *Borrowing $b > 0$ is sustainable if and only if there exists a bounded sequence of borrowings $(b_t)_t$ such that $b_0 = b$ and $b_t \leq \tau(b_{t+1}) \forall t$.*

Present borrowing is sustainable when its repayment does not rely on a sequence of future borrowings that is unbounded. We have¹⁶

Proposition 1 *Consider in turn the two cases (i) $\gamma < 1 + r$ and (ii) $\gamma \geq 1 + r$.*

(i) When $\gamma < 1 + r$, maximum sustainable borrowing (MSB) is finite. It equals

$$b_M \equiv \frac{\alpha\gamma}{1+r-\gamma}. \quad (10)$$

(ii) When $\gamma \geq 1 + r$, every borrowing $b > 0$ is sustainable.

The results in Proposition 1 are immediate from Remark 2 and (9). When $\gamma < 1 + r$, the increase in borrowing made possible by the borrowing multiplier is finite; there is a maximum level of borrowing that can be sustained through future borrowing; the function τ is a contraction that

¹⁶All Proofs are in the Appendix.

has the unique fixed point b_M .¹⁷ When $\gamma \geq 1 + r$, the increase in borrowing is infinite; any level of borrowing can be sustained.

It is instructive to compare our measure of maximum sustainable borrowing with Tanner's (2013). Tanner proposes as measure of maximum sustainable borrowing the expected present value of all future maximum primary surpluses, specifically

$$\alpha E \left[\sum_{s=1}^{\infty} \frac{\Pi_{t=1}^s g_t}{(1+r)^s} \right] = \frac{\alpha \bar{g}}{1+r-\bar{g}} \equiv b_E,$$

where the equality is true when $\bar{g} < 1 + r$.¹⁸ We call Tanner's (2013) measure b_E the 'equity-like' measure of government liabilities. Remark 1 implies that the condition $\gamma < 1 + r$ is weaker than the condition $\bar{g} < 1 + r$: a lower interest rate is necessary to ensure the convergence of a stream of debt payments that are fixed and subject to possible default than that of a stream of equity payments that are not. Remark 1 also implies

Remark 3 *Maximum sustainable borrowing b_M is smaller than Tanner's equity-like measure b_E :*

$$b_M = \frac{\alpha \gamma}{1+r-\gamma} < \frac{\alpha \bar{g}}{1+r-\bar{g}} = b_E. \quad (11)$$

Note that the constant debt-to-GDP ratio rule (Aaron, 1966) specifies a level of debt that is identical to Tanner's (2013) equity-like measure when interest-bearing debt is risk-free, the primary surplus is maximal, and growth is deterministic.¹⁹ Further note that an attempt at replicating equity's payoff with debt by setting a very high face value for debt would fail to equate b_M and b_E

¹⁷Note that

$$\begin{aligned} b_S \Gamma &= \frac{\alpha \gamma}{1+r} \left[1 + \frac{\gamma}{1+r} + \left(\frac{\gamma}{1+r} \right)^2 + \dots \right] \\ &= \frac{\alpha \gamma}{1+r} \frac{1}{1 - \frac{\gamma}{1+r}} \\ &= \frac{\alpha \gamma}{1+r-\gamma} \\ &\equiv b_M \end{aligned}$$

¹⁸Recall that g is the *gross* rate of growth.

¹⁹The constant debt-to-GDP ratio rule (Aaron, 1966) specifies debt-to-GDP ratio such that debt service requirement is covered exactly by the sum of primary surplus and new borrowing at that same ratio. When interest-bearing debt b_C is risk-free and the primary surplus is maximal, the constant debt-to-GDP ratio rule can be expressed as

$$b_C (1+r) y_t = \alpha y_{t+1} + b_C y_{t+1}.$$

even in the case of maximum recovery in default. This is because default precludes the raising of new debt intended to service existing debt: there is a sudden stop in default.

MSB b_M combined with (4) determines maximum sustainable debt (MSD) $d_M \equiv (\alpha + b_M) x_M$; x_M defined in Remark 1 is the minimum growth rate necessary to avoid default at MSD. MSB further determines the probability of default (PD) for a given level of debt d , $PD(d) \equiv F(d/(\alpha + b_M))$. This probability of default depends not only on what is owed, d , but also on the maximum that can be borrowed in the future, b_M .

We wish to determine the comparative statics of MSB, MSD, and PD. We shall assume lognormality of the growth rate for that purpose: $\ln(g) \sim N(\mu, \sigma^2)$; $\bar{g} = \exp(\mu + \sigma^2/2)$. We define

$$z \equiv \frac{\ln(x) - \mu}{\sigma}. \quad (12)$$

We thereby rewrite (6) as²⁰

$$\gamma = \max_z [1 - \Phi(z)] e^{\mu + \sigma z}, \quad (13)$$

where $\Phi(\cdot)$ denotes the standard normal cdf.²¹ By analogy to x_M , we define

$$z_M \equiv \arg \max_z [1 - \Phi(z)] e^{\mu + \sigma z}. \quad (14)$$

MSD and PD become²²

$$d_M = (\alpha + b_M) \exp(\mu + \sigma z_M) \quad (15)$$

and

$$PD(d) = \Phi\left(\frac{\ln(d) - \ln(\alpha + b_M) - \mu}{\sigma}\right) = \Phi(z_d) \quad (16)$$

respectively, with $z_d = [\ln(d) - \ln(\alpha + b_M) - \mu] / \sigma$. We show

When $y_{t+1}/y_t \equiv \bar{g}$, the preceding equation can be rewritten

$$b_C = \frac{\alpha \bar{g}}{1 + r - \bar{g}} = b_E.$$

²⁰To transform (6) into (13), rewrite x as $\exp(\mu + \sigma z)$ and $F(x)$ as $\Phi((\ln(x) - \mu)/\sigma) = \Phi(z)$.

²¹Note that $\gamma = \bar{g}\psi(z)$ where

$$\psi(z) = \max_z [1 - \Phi(z)] e^{\sigma z - \sigma^2/2} < 1.$$

²²Use the same transformations as in Footnote 20, at $x = x_M$ and $z = z_M$ for d_M and at $x_d = d/(\alpha + b_M)$ and $z_d = [\ln(d) - \ln(\alpha + b_M) - \mu] / \sigma$ for $PD(d)$.

Proposition 2 *Maximum sustainable government borrowing b_M is increasing in the mean growth rate μ and the MPS α , decreasing in growth rate volatility σ for $z_M < 0$, and decreasing in the risk-free interest rate r .*

The results are intuitive.²³ A government that achieves a higher primary surplus on GDP that is expected to grow faster can borrow more, for it is expected to have more income with which to service its debt. In contrast, a government whose GDP growth is more volatile can borrow less, for the greater likelihood of low growth realizations increases the probability of default, thereby decreasing lenders' willingness to lend to the government.²⁴ A government can borrow less when the risk-free interest rate is high, for a high risk-free rate raises lenders' opportunity cost of lending to the risky government, thereby decreasing lenders' willingness to lend to the government.

Proposition 3 *Maximum sustainable debt d_M is increasing in the mean growth rate μ and the MPS α and decreasing in the risk-free interest rate r for $z_M < 0$.*

The results $\partial d_M / \partial \mu > 0$, $\partial d_M / \partial \alpha > 0$, and $\partial d_M / \partial r < 0$ recall the results $\partial b_M / \partial \mu > 0$, $\partial b_M / \partial \alpha > 0$, and $\partial b_M / \partial r < 0$, whose intuition they share. Intuition and the result $\partial b_M / \partial \sigma < 0$ for $z_M < 0$ suggest that d_M should be decreasing in growth rate volatility σ for $z_M < 0$. This property is not always true, due to the offsetting role of the probability of default at MSD, $PD(d_M) = \Phi(z_M)$, the increase in which requires an increase in debt to be repaid absent default, d_M , as compensation for the larger probability of default in which no payment is received.²⁵ Figure 1 shows that d_M does in fact decrease in growth rate volatility σ for an interval that encompasses the volatilities in our data.

Proposition 4 *The probability of default $PD(d) = \Phi(z_d)$ is decreasing in the mean growth rate μ and the MPS α , increasing in growth rate volatility σ for $z_d < 0$, and increasing in the risk-free*

²³There is a slight abuse: strictly speaking, μ and σ are the mean and volatility of the log-growth rate, not of the growth rate. We nonetheless refer to these as the mean and volatility of the growth rate for simplicity.

²⁴More volatile GDP growth also results in a greater likelihood of *high* growth realizations. The condition $z_M < 0$ ensures that the detrimental effect of volatility on debt dominates. It is shown in the Proof of Proposition 2 to be equivalent to $\sigma < \sqrt{2/\pi}$, a condition that holds comfortably as $\sqrt{2/\pi} \approx 80\%$, well above growth volatility in our data.

²⁵Somewhat surprisingly, the probability of default $PD(d_M) = \Phi(z_M)$ is affected only by the volatility of growth σ ; it is not affected by μ , α , or r . This result stems from z_M 's exclusive dependence on σ , immediate from the first-order condition for (14), $[1 - \Phi(z_M)]\sigma = \phi(z_M)$. Changes in government creditworthiness that would be expected to be reflected both in the amount borrowed and in the probability of default therefore are reflected only in the former for μ , α , and r . This can be shown to be an artefact of lognormality.

interest rate r .

The direct effects of μ , α , σ , and r on PD are compounded by their indirect effects through MSB. For example, for a given face value of debt d , higher mean growth μ increases both expected GDP and (maximum) future borrowing proceeds b_M ; both effects increase the funds available for debt service, thereby decreasing the probability of default.

An alternative characterization of MSD d_M will prove useful in interpreting our calibration results below. Use (2) to define borrowing proceeds for given face value d

$$B(d) \equiv \frac{1}{1+r} \left[1 - F\left(\frac{d}{\alpha + b_M}\right) \right] d \quad (17)$$

Further define the (gross, implicit) interest rate at d to be

$$R(d) \equiv \frac{d}{B(d)}, \quad (18)$$

Maximizing borrowing proceeds $B(d)$ is equivalent to minimizing the average interest rate $R(d)/d = 1/B(d)$: $d_M = \arg \min_d R(d)/d$; it solves first-order condition

$$R'(d_M) = R(d_M)/d_M.$$

In words, maximum sustainable debt equates the marginal and average interest rates to attain the minimum average interest rate. This is somewhat similar to the manner in which a firm's optimal production equates the firm's marginal and average costs to attain the firm's minimum average cost.

5 Data

As noted in the Introduction, we limit our analysis to OECD countries and use OECD data for growth rates and primary surpluses and Reinhart and Rogoff data for debt-to-GDP ratios.²⁶ We exclude the Czech Republic, Estonia, Israel, Luxembourg, Slovakia, and Slovenia because they are not included in the Reinhart-Rogoff database; we exclude Chili, Japan, Mexico, Poland, and Turkey

²⁶See www.oecd.org/statistics/ and reinhartandrogoff.com. Note that OECD data ends in 2011 and Reinhart-Rogoff data in 2010.

because the OECD database includes fewer than the 10 years of government income and spending data we deem necessary to obtain a meaningful measure of the actual maximum primary surplus. We are thus left with the 23 countries shown in Table 1. We use all available years for growth rates and government income and spending. These are 1970-2011 for most countries for growth rates but range from 1965-2011 (Australia) to 2000-2011 (Korea) for government income and spending. We use the years 1980-2010 for the debt-to-GDP ratios.²⁷ This is because we seek to track the changes in countries' default probabilities over the same period of time and many countries have debt data starting only in 1980.²⁸

Table 1 shows the mean growth rate μ , the standard deviation of the growth rate σ , the 2010 debt-to-GDP ratio d_{2010} , and the historical maximum primary surplus MPS α for each of the 23 countries.²⁹ Korea had the highest mean growth rate at 5.75%, followed by Ireland at 3.30% and, Norway at 2.37%; Switzerland had the lowest mean growth rate at 1.01%, not far behind New Zealand at 1.12% and, to a lesser extent, Greece at 1.56%. Korea had not only the highest mean growth rate but also the most volatile at 7.39%; it was followed by Greece at 6.65% and Portugal at 6.13%; the three countries that had the most stable growth rates were France at 3.26%, Australia at 3.30%, and Spain at 3.36%. The least indebted countries in 2010 were Australia at 11%, New Zealand at 31%, and Switzerland at 39%; the three most indebted countries were Greece at 144%, Italy at 117%, and Iceland at 116%. Finally, Norway had by far the highest historical MPS at 20.25%, with Canada a distant second at 10.05%; the three countries with lowest historical MPS were Portugal at 0.23%, France at 1.36%, and Switzerland at 3.05%.

Table 2 shows aspects of the variation of country debt-to-GDP ratios over time. Debt increased for most countries between 1980 and 2010, although it decreased for some. For example, Australia had a debt-to-GDP ratio of 19% in 1980 and 11% in 2010. The decline was not monotonic, as Australia's debt reached a peak of 21% and a trough of 4.5%. Hungary saw a dramatic decline in its debt, from 156% in 1991 (the first year for which we have Hungarian debt data) to 73% in 2010,

²⁷There are three exceptions: Switzerland, with government income and spending data starting only in 1983, Canada, with data starting in 1988, and Hungary, with data starting in 1991.

²⁸We did not use alternative data sources such as those of the OECD, the World Bank, or the IMF because of surprising inconsistencies across sources.

²⁹We use general government debt, except for Australia, Denmark, Greece, and Hungary for which only central government debt is available. The difference between general and central government debt is generally small, perhaps because local government debt often is due central government. It can sometimes be quite large. For example, Canada's general government debt-to-GDP ratio in 2010 is 82%, its central government debt 54%.

not before having reached a peak of 172% and a trough of 56%. Some increases were quite modest (Switzerland from 38% in 1983 to 39% in 2010 by way of a 55% peak and a 31% trough; Sweden from 39% in 1980 to 42% in 2010 by way of a 73% peak and a 38% trough), others much less so (Greece from 25% in 1980 to 144% in 2010; Italy from 53% to 117% by way of a 120% peak). We compute the PD associated with these debt ratios in Section 6.

6 Calibration analysis

The present section calibrates each country's maximum sustainable debt d_M , maximum sustainable borrowing b_M , maximum borrowing absent the borrowing multiplier b_S , and equity-like measure of maximum government liability b_E . It computes each country's probability of default at MSD and in each year of the period 1980-2010.

An issue that arises is that of the length of period chosen, from the date t at which debt is raised to the date $t + 1$ at which it is repaid. We choose the length of the period to be 4 years.³⁰ All calibration variables therefore are measured over 4-year periods, both inputs (α , μ , σ , and r) and outputs (γ , b_M , d_M , $\Phi(z_M)$, $\Phi(z)$). We nonetheless present these variables on the much more intuitive annual basis. In accordance with our chosen period, we set the risk free interest rate r equal to the average real yield on the United States 4-year Treasury Bond over the period 1980-2010.³¹ Expressed on an annual basis, it is 3.54%.

Table 3 shows countries' MSD d_M under the assumptions that MPS equals 5% (Column (1)), 4% (Column (5)), and for historical MPS (Column (6)).³² There is a very wide range of MSD, extending from Portugal's 5% to Norway's 720%, both with historical MPS, both indeed reflecting these MPS, Portugal at the 0.23% minimum and Norway at the 20.25% maximum. Using the IMF's estimate of 5% MPS instead of historical MPS narrows the range between minimum and maximum MSD and changes the identity of the countries at these extremes. These are now Greece at 89% and Korea at 282%. Where MPS is held constant across countries, mean growth rate appears to play a paramount role in the determination of MSD: Korea has the highest mean growth rate at 5.75%, Greece the third-lowest at 1.56%. Interestingly, Korea has the most volatile growth rate at

³⁰This is precisely the duration of US Government debt in 2010. See www.oecd.org/statistics/. We use duration rather than maturity because ours is zero-coupon debt.

³¹This is of course the same period as for the debt-to-GDP ratios.

³²Columns (2), (3), and (4) are discussed in Section 7.

7.39% and Greece the second-most volatile at 6.66%. This suggests a less important role for the volatility of growth as compared to its mean in the determination of MSD.

Table 4 presents MSB b_M (Column (2)), maximum static borrowing b_S (Column (1)), and Tanner’s (2013) equity-like measure of maximum government liability b_E (Column (3)), for 5% MPS.³³ The comparison of b_M and b_S illustrates the importance of the borrowing multiplier Γ defined in Definition 2.³⁴ For example, Sweden’s ability repeatedly to rely on future borrowing to repay present borrowing increases its maximum borrowing from 17% to 99% with 5% MPS; Korea’s ability to do likewise increases borrowing from 18% to 236% with 5% MPS. The comparison of b_M and b_E illustrates the importance of debt’s fixed payments and default. If Sweden were able credibly to commit to delivering *all* its 5% MPS *forever*, it could obtain financing equal to 266% of GDP rather than the 99% it is limited to by debt’s fixed payments and default. The equivalent number for Korea is infinite, because Korea’s growth rate at 5.75% is higher than the risk-free interest rate at 3.54%.

Table 5 shows the probabilities of default at MSD, $PD(d_M)$, for MPS equal to 5% (Column (1)), 4% (Column (5)), and for historical MPS (Column (6)).³⁵ Norway has the lowest $PD(d_M)$ at 0.27%, Korea the highest at 0.81%.

Figures 2 to 5 provide further insights into our calibrations. Figure 2 shows borrowing proceeds $B(d)$ in (17) as a function of debt d for two countries in our sample, France and Greece. Borrowing proceeds exhibit a marked Laffer Curve property, peaking at these two countries’ MSD d_M , 147% for France and 89% for Greece with 5% MPS. Borrowing proceeds’ dramatic decline above MSD suggests that the probability of default $PD(d)$ increases extremely rapidly towards one above MSD. Figure 3 confirms that this is indeed the case. The transition of the probability of default from zero to one is steeper for France, reflecting its lower growth rate volatility (3.26% for France and 6.65% for Greece): low volatility concentrates the probability mass around the mean, thereby steepening the transition from low default probability to high.

Figure 4 shows the marginal and average interest rates for France and Greece. The MSD lies at the intersection of the two curves. France’s faster growing marginal interest rate is consistent

³³The results are reported for 5% MPS for conciseness. The patterns reported in Table 4 are true for 4% and historical MPS too.

³⁴Note that the seemingly unreasonably large difference between debt proceeds b_M and debt repayment d_M is due to the maturity of the loan being 4 years.

³⁵Columns (2), (3), and (4) of tables 5 and 6 are discussed in Section 7.

with its steeper transition from probability of default zero to one.

Our calibrations assume a fixed interest. Figure 5 shows France and Greece's MSD and MSB as functions of the interest rate r . MSB's greater sensitivity to r reflects the impact of the probability of default $PD(d)$, shown in Proposition 4 to be increasing in r : proceeds decrease faster than face value because of the greater probability of default.

Table 6 shows the probabilities of default $PD(d_{2010})$ associated with countries' 2010 debt-to-GDP ratios d_{2010} , for MSB b_M computed with 5% MPS (Column (1)), 4% MPS (Column (5)), and historical MPS (Column (6)). Greece has non-zero PD for all measures of MSB, France, Iceland, Italy, Portugal, and the United States for at least one measure; Ireland and Spain have zero PD, despite having received some form of financial support from the European Union and/or the International Monetary Fund.³⁶ In order better to understand how realistic the probabilities of default computed with MSB are, we consider a number of countries' probabilities of default associated with actual debt over the entire period 1980-2010.

Figures 6 and 7 show the variation of country debt-to-GDP ratios over the period 1980-2010, as well as the associated MSD (left panel) and PD (right panel) with 5%, 4%, and historical MPS for Belgium, France, Greece, Hungary, Iceland, Ireland, Italy, and Portugal.³⁷ Belgium's debt reached its 106% MSD with 4% MPS in the mid-nineteen-eighties, remaining above that level until the late nineteen-nineties. Its associated PD rose to a maximum of 93.5% in 1994, as its debt-to-GDP ratio reached 134%. Belgium's PD reverted to near zero as its debt reverted to below its MSD by 2000. France's debt reached its 40% MSD with historical MPS in 1992; its PD was 0.31%. One year later, debt was at 46% and the associated PD already at 66%, suggesting that changes in PD can be very slow below MSD and very rapid above. There are two reasons for such a pattern. One reason explains the steepness of the transition; it relates to our discussion of Figure 3 above: very low growth volatilities make for extremely steep transitions from zero to one default probabilities. The other reason explains the asymmetry of the transition, slow below MSD and rapid above; it relates to the result at the end of Section 4: marginal and average interest rates are equal at MSD at which the average interest rate reaches its minimum. The average interest rate therefore decreases in debt

³⁶PD of zero and 100% should be understood to mean near zero and near 100%.

³⁷We leave the discussion of MSD and PD computed with a time-varying interest rate for Section 7.2. We do not show the other 15 countries because these have zero PD, regardless of whether MSD is computed with 5%, 4%, or historical MPS. The United States in 2010 has near zero PD of 0.02% for 4% MPS. Bohn (2011) provides an extended analysis of the prospects for U.S. debt.

below MSD and increases above. This necessarily implies that the interest rate increases slowly in debt before MSD and rapidly above. What is true of the interest rate naturally is also true of the probability of default. By 1995, France's PD with historical MPS had reached 100% with debt at 55%. Its PD has remained at 100% since. That neither Belgium nor, especially, France defaulted despite these high PD suggests that financial markets consider neither a country's historical MPS, 1.36% in France's case, nor 4% MPS the proper measure of the country's ability to service its debt. Indeed, with 5% MPS viewed by the IMF (2011) as a more appropriate measure of that ability, both Belgium and France had near zero PD over the entire 1980-2010 period. The same observation to some extent applies to Portugal, whose very low historical MPS at 0.23% implies that its PD was 100% over the entire period under consideration, but whose PD was zero with 5% and 4% MPS. That Portugal received EU and IMF support in 2011 suggests that low historical MPS is not entirely to be neglected.

Whilst France and Portugal are instances of countries for which historical and assumed MPS imply very different PD, such is not the case for Greece. There was a marked increase in Greece's debt between 1989 and 1990, from 66% to 89%, the latter Greece's MSD with 5% MPS. Greece's debt not long afterwards increased to about 105%, around which it hovered until 2008, dramatically to increase to 127% in 2009 and 144% in 2010. These increases were reflected in Greece's PD, that computed with 4% MPS increasing fastest, that with 5% MPS slowest, together providing the upper and lower bounds for PD computed with historical MPS at 4.37%. By and large, the increase in Greece's PD to above 50% was associated with the earlier increase in debt for 4% and historical MPS; it was associated with the later increase for 5% MPS. That Greece's needed EU and IMF support in 2010 supports the view that 5% MPS may be the more appropriate measure of a country's ability to service its debt; as noted above for Portugal, it is not the only measure.

Hungary's debt experience was in many ways the mirror image of that of Greece. Hungary enters our dataset in 1991 with very high debt at 156%, which increases to 172% in 1993 before declining dramatically. An examination of Hungary's PD again illustrates the abruptness of the transition around MSD: Hungary's PD with 5% MPS was 93.48% in 1991, when debt was 156%; it had declined to 0.06% by 1996, when debt for its part had declined to 109%, slightly below Hungary's MSD of 114% with 5% MPS. That Hungary did not default, despite having 172% debt-to-GDP ratio in 1993, with associated PD 99.45% with 5% MPS, probably reflects investors' recognition of

the additional creditworthiness provided by Hungary's extensive state holdings, inherited from four decades of socialism.³⁸ Indeed, it was largely through the use of privatization proceeds for debt retirement purposes that Hungary was able to decrease its debt to the aforementioned 109%.

Greece and Portugal were not the only countries to receive financial support in the wake of the 2008-2009 financial crisis: Iceland received IMF and Nordic country support in 2008, Ireland EU and IMF support in 2010, and Spain EU support in 2012. Iceland's need for support is to some extent apparent in Figure 7, which shows the rapid increase in Iceland's debt from 29% in 2008 to 116% in 2010, past all measures of MSD but that with historical MPS. Two observations are in order: (i) unlike Greece, Iceland received financial support before rather than after the increase in debt that took its debt-to-GDP ratio past MSD; (ii) Iceland's PD rose to high levels only with 4% MPS (71.76% PD); the increase in Iceland's PD with 5% MPS was much more muted (2.62% PD), suggesting that Iceland should have had no need for support under what we have argued is the more appropriate measure of a country's ability to service its debt.³⁹ Observation (i) may be explained by noting that Iceland needed IMF and Nordic country support for the purpose of bailing out its overextended banks; consolidation of the banks' debt on the government's balance sheet may have occurred only in 2009 and 2010. Observation (ii) may reflect the governments and the markets' recognition of the abruptness of changes in PD above MSD; private sector financing above MSD, possible for Greece in the years before the crisis, may not have been possible for Iceland in the wake of the crisis.

Unlike the cases for Greece and Iceland to a lesser extent, our analysis gives no hint of Ireland and Spain's needs for financial support. In the case of Spain, this may be explained by support being granted in June 2012, 18 months after the end of the period under consideration. The case of Ireland points to a limitation of our model and our measure of maximum sustainable debt.

Finally, we consider Italy. Figure 7 nicely illustrates the differences in PD associated with different MPS. It is highest with 4% MPS, lowest at zero with 6.51% historical MPS. This is most apparent in the year 1994 in which debt reaches its maximum of 120%: PD is 93.48% with 4% MPS, 5.88% with 5% MPS, and zero with historical MPS; a small MPS difference can make large MSD

³⁸Note that *net* debt-to-GDP ratios generally do not include real assets such as Hungary's state holdings in 1993. We use gross ratios in our analysis because these are generally more directly comparable across countries and across time.

³⁹Note that Iceland's debt is well below its 188% MSD with maximum historical MPS.

and PD differences when maintained forever. That Italy did not default in the nineteen-nineties again confirms the primacy of 5% MPS. The recent increase in Italy's debt to above its 113% MSD with 5% MPS, combined with the abruptness of changes in PD around MSD and the possibility already mentioned in the case of Iceland that private sector financing above MSD may no longer be possible in the wake of the crisis, suggests that Italy's financial situation may not be entirely comfortable.

7 Extensions

7.1 Maximum recovery in default

We now abandon the assumption of zero recovery in default, which we replace by the assumption of maximum recovery: a defaulting country's creditors receive the country's entire primary surplus. The analysis proceeds much as above, with (1) becoming

$$b_t y_t = \frac{\Pr[(\alpha + b_{t+1}) y_{t+1} > d_t y_t] d_t y_t + \Pr[(\alpha + b_{t+1}) y_{t+1} \leq d_t y_t] E[\alpha y_{t+1} | (\alpha + b_{t+1}) y_{t+1} \leq d_t y_t]}{1 + r},$$

b_M being the fixed point not of (7) but of

$$b_t = \max_z \frac{e^\mu}{1 + r} \left[[1 - \Phi(z)] (\alpha + b_{t+1}) e^{\sigma z} + \Phi(z - \sigma) \alpha e^{\frac{\sigma^2}{2}} \right] \equiv \tau(b_{t+1}), \quad (19)$$

(24) becoming

$$[1 - \Phi(z_M)] (\alpha + b_M) \sigma = \phi(z_M) b_M, \quad (20)$$

and d_M now maximizing not (17) but

$$B(d) \equiv \frac{1}{1 + r} \left[\left[1 - F\left(\frac{d}{\alpha + b_M}\right) \right] d + \int_0^{d/(\alpha + b_M)} \alpha g dF(g) \right]. \quad (21)$$

Remark 4 *The RHS of (21) recalls Merton's (1974) risky debt pricing formula in the case where (i) the real rather than the risk-neutral probability is used and (ii) debt has maturity a single period.*⁴⁰

⁴⁰To see this, use

$$\int_0^{d/(\alpha + b_M)} g dF(g) = \bar{g} \Phi\left(\frac{\ln\left[\frac{d}{\alpha + b_M}\right] - \mu - \sigma^2}{\sigma}\right)$$

Table 3, Column (2) shows MSD d_M with 5% MPS and maximum recovery. MSD increases, reflecting investors' greater willingness to lend when default need not imply a zero payoff. The increase is very slight, because default occurs with very low probability at MSD. Table 5, Column (2) shows the corresponding PD at MSD $PD(d_M)$; $PD(d_M)$ increases, reflecting investors' lesser reluctance to contemplate less costly default; for $PD(d_M)$ as for d_M , the increase is very slight. Finally, Table 6, Column (2) shows the associated PD at 2010 debt, $PD(d_{2010})$; unlike $PD(d_M)$, $PD(d_{2010})$ decreases: higher MSD d_M is accompanied by higher MSB b_M ; this increases the proceeds from raising new debt for the purpose of servicing existing debt; it thereby decreases the probability of default.

7.2 Time-varying risk-free interest rate

We now assume that the risk-free rate at date t , denoted r_t , follows a Markov chain with finite support $r^1 < r^2 < \dots < r^N$ and transition probabilities

$$\Pi_{ij} = \Pr [r_{t+1} = r^j | r_t = r^i].$$

Maximum borrowing at date t is now state-dependent. Specifically, it is a vector $b_t = (b_t^1, \dots, b_t^N)$ where b_t^i denotes maximum borrowing at date t when $r_t = r^i$. By analogy to (3), we define

$$b_t^i = \max_{d_t^i} \frac{d_t^i}{1+r^i} \sum_j \Pi_{ij} \left[1 - F \left(\frac{d_t^i}{\alpha + b_{t+1}^j} \right) \right] \equiv \tau_i(b_{t+1}).$$

The borrowing operator $\tau = (\tau_1, \dots, \tau_N)$ is now a mapping from \mathbb{R}_+^N into itself.

The analogues to Definition 3 and Proposition 1 are

from the properties of the lognormal distribution to rewrite (21) as

$$\begin{aligned} & \frac{1}{1+r} \left[\Phi \left(\frac{-\ln \left[\frac{d}{\alpha + b_M} \right] + \mu}{\sigma} \right) d + \alpha \bar{g} \Phi \left(\frac{\ln \left[\frac{d}{\alpha + b_M} \right] - \mu - \sigma^2}{\sigma} \right) \right] \\ &= \frac{d}{1+r} \Phi \left(-\frac{1}{\sigma} \ln \left[\frac{d}{(\alpha + b_M) \bar{g}} \right] - \frac{1}{2} \sigma \right) + \frac{\alpha \bar{g}}{1+r} \Phi \left(\frac{1}{\sigma} \ln \left[\frac{d}{(\alpha + b_M) \bar{g}} \right] - \frac{1}{2} \sigma \right) \end{aligned}$$

where $\bar{g} = E[g] = \exp[\mu + \sigma^2/2]$. This is essentially Merton's (1974) equation (13), with $B = d$, $V = \alpha + b_M$, $\tau = 1$, and $r = \mu + \sigma^2/2$; the LHS variables are Merton's and the RHS ours. The single difference is due to the presence of the term $\alpha \bar{g}/(1+r)$ where Merton's formula would have $\alpha + b_M$. There is no b_M because there is no new borrowing in default; $\bar{g}/(1+r)$ would reduce to 1 if the risk-neutral probability were used (neglecting the difference between simple and continuous compounding).

Definition 4 Borrowing $b \in R_+^N$ is sustainable if and only if there exists a bounded sequence of borrowing vectors $(b_t)_t$ such that $b_0 = b$ and $b_t^i \leq \tau_i(b_{t+1}) \forall t, i$.

Proposition 5 If the borrowing operator τ is a contraction, then it has a unique fixed point $b_M \in R_+^N$ which defines MSB conditionally on the prevailing interest rate..

The condition guaranteeing that τ is a contraction is somewhat more restrictive than the condition $\gamma < 1 + r$ in Proposition 1; it is nonetheless satisfied for all countries in our dataset.

Table 3, Column (3) shows MSD d_M^i corresponding to the risk-free interest rate r^i prevailing in 2010, with 5% MPS. MSD decreases for all countries but Greece. Lenders are aware that a country's ability to issue future debt for the purpose of servicing present debt will be curtailed in high risk-free interest rate states and improved in low interest rate states (recall Proposition 2). The first effect decreases MSD and the second increases it. The first effect dominates for the vast majority of countries: debt's fixed payments induce risk averse behavior on the part of lenders by limiting their ability to profit from low interest rates. The Greek exception may be due to Greece's combination of low MSD (89%) and high probability of default at MSD (0.71%) in the case of constant risk-free interest rate: Greece may have less to lose and more to gain from a transition to a high or low interest rate state, respectively.⁴¹ Table 5, Column (3) shows the corresponding PD at MSD $PD^i(d_M^i)$.⁴² All $PD^i(d_M^i)$ decrease, the vast majority to zero: the desire to keep the maximum probability of default, $\max_i [PD^i(d_M^i)]$, low keeps all other probabilities of default very low. Table 6, Column (3) shows the associated PD at 2010 debt $PD^i(d_{2010})$. For those countries for which the probability of default for constant risk-free interest rate— $PD(d_{2010})$ in Column (1)—was not zero, $PD^i(d_{2010})$ decreases: persistence in interest rates and the historically low 2010 risk-free interest rate imply that the future interest rate is likely to be low, making future borrowing proceeds high and the probability of default low.

Figures 6 and 7 show maximum sustainable debt and the probability of default associated with actual debt for time-varying risk-free interest rate with 5% MPS (light, broken line). MSD increases over time, reflecting the more or less continual decline in the risk-free interest rate over the period

⁴¹To confirm this intuition, note that Portugal, which presents a somewhat less extreme form of Greece's low MSD-high PD at MSD combination, sees barely a change in its MSD.

⁴²Note that the probability of default depends on the present state, $PD^i(\cdot)$, because the transition probabilities do, Π_{ij} .

1980-2010. As MSD with time-varying rate is below that with constant rate and 5% MPS for the vast majority of countries (left panel), the probability of default with actual debt is correspondingly higher (right panel). This is clearly to be seen in Figure 6 for Belgium and Hungary and in Figure 7 for Ireland and Italy. Belgium’s PD increases earlier, decreases later, and remains at 100% longer for time-varying interest rate; Hungary’s PD decreases later; Ireland has a hitherto unobserved departure from zero PD over part of the nineteen-eighties; finally, Italy’s very small departure from zero PD over part of the nineteen-nineties is greatly magnified and expands to include the entirety of the nineteen-nineties, much of the nineteen-eighties, and a very small part of the ‘two-thousands.’ The exception is again Greece: its higher MSD with time-varying interest rate towards the end of our period results in the probability of default being lower for time-varying than for constant interest rate over much of the two-thousands.⁴³

7.3 Rare growth disasters

We now allow for the possibility of rare growth disasters (Rietz, 1988; Barro, 2006). Specifically, we follow Barro (2006) in extending our growth process from

$$\log(g) = \mu + u, \quad u \sim N(0, \sigma^2)$$

to

$$\log(g) = \mu + u - v, \quad u \sim N(0, \sigma^2)$$

and

$$v = \begin{cases} z & \text{with probability } p \\ 0 & \text{with probability } 1 - p \end{cases}$$

with

$$f(z) = \begin{cases} \alpha e^{-\alpha(z-z_0)} & \text{if } z \geq z_0 \\ 0 & \text{otherwise} \end{cases}$$

Table 3, Column (4) shows MSD d_M with 5% MPS where growth disasters may occur. There is a marked decline in MSD: Korea’s decreases from 282% to 117%, Greece’s from 89% to 59%.

⁴³The results for Belgium and Italy beg the question of why these two countries did not default. One possible answer is that effective government debt market segmentation prior to the introduction of the Euro in 1999 made it possible for Belgian Franc and Italian Lira (real) interest rates to remain below US dollar rates.

Lenders fearing a collapse in output that will leave governments with little revenue for debt service lend less than they otherwise would. Table 5, Column (4) shows the corresponding PD at MSD $PD(d_M)$. That probability increases, despite the decline in maximum sustainable debt: there are two ‘margins’ through which the possibility of growth disasters may be ‘accommodated,’ the amount lent and the probability of default; lenders use both margins, lending less and requesting a higher interest rate.⁴⁴ Table 6, Column (4) shows the associated PD at 2010 debt $PD(d_{2010})$. Its increase reflects both lower government revenues should a growth disaster occur and lower proceeds from issuing new debt to service existing debt. Note the dramatic increases for Iceland and Italy, whose MSD (69% and 74%, respectively) are now well below their 2010 debt (116% and 117%).

Figure 8 shows the probability of default associated with actual debt for those nine countries that have PD at 2010 debt greater than 1% in Column (4) of Table 6.⁴⁵ Should growth disasters similar in likelihood and magnitude to those calibrated by Barro (2006) have been considered a real possibility, Belgium, Greece, and Italy surely and Canada and Ireland possibly should have defaulted already in the nineteen-eighties or nineteen-nineties. That they did not suggests that lenders at the time rightly or wrongly considered growth collapses a thing of the past. The recent experience of Greece and Iceland but not that of Italy suggests that lenders may no longer be of that opinion.

8 Conclusion

We have developed a measure of maximum sustainable government debt for advanced economies. Our measure assumes that governments do their utmost not to default and that maturing debt can be serviced out of newly raised debt’s proceeds. This last assumption gives rise to a borrowing multiplier that increases a country’s sovereign debt capacity well above what it would be if the country’s government were limited to its current primary surplus for the purpose of servicing its debt. We have calibrated our measure for 23 OECD countries and have compared maximum sustainable debt (MSD) to actual government debt over the period 1980-2010. Our measure appears to capture at least part of a country’s debt capacity: countries whose actual debt increases beyond

⁴⁴Maximum sustainable borrowing MSB decreases still more than does MSD (not shown); the interest rate therefore increases.

⁴⁵We somewhat arbitrarily leave out the United Kingdom at $PD(d_{2010}) = 0.03\%$.

MSD generally encounter difficulty servicing their debt. A somewhat unexpected yet important result is that a country's probability of default increases slowly below MSD and very rapidly above. The steepness of the transition is a consequence of the very low volatility of growth rates, which concentrates the probability mass over a very short interval that sees the transition from low default probability to high. The asymmetry of the transition around MSD is a consequence of MSD being the level of debt at which the average interest rate reaches its minimum, somewhat similarly to optimal output being the level of output at which average cost is minimized. The average interest rate therefore decreases in debt below MSD and increases above. This necessarily implies that the interest rate increases slowly in debt before MSD and rapidly above. What is true of the interest rate naturally is also true of the probability of default.

Our measure is not without its limitations. In statistical parlance, it is subject to both Type I and Type II errors: some countries have debt markedly above MSD yet display little to no sign of financial distress; other countries are in need of financial support yet have debt below MSD. It can be quite sensitive to the values of its various parameters: Norway's MSD would increase from 720% to 988% if it were computed with the risk-free interest rate averaged over the postwar period (2.67%) rather than the period 1980-2010 (3.54%). Finally, it does not consider governments' implicit liabilities such as unfunded pensions and increased health care costs due to population ageing.⁴⁶

The present paper has focused entirely on the *maximum* levels of sovereign debt that can be sustained in advanced economies. Our companion paper (Collard, Habib, and Rochet, 2014) instead tries to explain the *actual* levels of sovereign debt in these economies. It abandons the classical assumption that governments choose the level of sovereign debt that maximizes some measure of social welfare; it proposes instead a "Public Choice" theory of sovereign debt, whereby government debt is chosen by self-interested politicians. In the extreme case where these politicians are completely myopic, and always borrow as much as they can without consideration for the future debt burden of their country, they always choose the maximum sustainable debt level characterized in the present paper. For less extreme objective functions, the model in Collard, Habib, and Rochet (2014) predicts debt levels that reflect the "impatience of politicians" and the quality of political institutions.

⁴⁶See Hagist, Moog, Raffelhueschen, and Vatter (2009) for an analysis of advanced economies' implicit liabilities.

Appendix

Proof of Proposition 1 (i) $\gamma < 1 + r$. Iterate (7) forward to obtain

$$b_t = \frac{\alpha\gamma}{1+r} \left[1 + \frac{\gamma}{1+r} + \dots + \left(\frac{\gamma}{1+r} \right)^\tau \right] + \left(\frac{\gamma}{1+r} \right)^\tau b_{t+\tau}. \quad (22)$$

Present borrowing b_t is sustainable when future borrowing $b_{t+\tau}$ is bounded $\forall \tau$. It is therefore the case that

$$\lim_{\tau \rightarrow \infty} b_t = \frac{\alpha\gamma}{1+r} \frac{1}{1 - \frac{\gamma}{1+r}} = \frac{\alpha\gamma}{1+r-\gamma} \equiv b_M.$$

Maximum borrowing b_M is sustained by the sequence $b_t = b_M \forall t$.⁴⁷ Borrowing b_B beyond b_M is not sustainable, for it requires unbounded future borrowing

$$\lim_{\tau \rightarrow \infty} \left(\frac{\gamma}{1+r} \right)^\tau b_{t+\tau} = b_B - b_M \Rightarrow \lim_{\tau \rightarrow \infty} b_{t+\tau} = \lim_{\tau \rightarrow \infty} \left(\frac{1+r}{\gamma} \right)^\tau (b_B - b_M) = \infty.$$

(i) $\gamma \geq 1 + r$. We show how to construct a bounded sequence that can sustain any $b > 0$.

Rewrite (7) as

$$b_{t+1} = \frac{1+r}{\gamma} b_t - \alpha$$

and iterate backward to obtain

$$b_t = \left(\frac{1+r}{\gamma} \right)^t b_0 - \alpha \left[\left(\frac{1+r}{\gamma} \right)^{t-1} + \dots + \left(\frac{1+r}{\gamma} \right)^2 + 1 \right]. \quad (23)$$

Since $b_t < b_0 \forall t$, (23) defines a bounded sequence $(b_t)_t$ that sustains any borrowing $b = b_0 > 0$.⁴⁸ ■

Proof of Proposition 2 Use the Envelope Theorem. The results for α and r are immediate from (10). The result for μ follows from $\partial\gamma/\partial\mu > 0$ from (13). The result for σ follows from $\text{sign}\{\partial\gamma/\partial\mu\} = \text{sign}\{z_M\}$ from (13). The condition $z_M < 0$ is equivalent to $\sigma < \sqrt{2/\pi}$. To see this, consider the FOC for z_M

$$[1 - \Phi(z_M)]\sigma = \phi(z_M). \quad (24)$$

⁴⁷This is consistent with maximum sustainable borrowing b_M being the fixed point $b_t = b_{t+1} = b_M$ in (7).

⁴⁸If

$$\left(\frac{1+r}{\gamma} \right)^t b_0 < \alpha \left[\left(\frac{1+r}{\gamma} \right)^{t-1} + \dots + \left(\frac{1+r}{\gamma} \right)^2 + 1 \right]$$

in (23), then $b_t = 0$: no new debt need be raised for the purpose of servicing existing debt; MPS α suffices for that purpose.

Define the hazard rate $h(z) \equiv \phi(z) / [1 - \Phi(z)]$ to rewrite (24) as

$$\sigma = h(z_M). \quad (25)$$

Since the normal distribution has increasing hazard rate, $h'(z) > 0$, the condition $z_M < 0$ is equivalent to $h(z_M) < h(0)$; but $h(z_M) = \sigma$ from (25) and $h(0) = \sqrt{2/\pi}$ by direct substitution. ■

Proof of Proposition 3 Differentiate (25) with respect to μ , α , σ , and r to obtain

$$h'(z_M) \frac{\partial z_M}{\partial \mu} = 0, \quad (26)$$

$$h'(z_M) \frac{\partial z_M}{\partial \alpha} = 0, \quad (27)$$

$$h'(z_M) \frac{\partial z_M}{\partial \sigma} = 1 > 0, \quad (28)$$

and

$$h'(z_M) \frac{\partial z_M}{\partial r} = 0. \quad (29)$$

Use (15) and (26) to write

$$\begin{aligned} \frac{\partial \ln(d_M)}{\partial \mu} &= \frac{\frac{\partial b_M}{\partial \mu}}{\alpha + b_M} + 1 + \sigma \frac{\partial z_M}{\partial \mu} \\ &= \frac{\frac{\partial b_M}{\partial \mu}}{\alpha + b_M} + 1; \end{aligned} \quad (30)$$

$\partial \ln(d_M) / \partial \mu > 0$ is then an immediate consequence of $\partial b_M / \partial \mu > 0$ established in Proposition 2; $\partial d_M / \partial \mu > 0$ follows.

Use (15) and (27) to write

$$\begin{aligned} \frac{\partial \ln(d_M)}{\partial \alpha} &= \frac{1 + \frac{\partial b_M}{\partial \alpha}}{\alpha + b_M} + \sigma \frac{\partial z_M}{\partial \alpha} \\ &= \frac{1 + \frac{\partial b_M}{\partial \alpha}}{\alpha + b_M}; \end{aligned} \quad (31)$$

$\partial \ln(d_M) / \partial \alpha > 0$ is then an immediate consequence of $\partial b_M / \partial \alpha > 0$ established in Proposition 2; $\partial d_M / \partial \alpha > 0$ follows.

Finally, use (15) and (29) to write

$$\begin{aligned}\frac{\partial \ln(d_M)}{\partial r} &= \frac{\frac{\partial b_M}{\partial r}}{\alpha + b_M} + \sigma \frac{\partial z_M}{\partial r} \\ &= \frac{\frac{\partial b_M}{\partial r}}{\alpha + b_M}\end{aligned}\tag{32}$$

$\partial \ln(d_M)/\partial r < 0$ is then an immediate consequence of $\partial b_M/\partial r < 0$ established in Proposition 2; $\partial d_M/\partial r < 0$ follows. ■

Proof of Proposition 4 From (16), default occurs with probability $\Phi([\ln(d) - \ln(\alpha + b_M) - \mu]/\sigma) = \Phi(z_d)$. Use $\partial b_M/\partial \mu > 0$, $\partial b_M/\partial \alpha > 0$, $\partial b_M/\partial \sigma < 0$ for $z < 0$, and $\partial b_M/\partial r < 0$, to conclude that $\partial z_d/\partial \mu < 0$, $\partial z_d/\partial \alpha < 0$, $\partial z_d/\partial \sigma > 0$ for $z_d < 0$, and $\partial z_d/\partial r > 0$, respectively. ■

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A Figures

Figure 1: Maximum Sustainable Debt as a Function of Growth Rate Volatility

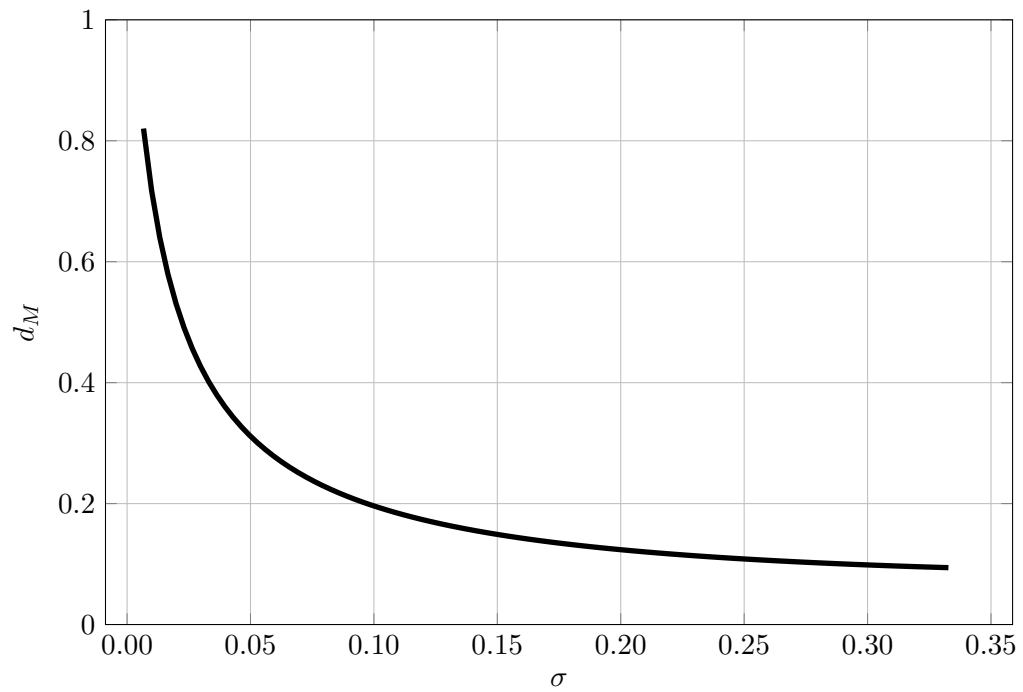


Figure 2: Borrowing proceeds as a function of nominal debt ($\alpha = 5\%$)

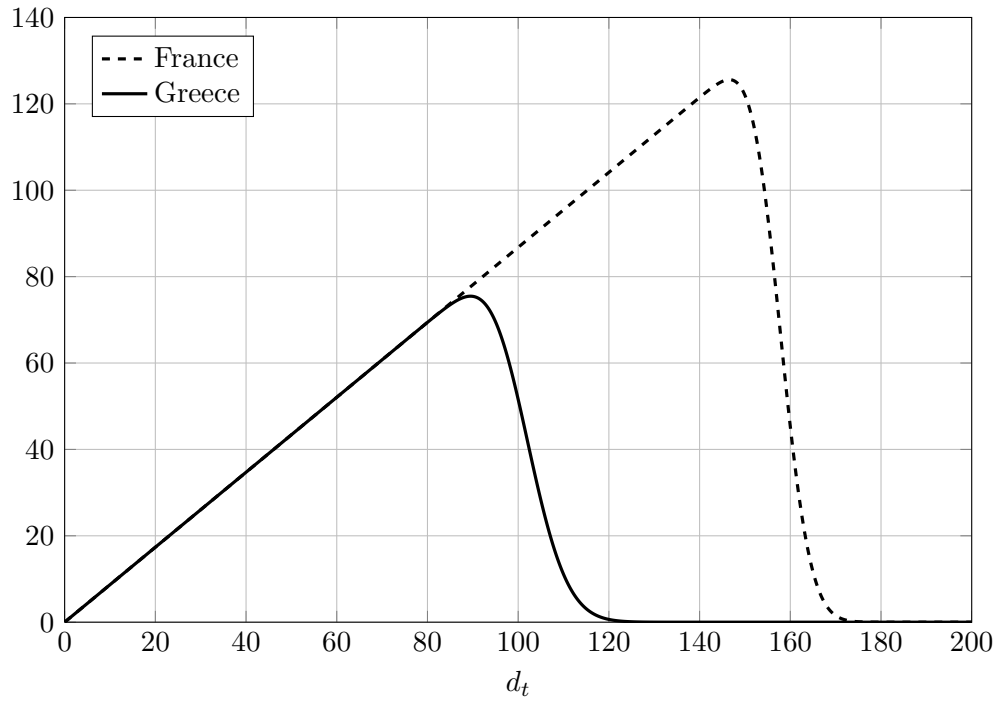


Figure 3: Default Probability ($\alpha = 5\%$)

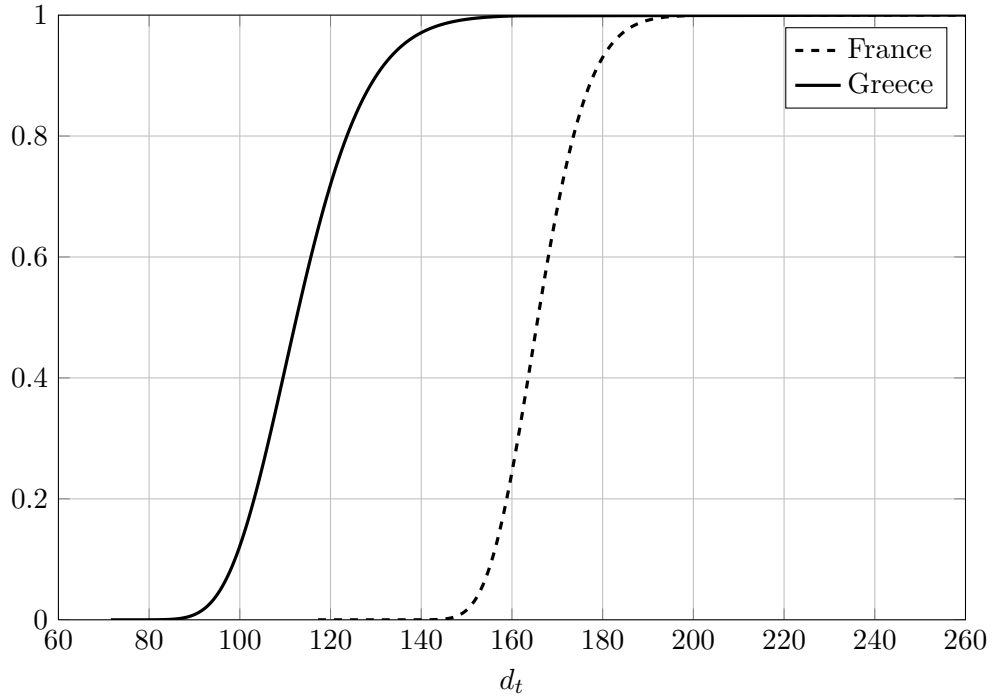


Figure 4: Marginal and Average Interest Rates ($\alpha = 5\%$)

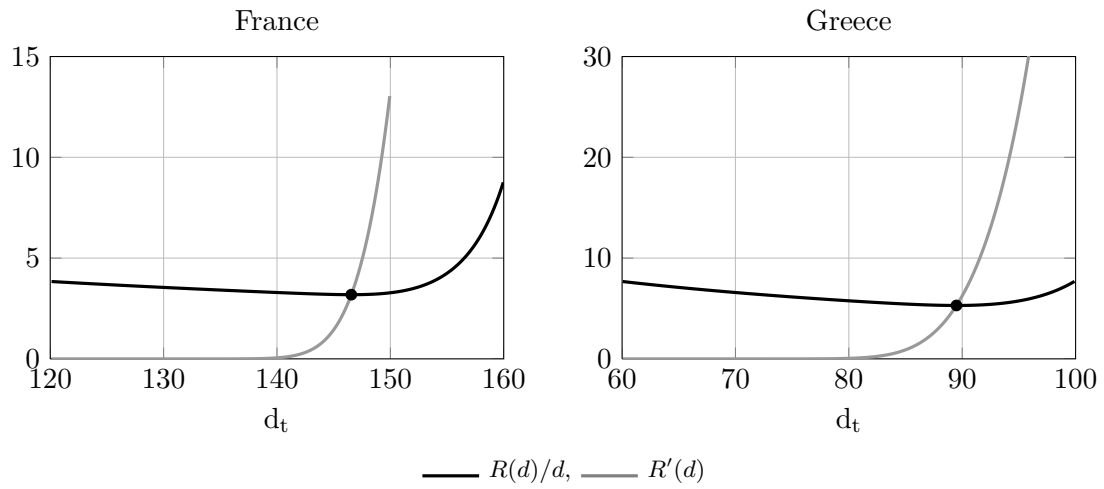


Figure 5: b_M and d_M as Functions of the Interest Rate

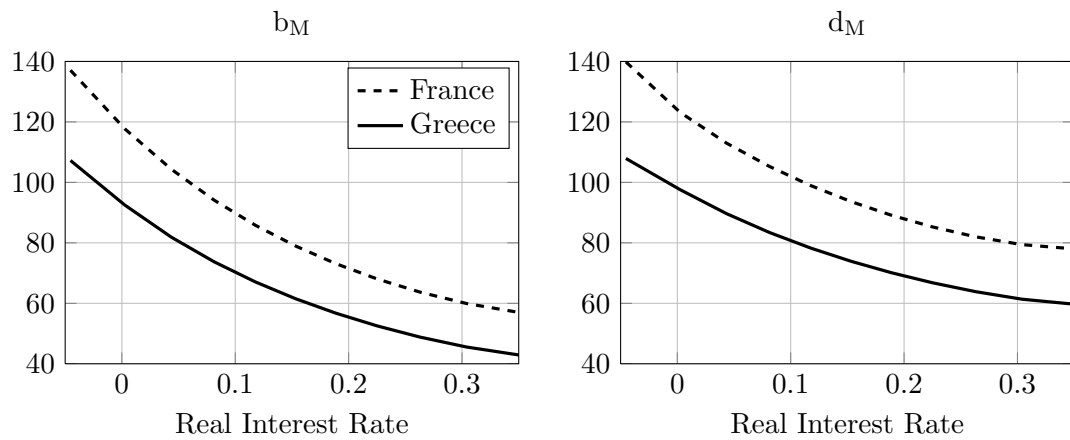
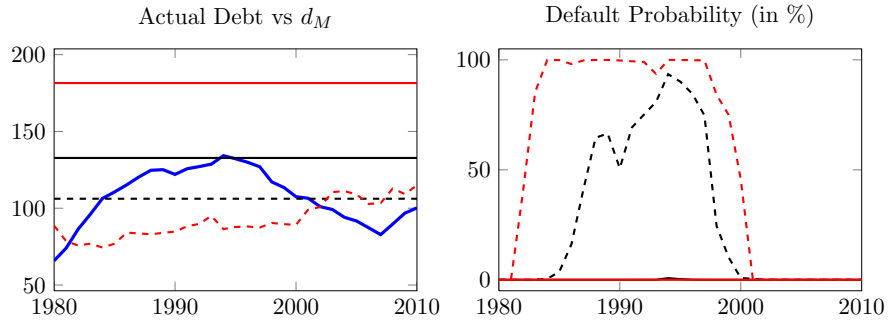
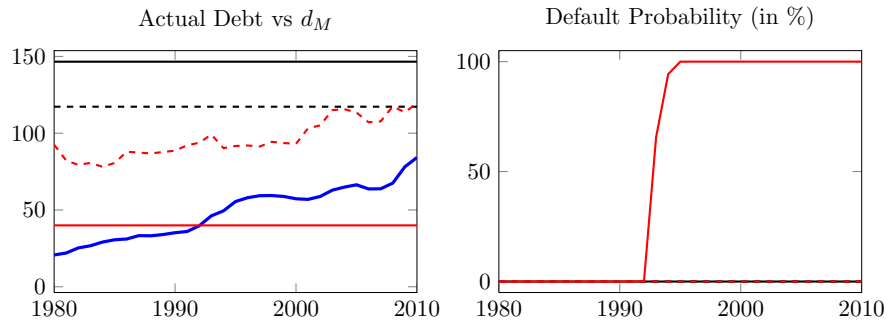


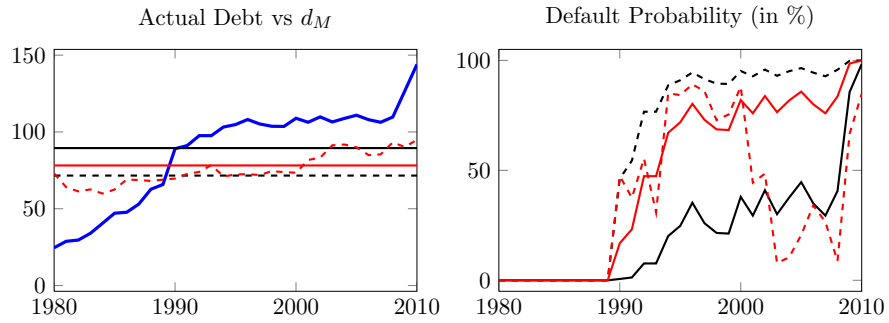
Figure 6: Actual Debt, Maximum Sustainable Debt, and Probability of Default over Time
Belgium



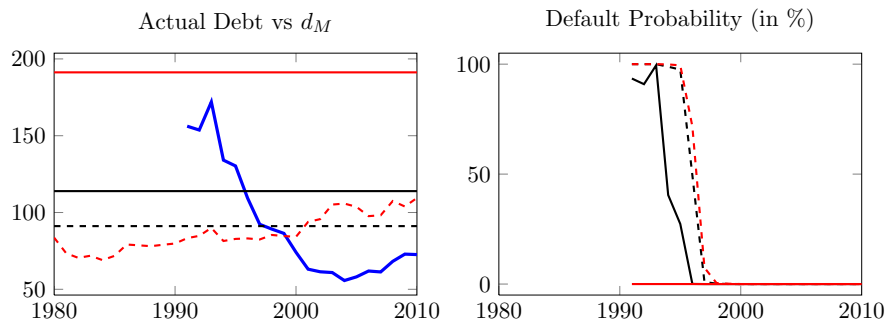
France



Greece

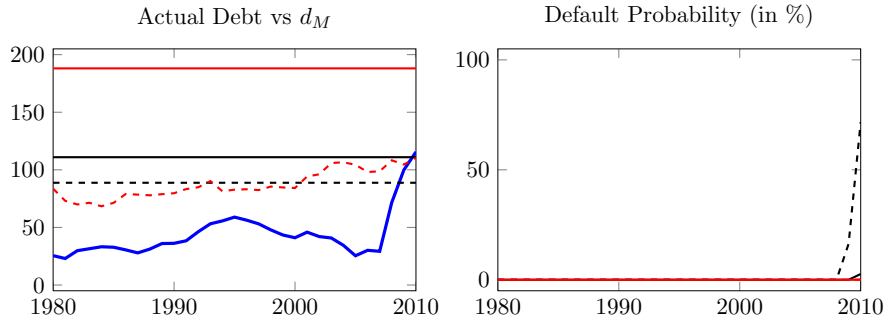


Hungary

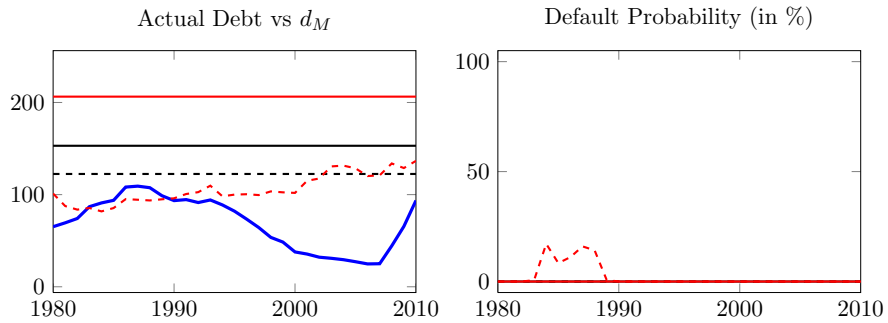


Note: — Observed debt/GDP ratio; — $\alpha = 5\%$, - - - $\alpha = 4\%$, — $\alpha = \max_t\{PS/Y\}$, - - - Time-Varying Interest Rate ($\alpha = 5\%$)

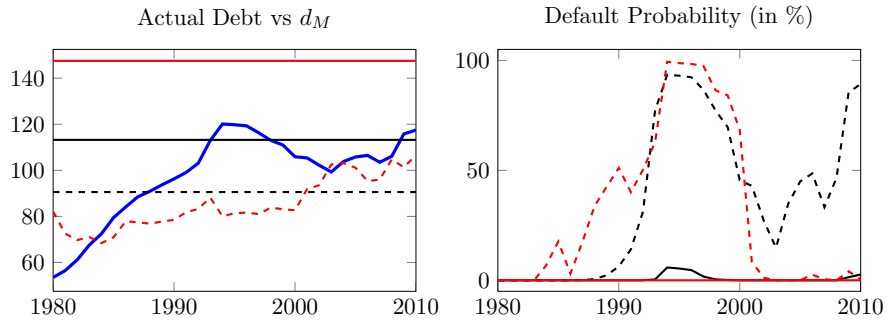
Figure 7: Actual Debt, Maximum Sustainable Debt, and Probability of Default over Time
Iceland



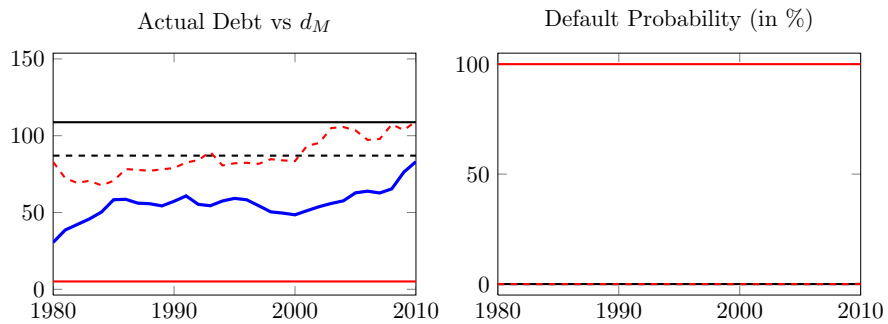
Ireland



Italy

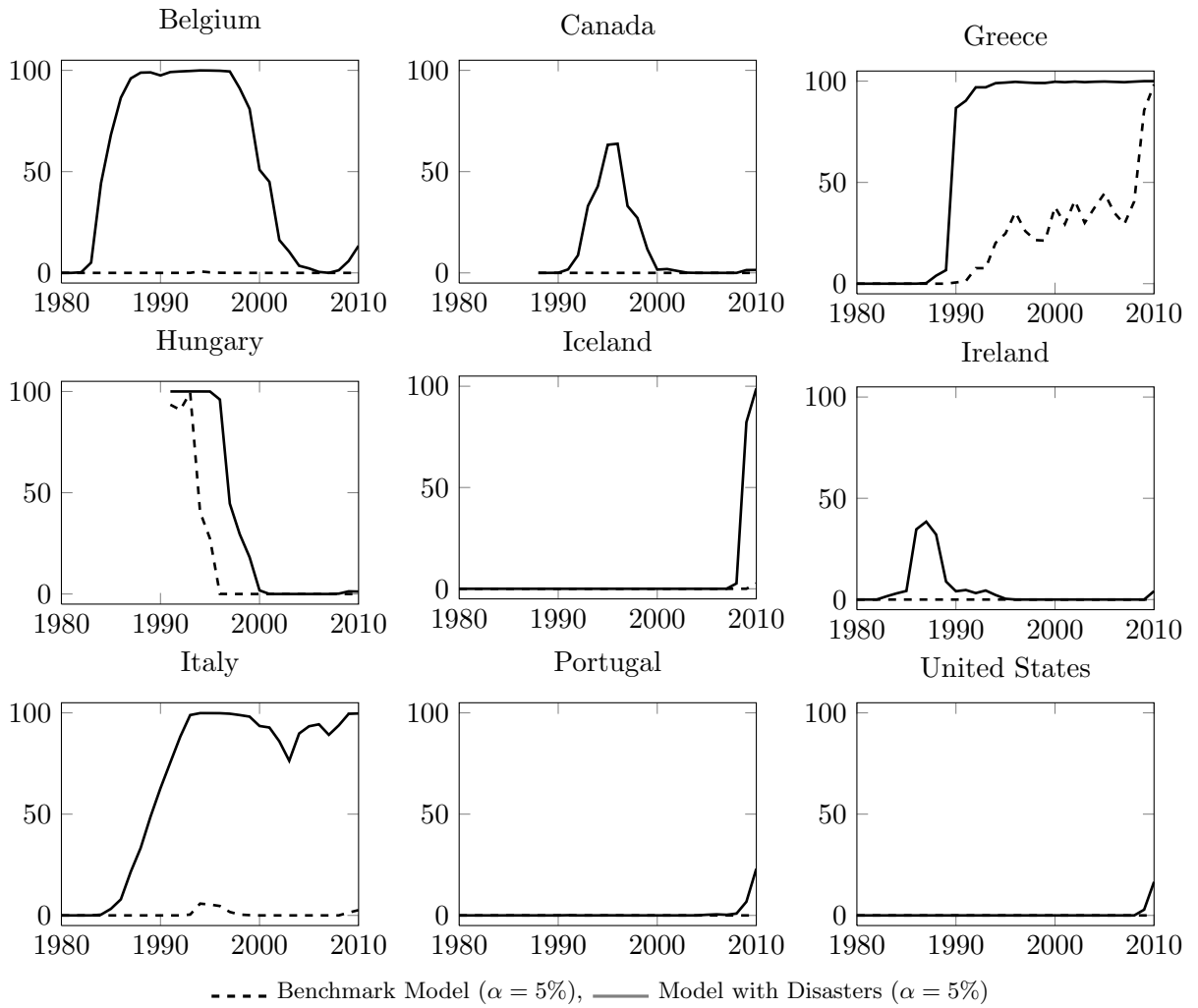


Portugal



Note: — Observed debt/GDP ratio; — $\alpha = 5\%$, - - - $\alpha = 4\%$, — $\alpha = \max_t\{PS/Y\}$, - - - Time-Varying Interest Rate ($\alpha = 5\%$)

Figure 8: Probability of Default: Benchmark vs Model with Disasters



B Tables

Table 1: Data (%)

Country	μ	σ	d_{2010}	MPS
Australia	1.91	3.30	11.30	4.16
Austria	2.17	3.65	70.00	3.32
Belgium	1.88	3.71	100.20	6.84
Canada	1.72	4.10	81.70	10.05
Denmark	1.74	4.41	40.60	7.10
Finland	2.21	5.91	50.00	9.82
France	1.99	3.26	84.20	1.36
Germany	1.89	3.88	78.80	4.34
Greece	1.56	6.65	144.00	4.37
Hungary	1.98	5.12	72.60	8.39
Iceland	2.25	5.92	115.58	8.47
Ireland	3.30	5.52	93.60	6.74
Italy	1.67	4.53	117.50	6.51
Korea	5.75	7.39	32.00	6.44
Netherlands	1.84	3.41	67.40	5.62
New Zealand	1.12	4.53	31.00	7.73
Norway	2.37	2.84	54.30	20.25
Portugal	2.26	6.13	83.10	0.23
Spain	1.99	3.36	63.50	4.01
Sweden	1.71	4.40	41.70	7.05
Switzerland	1.01	4.10	39.50	3.05
United Kingdom	1.93	4.17	76.70	6.31
United States	1.75	4.16	92.70	5.09

Note: MPS denotes the historical maximum primary surplus, $\alpha = \max_t \{PS/Y\}$.

Table 2: Debt-to-GDP Ratios (%)

	1980	min(D/Y)	E(D/Y)	max(D/Y)	2010
Australia	18.70	4.50	12.74	21.00	11.30
Austria	35.30	35.30	58.46	70.00	70.00
Belgium	65.80	65.80	107.71	134.20	100.20
Canada ^(a)	71.10	65.10	82.84	101.70	81.70
Denmark	35.00	27.90	57.06	79.80	40.60
Finland	11.00	11.00	34.32	57.70	50.00
France	20.70	20.70	48.65	84.20	84.20
Germany	30.00	30.00	52.05	78.80	78.80
Greece	24.60	24.60	86.57	144.00	144.00
Hungary ^(b)	156.20	55.70	91.69	172.00	72.60
Iceland	25.50	23.00	44.09	115.58	115.58
Ireland	65.20	24.80	68.86	109.20	93.60
Italy	53.50	53.50	97.77	120.10	117.50
Korea	14.10	8.90	20.70	36.80	32.00
Netherlands	45.40	45.40	63.13	77.20	67.40
New Zealand	45.00	17.40	42.82	71.60	31.00
Norway	47.30	28.90	42.51	60.50	54.30
Portugal	30.50	30.50	55.77	83.10	83.10
Spain	16.40	16.40	46.62	67.40	63.50
Sweden	39.30	37.60	54.67	73.20	41.70
Switzerland ^(c)	38.30	31.00	44.06	55.30	39.50
United Kingdom	46.10	31.30	44.65	76.70	76.70
United States	42.10	41.20	62.28	92.70	92.70

Note: (a) Column 1980 reports the number for 1988. (b) Column 1980 reports the number for 1991. (c) Column 1980 reports the number for 1983.

Table 3: Maximum Sustainable Debt d_M (%)

Country	$\alpha = 5.00\%$ (1)	MRR (2)	TVR (3)	CATA (4)	$\alpha = 4.00\%$ (5)	MPS (6)
Australia	142.25	144.25	115.79	95.37	113.80	118.49
Austria	145.14	147.43	119.43	93.86	116.11	96.40
Belgium	132.73	134.87	113.19	87.81	106.18	181.50
Canada	120.75	122.97	108.33	79.80	96.60	242.70
Denmark	116.99	119.33	107.42	76.51	93.59	166.11
Finland	110.18	113.32	108.90	68.31	88.15	216.32
France	146.56	148.58	117.65	97.77	117.24	39.98
Germany	130.13	132.35	112.70	85.48	104.10	112.88
Greece	89.49	92.51	94.49	58.57	71.59	78.18
Hungary	113.92	116.65	108.47	72.19	91.14	191.25
Iceland	110.96	114.12	109.52	68.60	88.77	188.07
Ireland	153.02	156.89	134.74	85.64	122.42	206.33
Italy	113.23	115.58	105.52	74.20	90.58	147.53
Korea	281.74	291.34	205.10	116.77	225.39	362.77
Netherlands	137.37	139.38	113.97	92.15	109.90	154.53
New Zealand	100.48	102.60	96.66	68.12	80.38	155.37
Norway	177.91	179.97	128.33	116.69	142.33	720.37
Portugal	108.78	112.02	108.66	67.30	87.02	5.10
Spain	144.16	146.22	117.10	95.79	115.33	115.57
Sweden	116.14	118.46	106.85	76.14	92.91	163.86
Switzerland	102.68	104.60	96.55	70.98	82.14	62.54
United Kingdom	126.22	128.57	111.97	81.95	100.97	159.36
United States	120.89	123.15	108.72	79.55	96.71	122.98

Note: MRR: Maximum Recovery Rate; TVR: Time-Varying Interest Rate; CATA: Model featuring catastrophes. In all three cases, α is set to 5%. In the case of a time-varying interest rate, maximum sustainable debt is computed assuming the interest rate prevailing in 2010. MPS: α is set to the historical maximum primary surplus, $\alpha = \max_t\{PS/Y\}$,

Table 4: Maximum Static Borrowing, Maximum Sustainable Borrowing, and Equity-Like Measure with $\alpha = 5\%$ (%)

Country	b_S (1)	b_M (2)	b_E (3)
Australia	17.18	121.89	298.41
Austria	17.23	124.17	358.45
Belgium	17.00	113.52	292.79
Canada	16.75	103.09	266.57
Denmark	16.66	99.74	271.11
Finland	16.47	93.27	378.83
France	17.25	125.60	314.61
Germany	16.95	111.22	296.20
Greece	15.81	75.47	248.65
Hungary	16.58	96.80	317.09
Iceland	16.49	93.92	390.40
Ireland	17.33	129.77	2361.16
Italy	16.57	96.48	259.98
Korea	18.44	236.70	∞
Netherlands	17.09	117.65	286.73
New Zealand	16.21	85.62	198.99
Norway	17.68	152.75	419.81
Portugal	16.43	91.98	394.61
Spain	17.21	123.49	314.18
Sweden	16.64	99.02	266.04
Switzerland	16.28	87.67	188.78
United Kingdom	16.87	107.73	303.23
United States	16.75	103.19	272.26

Table 5: Default Probability at Maximum Sustainable Debt $PD(d_M)$ (%)

Country	$\alpha = 5.00\%$ (1)	MRR (2)	TVR (3)	CATA (4)	$\alpha = 4.00\%$ (5)	MPS (6)
Australia	0.32	0.38	0.00	0.51	0.32	0.32
Austria	0.36	0.42	0.00	0.59	0.36	0.36
Belgium	0.36	0.43	0.00	0.61	0.36	0.36
Canada	0.41	0.50	0.00	0.71	0.41	0.41
Denmark	0.44	0.54	0.00	0.79	0.44	0.44
Finland	0.62	0.77	0.01	1.37	0.62	0.62
France	0.31	0.37	0.00	0.50	0.31	0.31
Germany	0.38	0.46	0.00	0.65	0.38	0.38
Greece	0.71	0.93	0.03	1.90	0.71	0.71
Hungary	0.53	0.65	0.00	1.03	0.53	0.53
Iceland	0.62	0.78	0.01	1.38	0.62	0.62
Ireland	0.57	0.67	0.00	1.19	0.57	0.57
Italy	0.46	0.57	0.00	0.83	0.46	0.46
Korea	0.81	0.89	0.01	2.77	0.81	0.81
Netherlands	0.33	0.39	0.00	0.54	0.33	0.33
New Zealand	0.46	0.58	0.00	0.83	0.46	0.46
Norway	0.27	0.31	0.00	0.41	0.27	0.27
Portugal	0.65	0.81	0.01	1.49	0.65	0.65
Spain	0.32	0.38	0.00	0.52	0.32	0.32
Sweden	0.44	0.54	0.00	0.79	0.44	0.44
Switzerland	0.41	0.51	0.00	0.71	0.41	0.41
United Kingdom	0.41	0.50	0.00	0.73	0.41	0.41
United States	0.41	0.51	0.00	0.72	0.41	0.41

Note: MRR: Maximum Recovery Rate; TVR: Time-Varying Interest Rate; CATA: Model featuring catastrophes. In all three cases, α is set to 5%. In the case of a time-varying interest rate, the default probability is computed assuming the interest rate prevailing in 2010. MPS: α is set to the historical maximum primary surplus, $\alpha = \max_t\{PS/Y\}$,

Table 6: Default Probability in 2010 $PD(d_{2010})$ (%)

Country	$\alpha = 5.00\%$ (1)	MRR (2)	TVR (3)	CATA (4)	$\alpha = 4.00\%$ (5)	MPS (6)
Australia	0.00	0.00	0.00	0.00	0.00	0.00
Austria	0.00	0.00	0.00	0.00	0.00	0.00
Belgium	0.00	0.00	0.00	13.32	0.00	0.00
Canada	0.00	0.00	0.00	1.43	0.00	0.00
Denmark	0.00	0.00	0.00	0.00	0.00	0.00
Finland	0.00	0.00	0.00	0.00	0.00	0.00
France	0.00	0.00	0.00	0.00	0.00	100.00
Germany	0.00	0.00	0.00	0.00	0.00	0.00
Greece	98.33	97.25	85.50	100.00	100.00	99.94
Hungary	0.00	0.00	0.00	1.17	0.00	0.00
Iceland	2.62	1.24	0.15	98.84	71.76	0.00
Ireland	0.00	0.00	0.00	4.09	0.00	0.00
Italy	2.62	1.28	0.85	99.71	89.24	0.00
Korea	0.00	0.00	0.00	0.00	0.00	0.00
Netherlands	0.00	0.00	0.00	0.00	0.00	0.00
New Zealand	0.00	0.00	0.00	0.00	0.00	0.00
Norway	0.00	0.00	0.00	0.00	0.00	0.00
Portugal	0.00	0.00	0.00	22.40	0.09	100.00
Spain	0.00	0.00	0.00	0.00	0.00	0.00
Sweden	0.00	0.00	0.00	0.00	0.00	0.00
Switzerland	0.00	0.00	0.00	0.00	0.00	0.00
United Kingdom	0.00	0.00	0.00	0.03	0.00	0.00
United States	0.00	0.00	0.00	16.47	0.02	0.00

Note: MRR: Maximum Recovery Rate; TVR: Time-Varying Interest Rate; CATA: Model featuring catastrophes. In all three cases, α is set to 5%. In the case of a time-varying interest rate, the default probability is computed assuming the interest rate prevailing in 2010. MPS: α is set to the historical maximum primary surplus, $\alpha = \max_t\{PS/Y\}$,