Forward Guidance at the Zero Lower Bound in a Model with Price-Level Targeting

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Outline

- Recently much attention on central bank forward guidance, short–term interest rate policy constrained by ZLB
  - Fed: policy rate remains at exceptionally low range as long as unemployment rate $\geq 6.5\,\%$, $\mathbb{E}_t[\pi_{t+1}], \mathbb{E}_t[\pi_{t+2}] \leq 2.5\,\%$ and longer–term inflation expectations remain anchored. (Dec. 2012)
  - BoJ: raises inflation target from $1\,\%$ to $2\,\%$. (Jan. 2013)
  - BoE: policy rate will not rise as long as unemployment rate $\geq 7\,\%$, $\mathbb{E}_t[\pi_{t+1.5}], \mathbb{E}_t[\pi_{t+2}] \leq 2.5\,\%$ and medium–term inflation is well anchored. (Aug. 2013)
  - ECB: policy rate remains at current or lower levels for an extended period of time conditional on the outlook for inflation and will be reviewed over time. (July 2013)

Theoretical Foundation

New-Keynesian model with Calvo Pricing (Eggertsson and Woodford, 2003; Eggertsson, 2011; Werning, 2012)

Related Literature

- Krugman(1998): Credible Commitment to being irresponsible (cash-in-advance model)
- Eggertsson/Woodford (2003): Commitment solution in a dynamic NK framework
- Werning (2012): Continuous-time version of NK model – *"simpler and more powerful"*

  Proposition 2: Sticky prices beneficial because they dampen deflation, this in turn mitigates the depression. In fact, the most favorable outcome is obtained when prices are completely rigid. At the other end of the spectrum, in the limit of perfectly flexible prices, the depression and deflation become unbounded.

  →Topsy Turvy world? Does any law of economics change sign at the zero bound?

Related Literature

- Cochrane (2013): Puzzling predictions as artifacts of equilibrium selection? Superior solutions with positive, but low long run steady rate of inflation; Problems due to reversal to price stability?

Challenges

- Barro/Gordon–type model: Discretion versus Commitment
- Commitment Solution: How to raise output above natural level?
  - inflation targeting emerges automatically under Calvo pricing
  - inflation targeting prone to dynamic inconsistency
  - price–level targeting suggested as remedy (Hatcher and Minford, 2014)
- Price–level targeting
  - optimal long–run target price level?
  - temporary or permanent deviation from price stability?
  - nominal indeterminacy under commitment?

What we do

Study optimal commitment in model where price–level targeting emerges endogenously through welfare function

Outline

A Simple Model

- only three periods: $t \in \{1, 2, 3\}$ with
- standard AD–curves

$$y_1 - y^\star = \mathbb{E}_1[y_2 - y^\star] - \sigma\big(i_1^S - [\mathbb{E}_1[p_2] - p_1] - \rho_1\big)$$
$$y_2 - y^\star = \mathbb{E}_2[y_3 - y^\star] - \sigma\big(i_2^S - [\mathbb{E}_2[p_3] - p_2] - \bar{\rho}\big)$$

A Simple Model

- monopolistic competitive firms
- deviation from Calvo assumption: ex–ante heterogeneity:
  - fraction α_1 having prices fixed at $p^\star$. Cannot adjust in $t = 1$ and $t = 2$ but can adjust at $t = 3$ with probability $1 - \lambda$ [long–run rigidity]
  - fraction α_2 fix their period 1 prices one period in advance [short–run rigidity]
  - remaining share of firms are free to adjust

$\Rightarrow$ AS–curves with nominal anchor $p^\star$

$$y_1 - y^\star = \frac{1}{\kappa_1}(p_1 - p^\star),\ \kappa_1 = \frac{1-\alpha_1-\alpha_2}{\alpha_1+\alpha_2}\left[\frac{1}{\sigma}+\varphi\right]$$

$$y_2 - y^\star = \frac{1}{\kappa_2}(p_2 - p^\star),\ \kappa_2 = \frac{1-\alpha_1}{\alpha_1}\left[\frac{1}{\sigma}+\varphi\right]$$

$$y_3 - y^\star = \frac{1}{\kappa_3}(p_3 - p^\star),\ \kappa_3 = \frac{1-\alpha_1\lambda}{\alpha_1\lambda}\left[\frac{1}{\sigma}+\varphi\right]$$

A Simple Model

- deviations from $p^\star$ induce welfare loss due to price dispersion
- however zero inflation not optimal in response to shocks due to firm heterogeneity

$\Rightarrow$ price–level targeting as optimal policy

$$\mathcal{L}_1 = \frac{1}{2} \times \mathbb{E}_1 \Bigg[(y_1 - y^\star)^2 + \frac{\theta}{\kappa_1}(p_1 - p^\star)^2 + \frac{1}{1+\rho_1}\Big\{ (y_2 - y^\star)^2 + \frac{\theta}{\kappa_2}(p_2 - p^\star)^2 \Big\} + \\ + \Big(\frac{1}{1+\rho_1}\Big)\Big(\frac{1}{1+\rho_2}\Big)\Big\{ (y_3 - y^\star)^2 + \frac{\theta}{\kappa_3}(p_3 - p^\star)^2 \Big\} \Bigg]$$

Policy Experiment

Discount–factor shock, $\rho_1 < 0$, shifts r_1^n below zero.
The shock has no persistence such that $\rho_2 = \rho_3 = \bar{\rho}$.

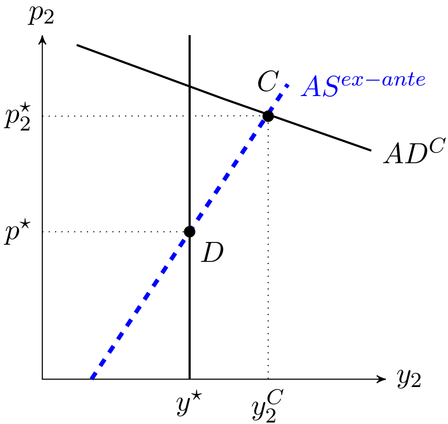
A Simple Model

- since i_1^S is constrained by the ZLB
- monetary authority conducts policy via announcement of price path $\{p_2, p_3\}$ to
- forward guide expectations and thus influence the real rate.

Dynamic Inconsistency

Assume $\mathbb{E}_2[p_3] = p^\star \Rightarrow \mathbb{E}_2[y_3 - y^\star] = 0$

$$\text{AD}: y_2 - y^\star = -\sigma(i_2^S - [\mathbb{E}_2[p_3] - p_2] - \bar{\rho}) = \sigma(\bar{\rho} + p^\star - p_2 - i_2^S)$$
$$\Rightarrow 0 \leq i_2^S \leq \bar{\rho} + p^\star - p_2$$
$$\text{AS}: y_2 - y^\star = \frac{1}{\kappa_2}(p_2 - \mathbb{E}_1[p_2]) \text{ (ex–ante)}$$

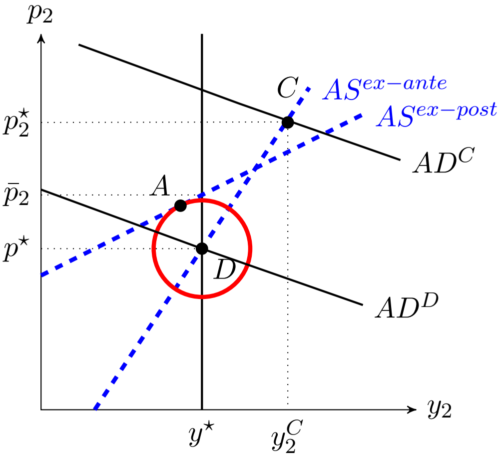


Dynamic Inconsistency

AD : $y_2 - y^\star = -\sigma(i_2^S - [\mathbb{E}_2[p_3] - p_2] - \bar{\rho}) = \sigma(\bar{\rho} + p^\star - p_2 - i_2^S)$

$\Rightarrow 0 \leq i_2^S \leq \bar{\rho} + p^\star - p_2$

AS : $y_2 - y^\star = \frac{1}{\kappa_1}\left(p_2 - \frac{1}{\alpha_1+\alpha_2}\left[\alpha_1 p^\star + \alpha_2 p_2^\star\right]\right)$ (ex–post)



Discretionary Solution

$$p_1^D - p^\star = \frac{\kappa_1 \sigma}{1+\kappa_1\sigma}\rho_1$$

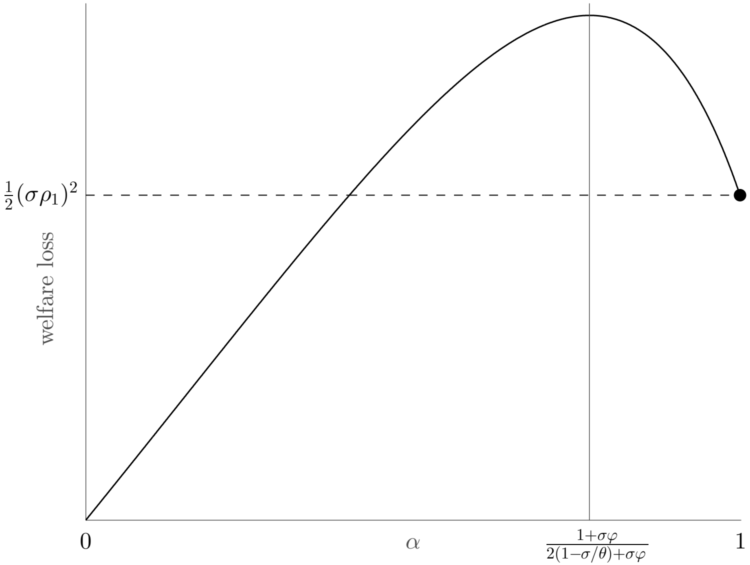
$$y_1^D - y^\star = \frac{\sigma}{1+\kappa_1\sigma}\rho_1$$

$$\Rightarrow \mathcal{L}^D = \frac{1+\theta\kappa_1}{(1+\sigma\kappa_1)^2}(\sigma\rho_1)^2$$

$$\frac{\partial \mathcal{L}^D}{\partial \alpha} > 0 \Leftrightarrow \alpha = \alpha_1 + \alpha_2 < \bar{\alpha} = \frac{1+\varphi\sigma}{2(1-\frac{\sigma}{\theta})+\varphi\sigma}$$

$$\Rightarrow \bar{\alpha} \geq 1 \Leftrightarrow \frac{\sigma}{\theta} \geq \frac{1}{2}$$

Discretionary Solution



A Simple Model

Assumption I

The central bank's announced price path $\{p_2, p_3\}$ is perfectly credible in the sense that

$$\mathbb{E}_t[p_{t+1}] = p_{t+1},\ t \in [1, 2]$$

Then

$$\begin{aligned} p_1 &= (\alpha_1 + \alpha_2)p^\star + (1 - \alpha_1 - \alpha_2)p_1^\star \\ p_2 &= (\alpha_1)p^\star + (1 - \alpha_1)p_2^\star \\ p_3 &= (\alpha_1\lambda)p^\star + (1 - \alpha_1\lambda)p_3^\star \end{aligned}$$

A Simple Model

to determine the optimal forward guidance path the monetary authority minimizes

$$\mathcal{L}_1 = \frac{1}{2} \times \mathbb{E}_1 \Bigg[(y_1 - y^\star)^2 + \frac{\theta}{\kappa_1}(p_1 - p^\star)^2 + \frac{1}{1+\rho_1}\Big\{(y_2 - y^\star)^2 + \frac{\theta}{\kappa_2}(p_2 - p^\star)^2\Big\} + \\ + \Big(\frac{1}{1+\rho_1}\Big)\Big(\frac{1}{1+\rho_2}\Big)\Big\{(y_3 - y^\star)^2 + \frac{\theta}{\kappa_3}(p_3 - p^\star)^2\Big\}\Bigg]$$

$$s.t.$$

$$y_1 - y^\star = [y_2 - y^\star] + \sigma[p_2 - p_1] + \sigma\rho_1$$

$$y_2 - y^\star = [y_3 - y^\star] - \sigma(i_2^S - [p_3 - p_2] - \bar{\rho})$$

$$y_1 - y^\star = \frac{1}{\kappa_1}(p_1 - p^\star)$$

$$y_2 - y^\star = \frac{1}{\kappa_2}(p_2 - p^\star)$$

$$y_3 - y^\star = \frac{1}{\kappa_3}(p_3 - p^\star)$$

Optimal Policy

Optimality Conditions

$$0 = \frac{1+\vartheta_1\kappa_1^2}{\kappa_1^2}(p_1 - p^\star) + \frac{1}{1+\rho_1}\frac{(1+\theta\kappa_2)\kappa_2\sigma(1+\kappa_1\sigma)}{\kappa_1\kappa_2^2\sigma(1+\kappa_2\sigma)}(p_2 - p^\star) + \dots$$

$$\dots + \frac{1}{1+\rho_1}\frac{1}{1+\bar{\rho}}\frac{(1+\theta\kappa_3)\kappa_3\sigma(1+\kappa_1\sigma)}{\kappa_1\kappa_3^2\sigma(1+\kappa_3\sigma)}(p_3 - p^\star), \quad (1)$$

$$p_1 - p^\star = \frac{\kappa_1\sigma(1+\kappa_2\sigma)}{\kappa_2\sigma(1+\kappa_1\sigma)}(p_2 - p^\star) + \frac{\kappa_1\sigma}{1+\kappa_1\sigma}\rho_1, \quad (2)$$

$$i_2^S = \bar{\rho} + \frac{1+\kappa_3\sigma}{\kappa_3\sigma}(p_3 - p^\star) - \frac{1+\kappa_2\sigma}{\kappa_2\sigma}(p_2 - p^\star) \quad (3)$$

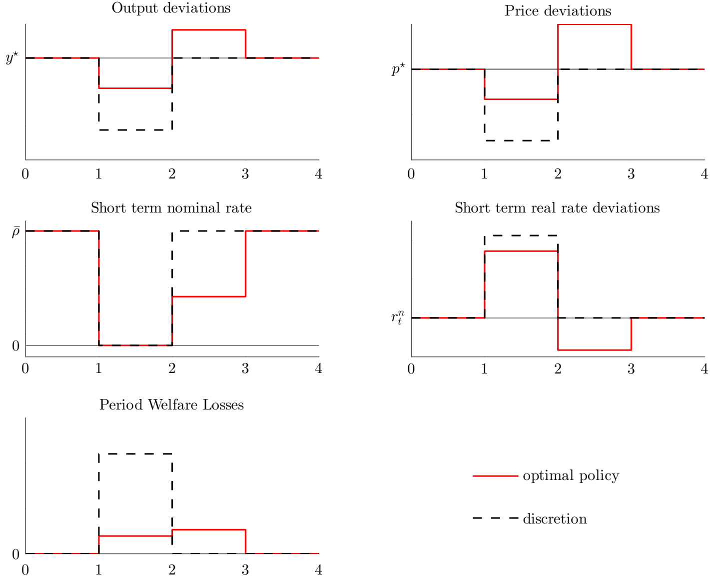
Optimal Policy

Assumption IIa: Unconstrained Forward Guidance

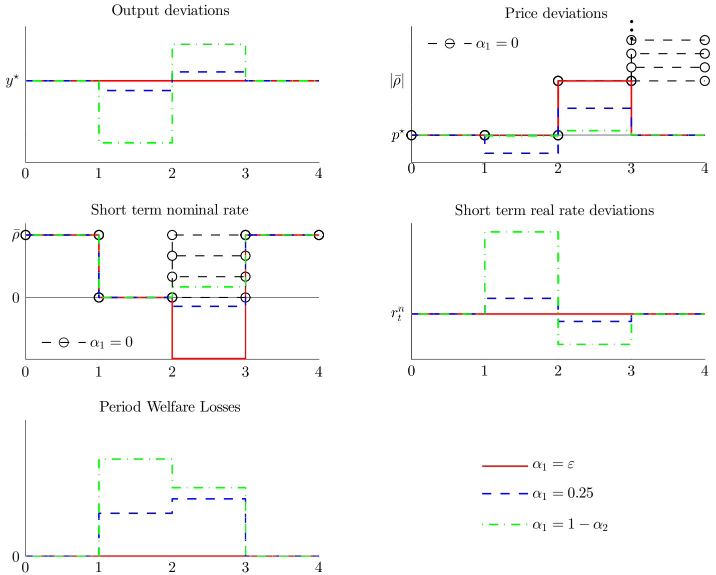
The discount factor shock ρ_1 is small enough such that under optimal policy the ZLB is not binding on $i_2^S \Leftrightarrow$

$$|\rho_1| \leq \left(1 + \frac{1}{1+\rho_1} \frac{1+\theta\kappa_2}{1+\theta\kappa_1} \left(\frac{1+\kappa_1\sigma}{1+\kappa_2\sigma}\right)^2\right) \bar{\rho}$$

Optimal Policy



Optimal Policy

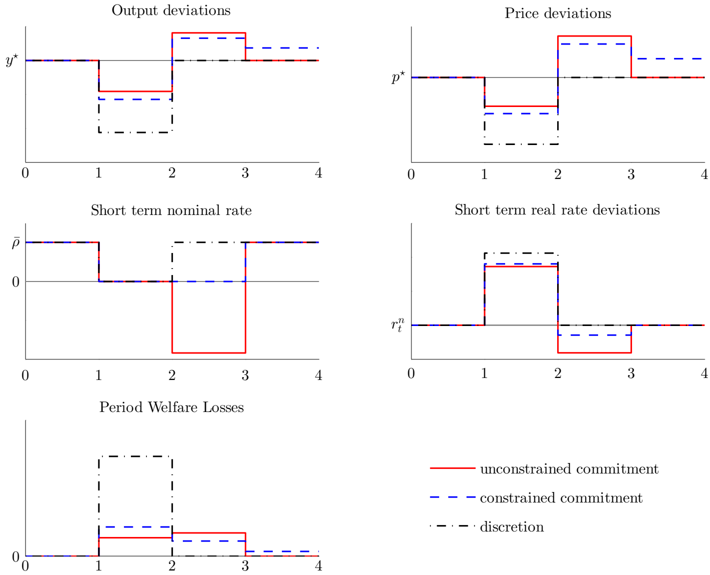


Optimal Policy

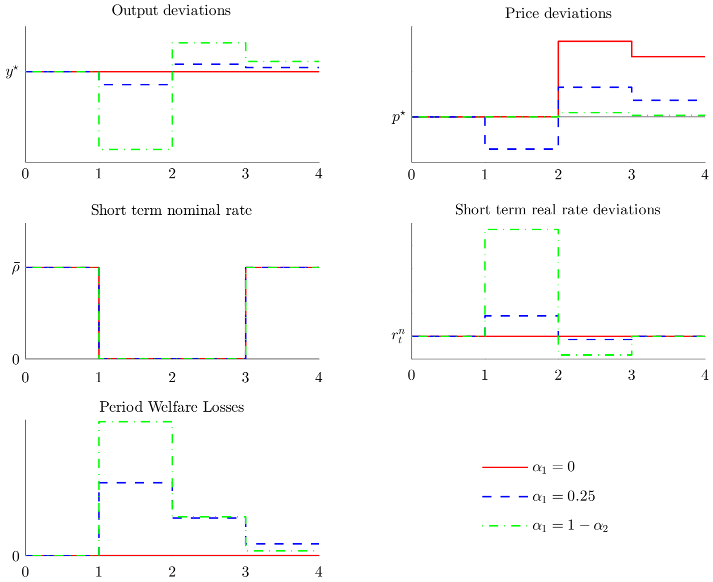
Assumption IIb: Constrained Forward Guidance

The discount factor shock ρ_1 is large enough and/or the degree of price stickiness is low such that under optimal policy the ZLB will be binding also in period 2, violating Assumption (IIa). In that case $i_2^S = 0$

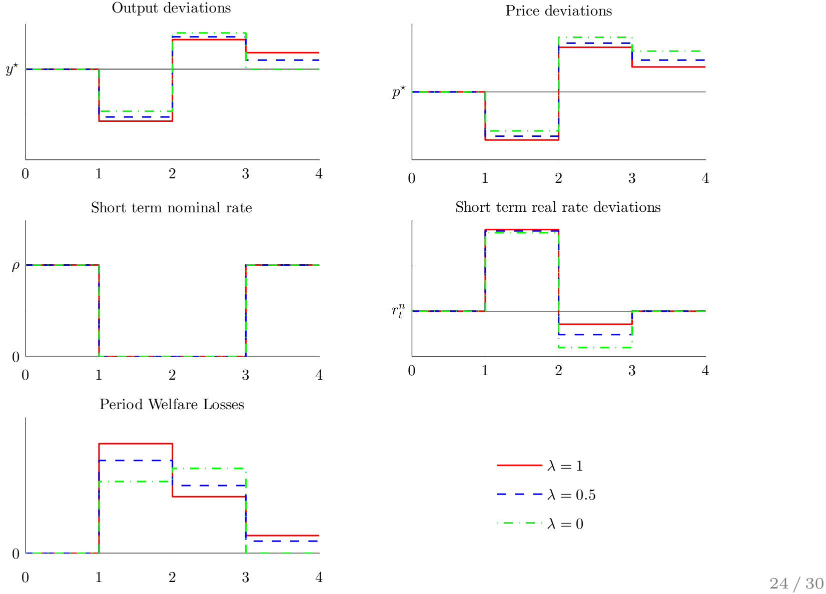
Optimal Policy



Optimal Policy



Optimal Policy



Extending the Model

- Add separable government spending on public good G to the utility function (Woodfoord, 2011)
- Effect of G in our model:

$$
\begin{aligned}
y_t - y^\star = {} & \mathbb{E}_1[y_{t+1} - y^\star] + \mathbb{E}_1[\hat{g}_t - \hat{g}_{t+1}] \\
& - \tilde{\sigma}\left[i_t^S - \rho_1 - \mathbb{E}_t[(p_{t+1} - p^\star) - (p_t - p^\star)]\right]
\end{aligned}
$$

with $\hat{g}_t \equiv \frac{G_t - G^\star}{Y^\star}$, $g^\star = \log(G^\star)$ and $\tilde{\sigma} \equiv \sigma(y^\star - g^\star)$.

1. direct demand effect: $\hat{g}_1 \uparrow$
2. marginal utility shifter: $\hat{g}_1 - \hat{g}_2 \uparrow$

Extending the Model

- Full commitment optimization problem:

$$\mathcal{L}_1^G = \frac{1}{2}\mathbb{E}_1\left[\sum_{t=1}^{3}\left(\prod_{j=1}^{t-1}\frac{1}{1+\rho_j}\right)\left\{\varphi(y_t-y^\star)^2+\eta_g g_t^2+\eta_u(y_t-y^\star-g_t)^2 + \frac{\theta(1+\varphi)}{\sigma\kappa_t}(p_t-p^\star)^2\right\}\right]$$

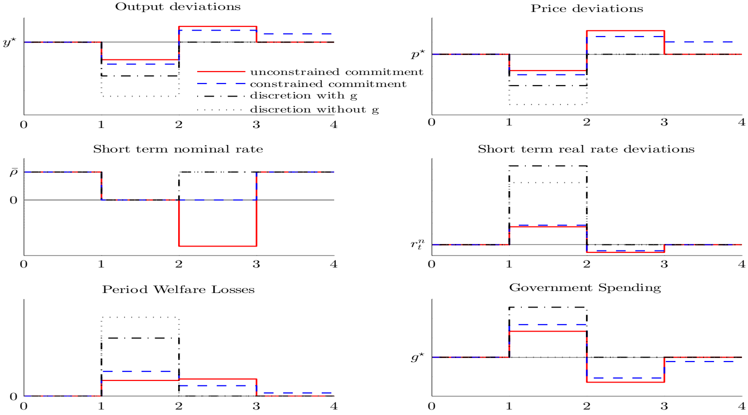
$$s.t.$$

$$p_1-p^\star = \frac{\kappa_1(\kappa_2+\tilde{\sigma})}{\kappa_2(\kappa_1+\tilde{\sigma})}\mathbb{E}_1[p_2-p^\star]+\frac{\kappa_1}{\kappa_1+\tilde{\sigma}}(g_1-\mathbb{E}_1[g_2])-\frac{i_1^S-\rho_1}{\kappa_1+\tilde{\sigma}}$$

$$p_2-p^\star = \frac{\kappa_2(\kappa_3+\tilde{\sigma})}{\kappa_3(\kappa_2+\tilde{\sigma})}\mathbb{E}_2[p_3-p^\star]+\frac{\kappa_2}{\kappa_2+\tilde{\sigma}}(g_2-\mathbb{E}_2[g_3])-\frac{\kappa_2\tilde{\sigma}}{\kappa_2+\tilde{\sigma}}[i_2^S-\bar{\rho}]$$

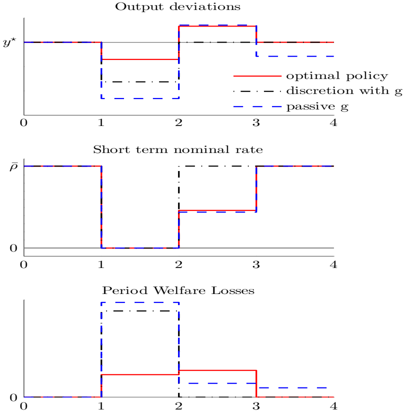
Government spending

Central bank keeps policy rate at the ZLB for longer ⇒ Optimal fiscal spending more front–loaded



Government spending

Welfare losses under procyclical austerity worse than under discretionary spending



Conclusion

- For discretionary case, potential non–linearity of price rigidity. But welfare loss is minimal under perfect price flexibility.
- Forward guidance can only steer expectations if it is credible.
- The announced price path must by accompanied by a consistently set nominal rate.
- Optimal policy eliminates price level indeterminacy except for the case $\alpha_1 = 0$.
- Unconstrained optimal forward guidance policy will bring the economy back to the initial equilibrium in period 3.

Conclusion

- Monetary policy should optimally be accompanied by countercyclical fiscal spending.
- Procyclical austerity induces higher welfare losses than discretionary policy with government spending.