Credit Ratings: Strategic Issuer Disclosure and Optimal Screening*

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Abstract

We study a model in which an issuer can manipulate information obtained by a credit rating agency (CRA) seeking to screen and rate its financial claim. Better CRA screening leads to a lower probability of obtaining a high rating but makes a high rating more valuable. Over an intermediate range of manipulation cost, improving screening quality can lead to more manipulation, dampening the CRA's incentive to screen. We further show that a CRA's own incentives to inflate ratings constrain its optimal screening intensity. Our model suggests that strategic disclosure by issuers may have played a role in recent ratings failures.

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1 Introduction

The failure of credit ratings to predict defaults of mortgage-backed securities in the lead-up to the financial crisis raises questions about the role of credit rating agencies (CRAs) in the economy. In principle, these "information intermediaries" create value by producing information in the form of ratings that allows investors to more accurately price assets. However, incentive problems may distort ratings and hence their usefulness. For example, there is evidence that CRAs inflated their ratings of mortgage-backed securities pre-crisis, allowing them to obtain more fees from issuers. An under-explored facet of the problem is that CRAs rely on issuers for much of the information on which they base their ratings, and issuers have incentives to distort this information in an effort to obtain more favorable ratings.

This paper explores how an issuer's incentives to manipulate the ratings process affect ratings accuracy and optimal CRA screening policies. We construct a model in which an informed CRA creates value by certifying assets of heterogeneous quality being sold by issuers to uninformed investors. Low-type issuers (those with a low-quality security to sell) can, at some cost, manipulate their information and try to mislead the CRA about their type. The CRA, in turn, wishes to screen out low-type issuers. The CRA chooses its screening intensity, and incurs a cost in doing so.

We show that improving the CRA's screening process has two opposing effects on the incentives of a low-type issuer to manipulate information. First, as expected, better screening implies a lower probability that a low-quality asset is rated as high, which discourages manipulation. This is the "reduced survival" effect. However, there is a second and countervailing effect—all else equal, better screening implies that the expected value and therefore price of a high-rated security is higher. This is the "price improvement" effect. When the screening intensity is low, the price improvement effect in fact dominates the reduced survival effect, so that an improvement in screening can lead to more (rather than less) manipulation by a low-type issuer. The converse happens when screening intensity is high. We further find that, surprisingly, over an intermediate range of issuer manipulation costs, the issuer's response causes the optimal level of screening by the CRA to increase with both average project quality and the manipulation cost itself.

In our base analysis, we assume that the CRA can commit to issuing ratings consistent with its information. We then consider a CRA that can strategically inflate ratings. The possibility of rating inflation creates a further friction—it can never be optimal for the CRA to engage in costly screening and then turn around and ignore its own information. Instead, the inability to commit to following its own signal leads the CRA to reduce its screening intensity ex ante to the point that it no longer gains from inflating ratings.

Many commentators have argued that CRAs were negligent, or possibly complicit with issuers, in the build-up to the financial crisis. Our model puts some focus back on the role of the issuer in misleading both CRAs and investors.[1] Our results suggest that in some situations, CRAs optimally limit screening to avoid exacerbating issuer misbehavior. Further, issuers' endogenous response to CRA screening makes it difficult to infer the degree of CRA diligence from observed rating accuracy. Finally, we show that penalizing issuers for distorting information can have the added benefit of also improving screening by the CRA.

In our model, an issuer has a project with either high or low value. The high (low) value project has a positive (negative) NPV. The issuer makes a report to a CRA about the value of its project. In equilibrium, an issuer with a high-value project truthfully reports a high value. The low-value issuer may misreport a high value in an attempt to manipulate the information of the CRA. The CRA observes a low signal if the low-type issuer reports truthfully, but may observe a high signal if the issuer manipulates. In the latter case, the probability that the CRA observes the high signal (i.e., that the issuer's manipulation is successful) decreases with the level of screening chosen by the CRA.[2] Having obtained its signal, the CRA assigns a high or low rating to the financial claim backed by the project.[3] We assume initially that the rating faithfully reflects the CRA's signal. The issuer then sells this claim, or asset, to investors operating in a perfectly competitive market. Finally, the asset's payoff is realized, and the game ends.

We assume that the CRA chooses its screening intensity before the issuer makes its report. The level of screening is a policy choice observable by issuers and investors alike.[4] The CRA's

[1]For example, in a written statement before the US SEC on November 21, 2002, Raymond W. McDaniel, President of Moody's Investors Service, states "Most issuers operate in good faith and provide reliable information to the securities markets, and to us. Yet there are instances where we may not believe that the numbers provided or the representations made by issuers provide a full and accurate story."

[2]We assume that the CRA's screening intensity is an ex ante observable policy choice. If screening intensity were unobservable, then issuers would conjecture the intensity level. Given a choice, the CRA would generally prefer that screening intensity be observable, as the CRA can then use its choice of screening intensity to influence the issuer's behavior.

[3]The discrete nature of the CRA's rating in our model is consistent with the actual ratings process. Goel and Thakor (2015) rationalize coarse (i.e., discrete) ratings. We assume that asset types and ratings are binary for simplicity. The general logic of the model would hold with more than two types and ratings, as long as issuers prefer higher ratings over lower ones.

[4]In practice, market participants gain knowledge of a CRA's screening policy through a variety of sources. As Begley (2015) points out, the CRAs publicly release guidelines on their rating methodologies. Participants can also learn from observing the rating outcomes for a variety of issuers.

payoff has three components. First, the CRA obtains a fixed proportion of the price of a high-rated asset; i.e., it obtains a share of the surplus in the transaction between the issuers and investors. Second, it incurs a direct technological cost based on the screening level it chooses. This cost is increasing and convex in screening intensity. Finally, the CRA is penalized for a rating error. At the end of the game, the true quality of an undertaken project is revealed. If the CRA assigned a high rating to a project that had a low value, it suffers an additional cost, reflecting expected regulatory sanctions, lawsuits, or reputation loss.

In our model, a low-rated asset is not sold, because the project is known to have negative NPV. A high-rated asset is sold at a price equal to the expected value of the project, given the equilibrium strategies of the CRA and the issuer. An increase in the CRA's screening intensity reduces the likelihood of surviving the screening process and successfully obtaining a high rating, weakening its incentives to manipulate. However, a high rating is more informative when investors know that the CRA is scrutinizing the asset more carefully. Thus, all else equal, an increase in screening intensity also has a price improvement effect: a high-rated asset has an increased price, strengthening incentives to manipulate.

When screening intensity is low, manipulation is likely to succeed, and therefore the price improvement effect dominates, so increased screening leads to more manipulation by the low-type issuer. When screening intensity is high, the price of a high-rated asset is high, and therefore the reduced survival effect dominates, so increased screening leads to less manipulation. The response of the low-type issuer to increased screening is therefore non-monotonic. This has important implications for the CRA; specifically, it pulls optimal screening intensity toward the extremes, either very high or very low.

The CRA's optimal choice of screening intensity varies with the underlying parameters of the model. We find three different types of equilibria. When the cost to the issuer of manipulating is high, the CRA screens with a high level of intensity, and the low-quality issuer never manipulates. When the cost to the issuer is moderate, the CRA screens with a relatively low intensity, and the low-quality issuer mixes between manipulating and not manipulating. When the cost to the issuer is low, the CRA screens with moderate intensity, and the low-quality issuer always manipulates. Thus the CRA's optimal screening intensity is non-monotonic in the cost of manipulation.

The comparative statics differ markedly across the three types of equilibria. For example, if the equilibrium features either zero or full manipulation by the low-quality issuer, optimal screening intensity decreases with the prior average project quality, and is invariant to the issuer's cost of manipulating. However, when the low-quality issuer mixes in equilibrium, optimal screening intensity actually increases with average project quality and the cost of manipulating. The divergence between these two cases is driven by the CRA taking into account the issuer's response to a small change in screening intensity, which is only relevant when the low-type issuer mixes.

In the last part of the paper, we allow the CRA to inflate its rating; that is, to issue a high rating even when it receives an unfavorable signal about the issuer's project. As before, a rating error is penalized at the end of the game. However, the CRA can benefit from inflating the rating because it only collects a fee if the issuer sells its security. This fee is proportional to the price of the high-rated security. Investors are fully rational, so the price depends on the equilibrium strategies of the CRA (including its rating assignment strategy) and issuer. A higher screening intensity results in a higher price conditional on a high rating, which strengthens the CRA's incentives to inflate the rating. However, this is counterproductive, as investors correctly account for the CRA's incentives, and discount the price accordingly, undoing the effects of more intense screening. As a result, the CRA never screens with sufficient intensity that it is tempted to inflate ratings ex post. In other words, the CRA's inability to commit to assigning ratings consistent with its information dampens the quality of screening ex ante.

To our knowledge, ours is the first paper to consider how the dependence of CRAs on issuers for much of the information on which they base ratings can impact ratings quality. Our analysis shows that issuers' incentives to distort such information can be especially difficult to counteract, as more diligent screening by CRAs in an effort to reduce the incidence of inflated ratings can actually strengthen these incentives.

Several factors have been shown to give rise to inflated credit ratings, including rating shopping by the issuer (Skreta and Veldkamp, 2009; Bolton, Freixas, and Shapiro, 2012; Sangiorgi and Spatt, 2015), a desire to attract more issuers and boost fee income (Frenkel, 2015; Bouvard and Levy, 2013), and a high income flow in a given period mitigating the incentive to maintain a reputation (Mathis, McAndrews, and Rochet, 2009; Bar-Isaac and Shapiro, 2013). Fulghieri, Strobl, and Xia (2014) show that the ability to issue unsolicited ratings may exacerbate the incentives to inflate ratings. We add to this literature by showing that CRAs may optimally limit the diligence of their screening in order to minimize their own future incentives to inflate ratings.

Our paper is also related to the literature on performance manipulation. Stein (1989), Goldman and Slezak (2006), and Crocker and Slemrod (2007), among others, show that concerns about short-term stock price performance can lead to "signal-jamming" equilibria in which managers take costly actions to manipulate their performance reports. Strobl (2013) studies a model in which the incentives of a manager worried about short-term stock price performance to manipulate earnings can increase with the stringency of disclosure requirements. The mechanism is the same as in our paper: More stringent disclosure requirements make earnings more credible, causing the stock price to respond more strongly to earnings, which can strengthen incentives to manipulate.[5] We extend this logic to the setting of a CRA scrutinizing the quality of an issuer's security, recognizing that similar incentives to manipulate information exist in this context. However, we go beyond the existing literature by (i) providing a fuller characterization of the conditions under which manipulation of the CRA's information increases or decreases with CRA scrutiny, (ii) characterizing the CRA's optimal screening policies, accounting for the response of the issuer's incentives, and (iii) considering how the CRA's own incentives to inflate ratings interact with the issuer's incentives to manipulate information in determining optimal screening intensity.

Our paper is also related to the economics of crime literature. For example, Tsebelis (1990) argues that an enforcement agency optimally responds to an increase in the penalty for crime by weakening enforcement, undoing the effect of the increased penalty.[6] While there is no specific penalty for being caught by the enforcing agent (i.e., the CRA) in our model, the cost of manipulation in our paper includes the expected penalty from being caught at a future date when investors find out the project has low quality. Further, the benefit of misreporting type in our model depends on the level of screening. As a result, in our model, when this cost goes up, the CRA may actually increase screening intensity, amplifying the effects of making crime more costly to commit.

Finally, our paper adds to the literature on the distorting effects of a principal's information about an agent on the agent's behavior. More information can dampen an agent's incentives to work hard in order to prove her worth (e.g., Holmström, 1999; Dewatripont, Jewitt, and Tirole, 1999). More transparency about an agent's actions can induce the agent to disregard her own information and to instead "herd" with more talented agents (Prendergast, 1993; Brandenburger and Polak, 1996; Prat, 2005). Song and Thakor (2006) show that a CEO

[5]A similar mechanism is at the heart of Cohn and Rajan's (2013) analysis of the impact of scrutiny by a board of directors of a manager in an activism contest, though the channel in their paper is posterior belief's about the manager's skill rather than a price.

[6]Cox (1994) shows that the Tsebelis result is not robust when agents have a heterogeneous benefit from committing a crime.

facing reputational concerns may limit the information she provides to a board of directors about project quality when less information induces the board to approve investment more often. Cohn and Rajan (2013) show that more information-gathering by the board about a CEO's skill exacerbates the agency problem caused by the CEO's reputational concerns. Our analysis complements these papers by identifying a new channel through which more information can have detrimental effects. In our model, a better screening technology influences the issuer's incentive to misbehave because of its effect on the value of a high rating.

2 Model

An issuer has a project that requires an upfront investment and generates future cash flows. The project has high quality with probability η and low quality with probability $1-\eta$. The high-quality project has an NPV $v_h > 0$ and the low-quality project has an NPV $v_\ell < 0$. The issuer privately observes the quality of its own project. If the issuer wishes to undertake the project, it issues a financial claim backed by the project. If it does not undertake the project, it obtains a payoff of zero. For convenience, we refer to an issuer with a high (low) quality project as a high (low) type issuer. We normalize v_h to 1 and v_ℓ to -1. The average NPV of the project is then $2\eta - 1$. We assume that $\eta \geq \frac{1}{2}$, so the average project has a positive NPV.

An issuer that wishes to sell a financial claim must obtain a rating from a credit rating agency (CRA). The CRA obtains a noisy signal about the type of the project and issues a rating. Investors observe the rating and decide whether they wish to buy the financial claim being issued, and if so at what price. Investors have no independent information about the type of the issuer's project. Based on the strategies of issuer and CRA, and on the observed rating, they update their beliefs over the type of the project. Investors are risk-neutral and perfectly competitive, so if they purchase the claim, they do so at the expected value of the asset given their posterior beliefs.

The game has two dates, 0 and 1. The sequence of events at date 0 is as follows:

- Stage 1. Nature chooses the type of the project, and the issuer privately observes this type.
- Stage 2. The credit rating agency (CRA) chooses a screening intensity α at a cost $c(\alpha)$, where c is strictly increasing and strictly convex. We assume that $c(0) =$

$c'(0) = 0$, so that both the total and marginal costs are zero if the CRA chooses to not screen at all.

Stage 3. The issuer reports his type (high or low) to the CRA. If the low type makes a high report, it incurs a cost $m > 0$. We refer to such misrepresentation of the issuer's type as "manipulation" and the associated cost as a "manipulation cost." This cost captures any upfront costs of distorting information as well as expected longer-run costs associated with sanctions for false disclosure and loss of reputation. No cost is incurred if the issuer reports its type truthfully, or if the high type makes a low report. Only the issuer knows whether it has incurred a cost at this stage.

Stage 4. If the issuer makes a high report, the CRA investigates the project to determine its actual quality. If project quality is high, the CRA obtains a high signal g_h in the investigation. If project quality is low, the CRA obtains a low signal g_ℓ with probability α, and a high signal g_h with probability $1 - \alpha$.

Stage 5. The CRA assigns a public rating, r_h or r_ℓ, to the financial claim backed by the project. Initially, we assume that the CRA commits to issue a rating that is faithful to its signal (i.e., the rating is r_h if its signal is g_h and r_ℓ if its signal is g_ℓ). In Section 4, we relax this assumption and allow the CRA to strategically inflate ratings ex post (i.e., issue rating r_h after observing signal g_ℓ).

Stage 6. Investors compute the expected NPV of the project, given the CRA's report (r_h or r_ℓ), its screening intensity α, and the exogenous parameters η, v_h, and v_ℓ. From an investor's perspective, the financial claim is an asset. If this NPV is positive, investors buy the asset at a price equal to the NPV. The CRA receives an exogenously-specified proportion ϕ of this price as a fee, and the issuer receives a proportion $1 - \phi$. If the expected NPV is negative, the security does not sell.

Stage 7. If the security is issued, the project is carried out.

At date 1, all parties find out the type of the project and earn their respective terminal payoffs. We normalize the discount rate between dates 0 and 1 to zero.

In addition, we assume that $m \leq 1 - \phi$. This provides an upper bound on the cost of manipulation for the low-type issuer. The maximal benefit from manipulation for the low-type issuer is $(1-\phi)v_h = 1-\phi$, recalling that $v_h = 1$. Therefore, if $m > 1-\phi$, the low-type issuer will never manipulate.

We consider perfect Bayesian equilibria of the game. The CRA only issues a low report if the issuer's project is indeed low quality. As a low-quality project has negative NPV, the issuer will not sell a security that has received a low rating, and all parties receive a payoff of zero. A high-rated security will have positive NPV in expectation (because $\eta > \frac{1}{2}$ by assumption). As the issuer values the project at zero unless financing is raised, it will issue a financial claim when the rating is high. Let p denote the price of a high-rated security. Investors form rational expectations and compete in a perfectly competitive market, so they earn zero profits in expectation. Therefore p is equal to the expected NPV of the project given investors' information. The high-type issuer obtains a payoff of $(1-\phi)p$, because it always receives a high rating. The low-type issuer can only receive a high rating if it manipulates. If it manipulates, the low-type issuer obtains a payoff of $(1-\phi)p - m$ if it receives a high rating and of $-m$ if it receives a low rating.

The payoff to the CRA consists of three components. It incurs a cost $c(\alpha)$ when it chooses a screening intensity α. It receives a payment ϕp from the issuer if it assigns a high rating and zero if it assigns a low rating. Finally, at date 1, if the CRA had assigned a high rating and the project is revealed to be of low quality, the CRA suffers a penalty λ. This penalty captures the notion that the CRA may suffer expected losses in the future if it assigns high ratings to poor-quality securities, possibly due to a loss of investor confidence and/or lawsuits from investors or the Justice Department.[7]

The high-type issuer reports high, because it has no benefit from reporting low. The low-type issuer, on the other hand, must decide whether to manipulate (report high) or not (report low). The potential benefit from manipulation is that if it receives a high rating, it can sell the asset at a positive price. The low-type issuer trades off this benefit against the cost of manipulating, m. We allow the low-type issuer to play a mixed strategy over its two possible reports, denoting by σ the probability that the low type manipulates (i.e., makes a high report to the CRA).

A perfect Bayesian equilibrium of this game is then determined by (α, σ). The price of the high-rated security, p, in turn depends on investors' observation of α and their beliefs about σ. In equilibrium, these beliefs must be correct. In addition, the CRA forms an expectation about σ before choosing α, and this expectation must be correct in equilibrium as well.

[7]For example, in 2015, the Justice Department reached a settlement with S&P over inflated ratings on mortgage-backed securities in the period 2004-07.

3 Equilibrium Strategies of Issuer and CRA

Observe that the screening level of the CRA, α, is chosen at Stage 2 of the game and σ at Stage 3. We first consider the continuation game starting at stage 3, at which point α is fixed, and then discuss the optimal choice of α.

In equilibrium, the price of a high-rated security is

$$p(\alpha, \sigma) = \frac{\eta v_h + (1-\eta)\sigma(1-\alpha)v_\ell}{\eta + (1-\eta)\sigma(1-\alpha)} = \frac{\eta - (1-\eta)\sigma(1-\alpha)}{\eta + (1-\eta)\sigma(1-\alpha)}. \tag{1}$$

Observe that p is decreasing in σ and increasing in α. A higher probability of manipulation by the low-type issuer leads to a lower price: conditional on a high rating, it is less likely that the project has high quality. Similarly, a higher screening intensity increases the likelihood that the project has high quality conditional on a high rating, leading to a higher price.

3.1 Low-type issuer's manipulation decision

If the issuer receives a low rating, it does not sell its asset, because the project is known to have a negative NPV. If it receives a high rating, it sells the asset at a price $p(\alpha, \sigma)$ and receives proceeds $(1-\phi)p(\alpha, \sigma)$. A low-type issuer that makes a high report (manipulates) receives a high rating with probability $1-\alpha$. Therefore, it strictly prefers to manipulate if $(1-\alpha)(1-\phi)p(\alpha, \sigma) > m$, or $\frac{m}{1-\phi} < (1-\alpha)p(\alpha, \sigma)$. Now, if $\frac{m}{1-\phi} \leq (1-\alpha)p(\alpha, 1)$, then $\frac{m}{1-\phi} < (1-\alpha)p(\alpha, \sigma)$ for all $\sigma < 1$, because p is strictly decreasing in σ. Therefore, if the manipulation cost is sufficiently low, the low-type issuer always prefers to make a high report, and $\sigma = 1$ must hold in equilibrium. Noting that $p(\alpha, 0) = 1$, a similar logic implies that if $\frac{m}{1-\phi} \geq (1-\alpha)p(\alpha, 0) = 1-\alpha$: the low-type issuer always prefers to make a truthful report, and $\sigma = 0$ in equilibrium.

If $(1-\alpha)p(\alpha, 1) < \frac{m}{1-\phi} < (1-\alpha)p(\alpha, 0)$, then a pure strategy equilibrium ($\sigma = 0$ or $\sigma = 1$) is not sustainable, and the low type mixes between reporting high and reporting low. In this case, the equilibrium value of $\sigma \in (0, 1)$ makes the low-type issuer indifferent between the two actions (i.e., $(1-\alpha)(1-\phi)p(\alpha, \sigma) = m$). Substituting the expression for p from (1) into this equality and solving yields the low-type issuer's equilibrium strategy σ when $\sigma \in (0, 1)$. Let $\sigma^*(\alpha)$ denote the equilibrium strategy. We can now fully characterize the low-type issuer's strategy in the continuation game at stage 3, taking α as given.

Lemma 1. *In any equilibrium of the continuation game at stage 3, the strategy of the low-type* *issuer is as follows:*

(i) If $\frac{m}{1-\phi} \leq (1-\alpha)p(\alpha,1)$*, then* $\sigma^*(\alpha) = 1$*, so that the low-type issuer makes a high report to the CRA.*

(ii) If $\frac{m}{1-\phi} \geq 1-\alpha$*, then* $\sigma^*(\alpha) = 0$*, so that the low-type issuer makes a low report to the CRA.*

(iii) If $\frac{m}{1-\phi} \in \big((1-\alpha)p(\alpha,1), 1-\alpha\big)$*, then the low-type issuer mixes between making a high report and a low report to the CRA, making a high report with probability*

$$\sigma^*(\alpha) = \frac{\eta}{1-\eta}\left(\frac{1-\alpha-\frac{m}{1-\phi}}{(1-\alpha)(1-\alpha+\frac{m}{1-\phi})}\right). \tag{2}$$

Observe that the expression in equation (2) decreases with $\frac{m}{1-\phi}$, which may be interpreted as the relative cost of manipulation. Naturally, the low-type issuer manipulates less often when this cost is high. Observe also that this expression equals one when $\frac{m}{1-\phi} = (1-\alpha)p(\alpha,1)$ and zero when $\frac{m}{1-\phi} = 1-\alpha$. Therefore, in equilibrium, the probability that the low-type makes a high report is continuous, equaling one when the manipulation cost is below a low threshold and zero when it is above a high threshold. Between the two thresholds, the probability decreases monotonically with the relative manipulation cost.

Many of the insights from our analysis come from exploring the case in which the low-type mixes between making a high report and a low report in equilibrium (i.e., $\sigma^* \in (0,1)$). We begin by showing in Proposition 1 that, in this case, an increase in the screening intensity α can either decrease or increase the manipulation probability by the low-type issuer. Proposition 2 then provides conditions under which there exist values of α such that $\sigma^* \in (0,1)$. Finally, in Proposition 4 in Section 3.2, we identify conditions under which the CRA will choose an α^* such that $\sigma^* \in (0,1)$.

To start, fix a value of α. Suppose that parameter values are such that, in equilibrium, the low-type issuer mixes between reporting high and low, that is, $\sigma^*(\alpha) \in (0,1)$. We first show that the low-type issuer's tendency to manipulate increases with α when the screening intensity is at a low level, and decreases in α when it is at a high level. In many ways, this result represents the main insight of the paper, and it plays a significant role in shaping the equilibrium of the overall game. Define $\bar{\alpha} = 1 - \frac{m}{1-\phi}(\sqrt{2}+1)$.

Proposition 1. *Suppose that* $\sigma^* \in (0,1)$*. Then,* $\frac{d\sigma^*}{d\alpha} > 0$ *if* $\alpha < \bar{\alpha}$ *and* $\frac{d\sigma^*}{d\alpha} < 0$ *if* $\alpha > \bar{\alpha}$*. That is, if the CRA's screening intensity is sufficiently low (high), the low-type issuer manipulates more often as the screening intensity increases (decreases).*

To see why manipulation can either decrease or increase with the CRA's screening intensity, observe that the expected benefit to the low-type issuer of successful manipulation is $(1-\alpha)(1-\phi)p(\alpha,\sigma^*)$, that is, the probability of surviving screening times the payoff obtained by a high-rated issuer. The partial derivative of this expected benefit with respect to screening intensity α (holding σ^* fixed) is

$$-(1-\phi)p + (1-\alpha)(1-\phi)\frac{\partial p}{\partial \alpha}. \tag{3}$$

The first term reflects the reduced survival effect—the reduced probability of successful manipulation as α increases—and is clearly negative. The second term reflects the price improvement effect—the impact of increased screening intensity on the price of a high-rated security. This term is positive, as investors' confidence that a high-rated security is indeed high quality increases with screening intensity. Thus, an increase in screening intensity has two countervailing effects on the low-type issuer's expected benefit from manipulation.

Which of these two effects dominates depends on the level of screening intensity. Suppose that screening is strong (α is high). In this case, the price of the high-rated security is large, so the reduced survival effect is large. Conversely, $1-\alpha$ is low, so the price improvement effect is small. Thus an increase in screening intensity is accompanied by a decrease in manipulation. The opposite is true when screening is weak (α small), as the reduced survival effect is small and the price improvement effect is large.

Overall, Proposition 1 suggests an inverted u-shaped relationship between the incidence of manipulation and the level of screening intensity. Of course, Proposition 1 only applies if there exist values of α for which $\sigma^* \in (0,1)$. In the next proposition, we identify parameter values under which this set of values of α is non-empty.

Proposition 2. *(i) There exists an* $\bar{\eta} > \frac{1}{2}$ *and manipulation cost thresholds* M_1 *and* M_2 *such that, for every* $\eta < \bar{\eta}$ *and* $\frac{m}{1-\phi} \in (M_1, M_2)$*, there is a range of screening intensity* $[0, \alpha_1)$ *with the following property: For all* $\alpha < \alpha_1$*,* $\sigma^* \in (0,1)$ *and* $\frac{d\sigma^*}{d\alpha} > 0$*. That is, the manipulation probability of the low-type issuer increases in the screening intensity.*

(ii) For all η and $\frac{m}{1-\phi} < 1$, there exists a screening intensity threshold $\alpha_2 < 1 - \frac{m}{1-\phi}$ such that, if $\alpha > \alpha_2$, then $\sigma^* \in (0,1)$. Here, $\frac{d\sigma^*}{d\alpha} < 0$. That is, the manipulation probability decreases in the screening intensity.

Proposition 1 shows that, when screening intensity is low, the probability of manipulation increases with an increase in screening intensity. However, extra conditions are required to ensure that $\sigma^* \in (0,1)$ for low values of α. First, if η is very large, the price conditional on a high rating (and hence the benefit of successful manipulation) is high even when the screening intensity α is low. Therefore, if $\frac{m}{1-\phi}$ is low, the low-type issuer manipulates with probability one. Alternatively, when $\frac{m}{1-\phi}$ is very high, σ^* decreases with α even when α is low. Thus, $\frac{m}{1-\phi}$ must be in an intermediate range to sustain an equilibrium with the desired features in part (i) of Proposition 2. Conversely, part (ii) holds for all values of η and $\frac{m}{1-\phi}$.

Under the conditions in Proposition 2 (i), manipulation increases with α for low screening levels and decreases with α for high screening levels (less than $1-\frac{m}{1-\phi}$). In this case, depending on parameter values, there may or may not exist an intermediate range of α for which the low-type manipulates with probability one.

How does the overall accuracy of ratings change as the screening intensity increases? When σ^* declines in α, it is immediate that the accuracy improves. In Lemma 2, we show that increasing the screening intensity results in more accurate ratings even when $\frac{d\sigma^*}{d\alpha} > 0$. That is, the increased manipulation by the low-type issuer does not fully unwind the effects of better screening. As a result, the price of a high-rated security strictly increases in the screening intensity α, even after taking into account the response of the low-type issuer.

Let $q(\alpha, \sigma^*) = (1-\eta)\sigma^*(1-\alpha)$ denote the probability that the CRA's rating is incorrect (i.e., that the rating is high while true asset quality is low).

Lemma 2. *Consider a screening intensity α at which $\sigma^*(\alpha) > 0$. Then, taking into account the issuer's optimal response:*

(i) The probability of a rating error decreases in α. That is, $\frac{dq}{d\alpha} < 0$.

(ii) The price of a high-rated security increases in α. That is, $\frac{dp}{d\alpha} > 0$.

This Lemma is clearly true when $\sigma^* = 1$, as the only effect of an increase in screening intensity in this case is a reduction in the likelihood that the CRA is fooled. Contrary to the statement of part (ii), suppose that we have $\sigma^* \in (0,1)$ and $\frac{dp}{d\alpha} \leq 0$. Then, an increase in α leads to both a decrease in the probability of successful manipulation and a weak decrease in the price conditional on a high rating. These effects pull in the same direction, dampening the low-type issuer's incentives to manipulate and resulting in a decrease in σ^*. However, if α increases and σ^* decreases, then q must decrease and p must increase, contradicting the supposition.

The rating process adds social value in our model to the extent that low-quality projects are weeded out and fail to get financing. All high-quality projects obtain a high rating and are financed. Investment efficiency, and therefore welfare, is thus inversely related to the proportion of low-quality projects that also receive a high rating, or in other words, to the rating error q. Lemma 2 shows that investment efficiency strictly increases in the screening intensity α, even after taking into account the endogenous response of the low-type issuer.

3.2 CRA's optimal screening level

We now turn to stage 2 to determine the optimal choice of the screening intensity, α, by the CRA. Recall that the CRA earns revenue ϕp each time it assigns a high rating, and suffers a penalty λ whenever a high-rated asset is subsequently revealed to have low quality. In addition, it incurs a direct cost of screening, $c(\alpha)$. Its payoff may therefore be written as follows:

$$\Psi(\alpha) = \eta\phi p(\alpha, \sigma^*) + (1-\eta)\sigma^*(1-\alpha)(\phi p(\alpha, \sigma^*) - \lambda) - c(\alpha). \tag{4}$$

The first term in equation (4) captures the revenue from high-quality assets (which always obtain a high rating). The second term captures the payoff from low-quality assets. These assets obtain a high rating with probability $\sigma^*(1-\alpha)$. Further, each time a low-quality asset is assigned a high rating, the issuer obtains a payoff ϕp at date 0, but suffers the penalty λ at date 1. Finally, the last term in equation (4) is the direct technological cost of screening. The CRA's objective is to choose α to maximize $\Psi(\alpha)$.

Substituting $p(\alpha, \sigma^*) = \frac{\eta-(1-\eta)\sigma^*(1-\alpha)}{\eta+(1-\eta)\sigma^*(1-\alpha)}$ into equation (4) and simplifying, we can write the CRA's payoff as

$$\Psi(\alpha) = \eta\phi - (1-\eta)\sigma^*(1-\alpha)(\phi + \lambda) - c(\alpha). \tag{5}$$

Examining this expression is illustrative. The CRA earns a fee equal to a fraction ϕ of the expected value of the securities being sold. If only high-quality assets earn high ratings, the price is equal to one. The CRA's fee is then ϕ, and its expected fee income is $\eta\phi$, the first term in equation (5). Assigning high ratings to low-quality assets reduces the total value of all assets being sold, as low-quality assets have a negative NPV. This dilutive effect hurts the CRA both because it decreases its expected fee income and because it results in less accurate ratings ex post, exposing it to higher penalties for inaccuracy. These two effects are reflected in the presence of the $(\phi + \lambda)$ term in equation (5).

From equation (5), it follows that $\Psi'(\alpha) = (1-\eta)(\phi+\lambda)\left[\sigma^* - (1-\alpha)\frac{d\sigma^*}{d\alpha}\right] - c'(\alpha)$. Therefore, the first-order condition for the CRA's problem is

$$c'(\alpha) = (1-\eta)(\phi+\lambda)\left[\sigma^* - (1-\alpha)\frac{d\sigma^*}{d\alpha}\right]. \tag{6}$$

The left-hand side of this equality is the marginal cost of increasing screening intensity, while the right-hand side is the marginal benefit of reduced ratings inaccuracy. It follows from Lemma 2 that the marginal benefit is always positive as long as $\sigma^* > 0$. If $\frac{d\sigma^*}{d\alpha} > 0$, then the low-type issuer's response to an increase in screening intensity partly undoes the effect of that increase on ratings accuracy. This blunts the incentives of the CRA to improve screening and leads to a lower level of screening than it would choose absent this feedback effect. Similarly, if $\frac{d\sigma^*}{d\alpha} < 0$, then the feedback effect boosts the benefit of an increase in screening intensity and causes the CRA to choose a higher screening intensity than it otherwise would.

Recall from Proposition 1 that, at low levels of screening intensity, the issuer manipulates more often as screening improves (i.e., $\frac{d\sigma^*}{d\alpha} > 0$), whereas at high levels of screening intensity the opposite occurs. This leads to the following observation:

Remark 1. *The issuer's response to screening tends to pull the CRA's choice of screening intensity towards the extremes.*

To simplify the remainder of the analysis, we make the following assumptions.

Assumption 1. *(a)* $\frac{\lambda}{\phi} \leq \frac{2\eta-1}{1-\eta}$*, and (b)* $\eta\phi < c(1)$*.*

Part (a) ensures that the CRA experiences a non-negative expected payoff when it chooses not to screen at all (i.e., sets α to zero). This represents a sufficient (though not necessary) condition for the CRA to earn a non-negative profit at its optimal choice of α, and therefore allows us to avoid concerns about the CRA's participation constraint being satisfied. Part (b) ensures that the CRA will never choose a perfect screening level, even when $m = 0$.

We first demonstrate that the CRA optimally chooses a strictly interior level of screening.

Lemma 3. *The optimal screening intensity α^* lies strictly between zero and one.*

As $c'(0) = 0$ and ratings accuracy increases with α (see Lemma 2 (i)), it follows that the optimal screening intensity will be strictly positive. From equation (5), it follows that $\Psi(1) = \eta\phi - c(1) < 0$ by Assumption 1 (b). Therefore, the optimal screening intensity remains strictly less than one.

We now characterize the CRA's optimal choice of screening intensity, α^*. There can be three kinds of equilibrium.

1. *Zero Manipulation Equilibrium*

   Here, the low-type issuer does not manipulate; i.e, $\sigma^*(\alpha^*) = 0$. It is immediate from Lemma 1 (ii) that this equilibrium is implemented by a screening intensity $\alpha^* = 1 - \frac{m}{1-\phi}$.

2. *Partial Manipulation Equilibrium*

   Here, the low-type issuer mixes between manipulating and not manipulating; i.e., $\sigma^*(\alpha^*) \in (0, 1)$. In this equilibrium, the optimal screening intensity is the solution to the CRA's first-order condition. Substituting σ^* from equation (2) and p from equation (1) into the first-order condition in equation (6) and simplifying, we obtain that the equilibrium screening intensity α^* satisfies the implicit equation

   $$c'(\alpha) = \frac{2\eta(\phi + \lambda)\,\frac{m}{1-\phi}}{\left(1 - \alpha + \frac{m}{1-\phi}\right)^2}. \tag{7}$$

3. *Full Manipulation Equilibrium*

   Here, the low-type issuer manipulates with probability one; i.e., $\sigma^*(\alpha^*) = 1$. A necessary condition for the existence of this equilibrium is that the relative cost of manipulation, $\frac{m}{1-\phi}$, is not too high. Note that $\sigma^* = 1$ implies that $\frac{\partial \sigma^*}{\partial \alpha} = 0$. Substituting these values into equation (6), in this equilibrium, the optimal screening intensity satisfies

   $$c'(\alpha^*) = (1 - \eta)(\phi + \lambda). \tag{8}$$

We show in Proposition 3 that full manipulation equilibria occur when the relative manipulation cost faced by the low-type issuer, $\frac{m}{1-\phi}$, is sufficiently low. Conversely, zero manipulation equilibria occur when this cost is sufficiently high. Of course, the thresholds exhibited in the proposition depend on the other parameters. For convenience, this dependence is suppressed in the notation.

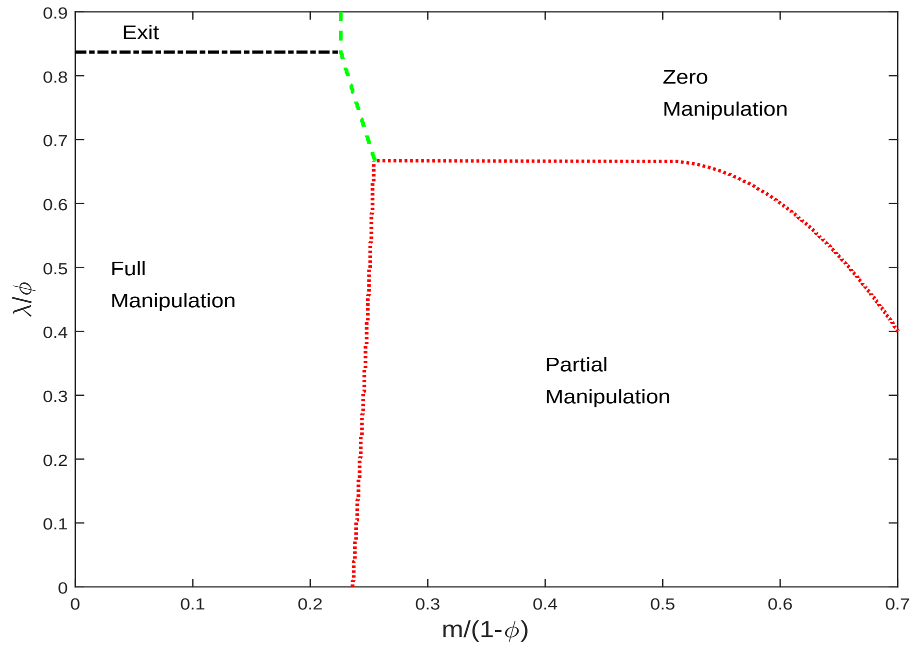
Proposition 3. *(i) There exists a threshold* $\underline{M} > 0$ *such that, if* $\frac{m}{1-\phi} < \underline{M}$*, the equilibrium features full manipulation; that is,* $\sigma^*(\alpha^*) = 1$*. The screening intensity chosen by the CRA is* $\alpha^* = (c')^{-1}((1-\eta)(\phi+\lambda))$*.*

(ii) There exists a threshold $\overline{M} < 1$ *such that, if* $\frac{m}{1-\phi} > \overline{M}$*, the equilibrium features zero manipulation; that is,* $\sigma^*(\alpha^*) = 0$*. The screening intensity chosen by the CRA is* $\alpha^* = 1 - \frac{m}{1-\phi}$*.*

Next, we consider partial manipulation equilibria. These equilibria are supported for intermediate values of the relative manipulation cost faced by the low-type issuer. Moreover, as we show in the proof of Proposition 4, a partial manipulation equilibrium can occur only if the term $\phi + \lambda$ is sufficiently low. As we illustrate in a numerical example in Figure 1 below, when $\phi + \lambda$ exceeds a particular threshold, partial manipulation equilibria cease to exist.

Proposition 4. *There exist thresholds* $L, M_3,$ *and* M_4*, where* $L > 0$ *and* $M_4 > M_3 > 0$ *such that, if* $\phi + \lambda < L$ *and* $\frac{m}{1-\phi} \in (M_3, M_4)$*, the equilibrium features partial manipulation; that is,* $\sigma^*(\alpha^*) \in (0,1)$*. Here, the screening intensity* α^* *satisfies the first-order condition* $c'(\alpha^*) = \frac{2\eta(\phi+\lambda)\frac{m}{1-\phi}}{\left(1-\alpha^*+\frac{m}{1-\phi}\right)^2}$*.*

To illustrate the results in Propositions 3 and 4, and to demonstrate that the parameter regions considered in those propositions are only a subset of the regions in which the respective equilibria obtain, we consider a numerical example. Set $\eta = 0.6$, $\phi = 0.1$, and $c(\alpha) = 0.1\alpha^2$. We determine the equilibrium for different values of $\frac{\lambda}{\phi}$ and $\frac{m}{1-\phi}$, and exhibit the results in Figure 1. In the example, $\frac{2\eta-1}{1-\eta} = 0.5$. Therefore, given Assumption 1 (a), our analytic results in this section hold if $\frac{\lambda}{\phi} \leq 0.5$. Nevertheless, as can be seen, all three kinds of equilibria (with zero, partial and full manipulation) continue to exist for many higher values of $\frac{\lambda}{\phi}$. As $\frac{\lambda}{\phi}$ becomes sufficiently high, partial manipulation equilibria do not exist for any value of $\frac{m}{1-\phi}$ (note that, keeping ϕ fixed, an increase in $\frac{\lambda}{\phi}$ represents an increase in $(\phi + \lambda)$).



This figure illustrates the equilibrium regions in the overall game based on a numerical example. We set $\eta = 0.6$, $\phi = 0.1$, and $c(\alpha) = 0.1\alpha^2$. In the 'Exit' region, there is no value of α at which the CRA earns a positive payoff.

Figure 1: Equilibria in the Whole Game: Numerical Example

The comparative statics in a partial manipulation equilibrium are of particular interest. One might imagine that an increase in either η, which captures the average quality of projects in the economy, or m, the issuer's manipulation cost, would reduce the need for screening and hence lead to a lower optimal screening intensity. However, the following proposition shows that the exact opposite happens in a partial manipulation equilibrium.

Proposition 5. *In a partial manipulation equilibrium, the CRA's optimal screening intensity increases in each of the following variables: η, the prior probability of a high-quality project, m, the manipulation cost to the low-type issuer, and ϕ, the CRA's share of the proceeds from the issuance of a high-rated asset. That is, in such an equilibrium, $\frac{\partial \alpha^*}{\partial \eta} > 0$, $\frac{\partial \alpha^*}{\partial m} > 0$, and $\frac{\partial \alpha^*}{\partial \phi} > 0$.*

As one would expect, the direct effect of a small increase in the quality of projects, η, is to shrink the set of issuers with bad projects, and hence the social importance of screening.

However, all else equal, it also increases the price of a high-rated security. It therefore provides the low-type issuer with a greater incentive to manipulate. In a partial manipulation equilibrium, $\sigma^* < 1$, so the low-type issuer responds by increasing the rate of manipulation. The endogenous response by the low-type issuer then leads the CRA to increase its screening intensity.

A small increase in m, the cost of manipulation, directly curtails manipulation and hence the need to screen. However, in a partial manipulation equilibrium, two effects offset this. First, the price at which a high-rated asset can be sold increases as the low-type issuer manipulates less often. This strengthens incentives to manipulate and hence partly (though not fully) offsets the direct effect of the increased manipulation cost. Second, the sensitivity of issuer manipulation to screening intensity decreases. This pushes up the optimal screening intensity, as an increase in α now has a greater effect on the final outcome. The latter two effects dominate the direct effect, and the CRA screens more intensely when the cost to the issuer of manipulation increases. The implication is that, in this parameter region, imposing extra costs of manipulation on the low-type issuer can lead to improved screening by the CRA.

A small increase in ϕ, the CRA's share of the issuance proceeds, has a similar effect to a small increase in m. In a partial manipulation equilibrium, a change in ϕ affects the optimal screening intensity through its effect on the low-type issuer. An increase in ϕ leads to an increase in the relative cost of manipulation, $\frac{m}{1-\phi}$, and therefore has all the same effects as mentioned in the previous paragraph.

These comparative statics are very different in the full manipulation and zero manipulation regions. In the full manipulation region, the optimal screening intensity satisfies $c'(\alpha) = (1-\eta)(\phi + \lambda)$. It is immediate that an increase in η leads to a lower screening intensity, an increase in m has no effect, and an increase in ϕ increases the screening intensity. In the zero manipulation region, we have $\alpha^* = 1 - \frac{m}{1-\phi}$. Now, an increase in η has no effect, whereas an increase in m reduces the optimal screening intensity. Similarly, an increase in ϕ now leads to a reduction in screening intensity because the issuer has less incentive to manipulate.[8]

[8] Note that the comparative statics outlined above hold for small changes in the parameters, specifically, changes small enough to avoid causing a change from one type of equilibrium (full, partial, or zero manipulation) to another.

4 Strategic Rating Inflation by the CRA

Thus far, we have assumed that the CRA commits to following its signal when it issues its rating. In this section, we relax this assumption and consider the possibility that the CRA may engage in conscious rating inflation. That is, the CRA may assign a high rating even though it received a low signal. The CRA clearly has no incentive to assign a low rating when it obtains a high signal—in equilibrium, a low-rated security is not issued, and the CRA receives zero revenue.

Rating inflation reduces the accuracy of ratings. At a fixed screening intensity α^*, this reduction in rating accuracy unambiguously harms the CRA. First, the CRA incurs a penalty for inaccurate ratings, and second, lower accuracy reduces the equilibrium price of a high-rated security, and therefore the CRA's fee from the issuance proceeds. Thus, if the CRA were able to commit to always follow its signal in assigning a rating, it would optimally choose to do so. However, if it cannot commit, the CRA may have an incentive to misreport, and issue a high rating even if it observes a low signal.

In principle, the CRA can consciously inflate its rating in two cases: when the issuer makes a low report, or when a low-type issuer makes a high report but the CRA's screening successfully identifies the issuer as a low type. We rule out the former case by assumption in order to avoid the perverse case in which the issuer honestly discloses that its project is of low quality, but the CRA effectively overturns the issuer and certifies the project as high quality.[9]

Let β denote the probability that the CRA assigns a low rating when it sees a low signal. Thus, the CRA inflates with probability $1-\beta$ when it has the option to do so. In the previous section, we assumed that $\beta = 1$.

Suppose the low-type issuer makes a high report. Then, with probability α, the CRA obtains a low signal. With further probability β, the CRA assigns a low rating. Therefore, conditional on making a high report, the probability that a low-type issuer receives a low rating is $\alpha\beta$, and the probability it receives a high rating is $1-\alpha\beta$. Therefore, in equilibrium, the price of a high-rated security is now

$$\hat{p}(\alpha, \sigma, \beta) = \frac{\eta v_h + (1-\eta)\sigma(1-\alpha\beta)v_\ell}{\eta + (1-\eta)\sigma(1-\alpha\beta)} = \frac{\eta - (1-\eta)\sigma(1-\alpha\beta)}{\eta + (1-\eta)\sigma(1-\alpha\beta)}. \tag{9}$$

[9]An issuer disclosing it has a low-quality project is effectively saying that it does not want to issue financing for its negative NPV project. Thus, it is odd to consider a situation in which the CRA nevertheless insists that financing be issued.

Observe that, given the expression for the price when $\beta = 1$ (as shown in equation (1)), it follows that $\hat{p}(\alpha, \sigma, \beta) = p(\alpha\beta, \sigma)$.

Now, suppose the issuer makes a high report and the CRA obtains a low signal. The CRA knows the issuer type is low. If it assigns a low rating, the security is not issued and the CRA receives zero. If it assigns a high rating, the CRA receives $\phi\hat{p}$, but then incurs the penalty λ when the market finds out the project has low cash flow. Therefore, the CRA optimally assigns a low rating if $\phi\hat{p} < \lambda$ and a high rating if $\phi\hat{p} > \lambda$. That is, the CRA's optimal strategy depends on the relative penalty for inflating ratings, $\frac{\lambda}{\phi}$.

We can now fully characterize the CRA's equilibrium strategy at stage 5, taking the screening intensity α and the low-type issuer's manipulation strategy σ as given. Let β^* denote the CRA's equilibrium strategy and, as in Section 3, let $p(\alpha, \sigma)$ denote the price when $\beta = 1$. Recall from equation (1) that $p(\alpha, \sigma) = \frac{\eta-(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)}$. Note that $p(0, \sigma) = \frac{\eta-(1-\eta)\sigma}{\eta+(1-\eta)\sigma}$ is strictly less than $p(\alpha, \sigma)$ whenever $\alpha > 0$ and $\sigma > 0$.

Lemma 4. *In any equilibrium of the continuation game at stage 5:*

(i) If it receives a high signal, the CRA assigns a high rating.

(ii) If it receives a low signal, the CRA always assigns a low rating (i.e., sets $\beta^* = 1$) if $\frac{\lambda}{\phi} \geq p(\alpha, \sigma)$ and a high rating (i.e., sets $\beta^* = 0$) if $\frac{\lambda}{\phi} \leq p(0, \sigma)$. If $p(0, \sigma) < \frac{\lambda}{\phi} < p(\alpha, \sigma)$, the CRA mixes between assigning a high and low rating, assigning a high rating with probability

$$\hat{\beta}(\alpha, \sigma) = \frac{1}{\alpha}\left(1 - \frac{\eta\left(1 - \frac{\lambda}{\phi}\right)}{(1-\eta)\left(1 + \frac{\lambda}{\phi}\right)\sigma}\right). \tag{10}$$

Observe that β^* as given by equation (10) increases with $\frac{\lambda}{\phi}$. Naturally, the CRA inflates its report less often when the relative penalty for ratings inaccuracy is higher. Note also that β^* is zero when $\frac{\lambda}{\phi} = p(0, \sigma)$, and increases continuously as α increases, reaching one when $\frac{\lambda}{\phi} = p(\alpha, \sigma)$. If either $\sigma = 0$ or $\alpha = 0$, the two thresholds collapse to the same point.[10]

Next consider the strategy of the low-type issuer when the CRA can inflate the rating. If the issuer makes a high report, it obtains a low rating with probability $\alpha\beta$ and a high rating

[10]If $\sigma = 0$ or $\alpha = 0$, a high report by the low-type issuer always results in a high signal for the CRA. As a result, the strategy of the CRA given a high report and a low signal is irrelevant, and any value of β can be an equilibrium.

with probability $1 - \alpha\beta$. Thus the low-type issuer's strategy is the same as shown in Lemma 1, except that α is replaced by $\alpha\beta$ in each of the expressions. For example, when the low-type issuer mixes between manipulating and truthfully revealing its type, we now have

$$\hat{\sigma}(\alpha, \beta) = \frac{\eta}{1-\eta} \left(\frac{(1-\alpha\beta) - \frac{m}{1-\phi}}{(1-\alpha\beta)\left[(1-\alpha\beta) + \frac{m}{1-\phi}\right]} \right). \tag{11}$$

Further, if the RHS of equation (11) is less than zero, then $\hat{\sigma} = 0$, and if the RHS exceeds one, then $\hat{\sigma} = 1$. Note also that when $\beta = 1$, the expression for $\hat{\sigma}$ in equation (11) reduces exactly to the expression for σ^* in equation (2).

Finally, we can write the payoff of the CRA as

$$\hat{\Psi}(\alpha) = \eta\phi - (1-\eta)(\phi + \lambda)\sigma^*(1 - \alpha\beta^*) - c(\alpha), \tag{12}$$

where both σ^* and β^* depend on α, and are determined in the equilibrium of the continuation game after α has been chosen. That is, σ^* and β^* jointly satisfy equations (10) and (11). If $\alpha = 0$, the value of β is irrelevant, and as in the previous section, the low-type issuer has a unique best response σ^*. When $\alpha > 0$, we establish in Lemma 5 in the appendix that the equilibrium in the continuation game is unique.

Observe that the price of the high-rated security in equation (9), the issuer's equilibrium strategy in equation (11), and the CRA's payoff in equation (12) all depend only on the overall probability that a low-type issuer obtains a high rating, $\alpha\beta$, and not separately on the screening intensity α or the rating strategy β. This insight yields the following result.

Proposition 6. *(i) In any equilibrium with positive screening, the CRA truthfully reports its signal. That is, if $\alpha^* > 0$, then $\beta^* = 1$.*

(ii) In any equilibrium, the low-type issuer manipulates with positive probability. That is, $\sigma^* > 0$.

That is, in equilibrium, the CRA will never choose a positive screening intensity and then engage in rating inflation. The CRA perfectly offsets the negative effect of slightly weaker screening on ratings accuracy by inflating less often. As a result, the issuer's strategy, the resulting price of the security, and the expected penalty from a rating error do not change. However, the CRA strictly benefits from the lower cost incurred in screening.

Part (ii) of the proposition implies that there is no zero manipulation equilibrium. Recall from Section 3 that zero manipulation is induced by increasing the screening intensity to the level $1 - \frac{m}{1-\phi}$. As the screening intensity increases toward that level, the price of the high-rated security increases toward $v_h = 1$. Therefore, the CRA has a strong incentive to inflate the rating when it obtains a low signal, and therefore to stop increasing the screening intensity.

We now have two new types of equilibria that emerge only because the CRA cannot commit to assigning a rating consistent with its signal. Continuing the numbering from the previous section, these are as follows.

4. *Commitment Failure Equilibrium*

   In a commitment failure equilibrium, the CRA does not inflate its ratings in equilibrium ($\beta^* = 1$). However, the optimal value of α^* is less than the value the CRA would choose if it could commit to honest ratings. If the CRA were to increase α beyond the chosen level, it would then engage in conscious rating inflation. As that would lead to the same rating quality, but at a higher cost, it optimally restrains its screening.

   There are two subtypes of such an equilibrium. In the first kind, $\sigma^* = 1$ in equilibrium, so there is full manipulation by the issuer. However, $\alpha^* < c'^{-1}((1-\eta)(\phi + \lambda))$, which is the optimal screening intensity in the full manipulation equilibrium in Section 3. In the second, there is partial manipulation by the issuer, with $\sigma^* \in (0, 1)$. However, α^* is again lower than in the partial manipulation equilibrium in Section 3.

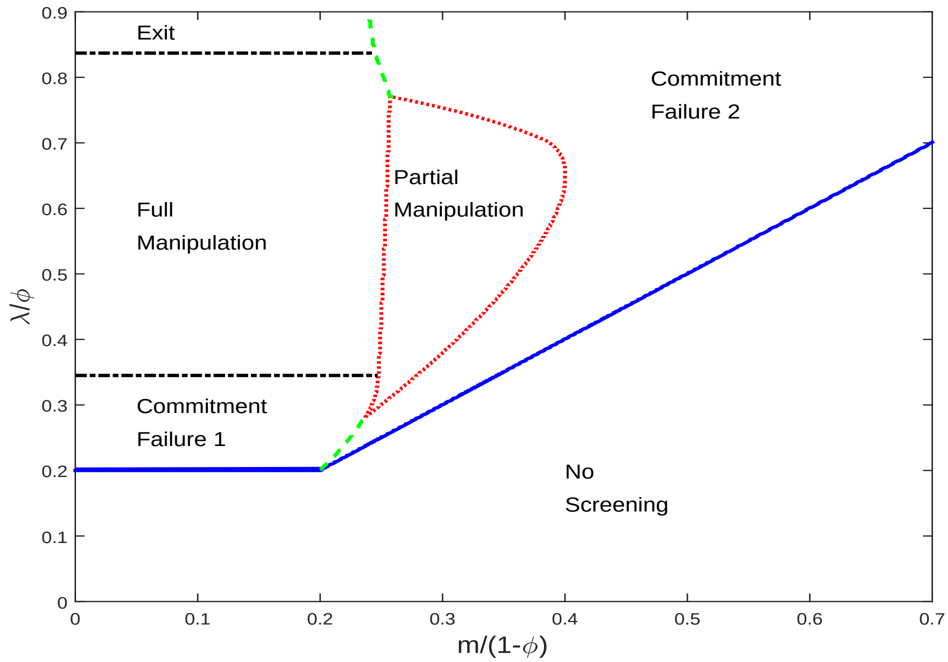
5. *No-Screening Equilibrium*

   As we show in Proposition 7 below, when the relative penalty for rating errors, $\frac{\lambda}{\phi}$, is sufficiently low, in equilibrium the CRA chooses to stop screening altogether, so $\alpha^* = 0$. In such an equilibrium, the price of the security exceeds $\frac{\lambda}{\phi}$, so at any positive value of α, the CRA would simply set $\beta^* = 0$. Note that although the CRA is not screening, for high values of $\frac{m}{1-\phi}$ the low-type issuer will set $\sigma^* < 1$. Therefore, the rating process can still be informative, with some low-type issuers choosing to voluntarily withdraw from the market rather than obtain a high rating.

A no-screening equilibrium exists for all values of $\frac{m}{1-\phi}$ when $\frac{\lambda}{\phi} \leq 2\eta - 1$. It also exists for values of $\frac{\lambda}{\phi}$ greater than $2\eta - 1$, as long as $\frac{\lambda}{\phi} \leq \frac{m}{1-\phi}$.

Proposition 7. *Suppose that* $\frac{\lambda}{\phi} \leq \max\left\{2\eta - 1, \frac{m}{1-\phi}\right\}$. *Then, in equilibrium, there is no screening; that is,* $\alpha^* = 0$.

We illustrate the equilibria in different regions of the parameter space in Figure 2. Set $\eta = 0.6, \phi = 0.1$, and $c(\alpha) = 0.1\alpha^2$. As can be seen from the figure, there is no region with zero manipulation in equilibrium. This is consistent with part (ii) of Proposition 6.



This figure illustrates the equilibrium regions in the overall game when the CRA can inflate ratings, based on a numerical example. We set $\eta = 0.6$, $\phi = 0.1$, and $c(\alpha) = 0.1\alpha^2$. In the 'Exit' region, there is no value of α at which the CRA earns a positive payoff. In the 'Commitment Failure 1' region, $\sigma^* = 1$, and in the 'Commitment Failure 2' region, $\sigma^* \in (0, 1)$.

Figure 2: Equilibrium Regions when CRA Chooses Optimal Rating Strategy

5 Implications

This section discusses some implications of our model. The first three implications we consider follow from Proposition 5, which provides comparative statics in a partial manipulation equilibrium. We expect many issuers to sometimes (but not always) manipulate their information. Further, the comparative statics in a partial manipulation equilibrium are counter-intuitive, and so are distinctive to our model, potentially allowing for sharp tests.

Proposition 5 relates changes in the screening intensity α to changes in the prior probability that an asset has high quality (η), the issuer's manipulation cost (m), and the CRA's share of proceeds from the sale of an asset (ϕ). Screening intensity can be measured in several different ways: by the number of analysts working for a CRA per security rated, the CRA's expenses per security rated, and the length and detail of reports accompanying ratings.[11] One might also imagine using proxies for λ, the cost to the CRA of inaccurate ratings, as indirect proxies for α, as α increases with λ in partial and full manipulation equilibria, but λ does not directly impact the issuer's strategy. Such proxies could include the severity of any sanctions for inaccurate ratings, the likelihood of investor lawsuits, or the importance of the CRA's reputation.

The first implication is that, in many situations, an increase in m, the cost of manipulation for the issuer, will be accompanied by an increase in the intensity of CRA screening. Conventional wisdom holds that when the cost of manipulation increases, the issuer is less inclined to manipulate and therefore the CRA does not need to screen as intensely. However, such an inference ignores the fact that the issuer's tendency to manipulate becomes less sensitive to screening intensity as the cost of manipulation rises.

One source of variation in manipulation cost is the novelty of the security. The cost of manipulation is likely to be lower for newer securities, as the CRA (and the rest of the market) will learn about the features of a security over time. Another source of variation is the complexity of a security. Manipulation is easier (and hence less costly) when a security is more complex. Expected manipulation costs should also increase with the ex post legal sanctions for committing fraud, the prosecutorial resources available to the regulator, and the willingness of courts to convict in fraud cases. For example, the 2002 Sarbanes-Oxley Act can be interpreted as a large positive shock to the cost of manipulating corporate debt ratings, as manipulation of accounting information is an important means of influencing CRA beliefs.[12]

Conversely, the 2010 Dodd-Frank Act can be interpreted to have reduced the cost of manipulation, in the sense of allowing issuers to hide more information from a CRA. Prior to the act, CRA's were exempt from Regulation Full Disclosure or Reg FD, which prohibited selective disclosure of information to some market participants. The Act extended Reg FD

[11] It is important to note that one cannot use the ex post accuracy of ratings directly as a measure of screening intensity—accuracy in the model is a function of both the CRA's screening intensity, α, and the low-type issuer's manipulation probability, σ.

[12] As Begley (2015) shows, some firms even incur real costs (such as reducing R&D expenditures) to reduce their debt-to-EBITDA ratio in the year before issuing a new security, in an attempt to obtain a more favorable credit rating.

to cover CRAs as well, so that any information disclosed by an issuer to a CRA must also be disclosed to investors at large.

A second implication is that, all else equal, an increase in η, the prior probability that an asset is high quality, should be accompanied by an increase in CRA screening intensity. This implication also runs counter to conventional wisdom. This parameter reflects the overall level of quality of assets for which ratings might be sought. This could be captured at a broad level through measures of the state of the economy such as GDP or the aggregate market-wide Tobin's Q.

The first two implications together suggest possible predictions about the variation in CRA screening intensity across different asset classes at different times. For example, one could reasonably argue that some of the mortgage-backed securities issued in the mid-2000s were novel, making information about their quality relatively easy to manipulate (i.e., low m), and that lax underwriting standards may have made the quality of these securities low (i.e., low η). In our model, these factors would lead to a low screening intensity on the part of the CRA.

The third implication relates to the impact of an increase in ϕ, the CRA's share of asset sale proceeds, on screening intensity. Here, the implications are more subtle. If the CRA lacks the wiggle room to consciously inflate ratings, then an increase in its share of proceeds results in greater screening intensity. However, if the CRA can inflate ratings, as in Section 4, then an increase in ϕ makes rating inflation more profitable, and the CRA responds by reducing its screening intensity instead. Thus examining the sensitivity of a CRA's screening intensity to its share of proceeds would be useful for distinguishing between these two possibilities, and hence would be informative about how much scope CRAs have to consciously inflate ratings.

One could measure a CRA's share of sales proceeds fairly directly by simply calculating the CRA's fees as a percentage of the value of the securities it rates. In addition, variation in competition among CRAs over time or across markets would also provide a source of variation in CRA bargaining power vis-à-vis issuers and hence their share of proceeds. Consistent with reduced competition leading to lower screening intensity, Becker and Milbourn (2011) find that ratings become more favorable and less accurate in response to an increase in competition due to the material entry of Fitch as a third major rating agency.

A fourth implication relates to the effect of screening intensity, α, on the extent of manipulation by the low-type issuer, σ, in a partial manipulation equilibrium. The model predicts (Proposition 1) that the low-type issuer manipulates more often when α increases, as long as the screening intensity is sufficiently low. The converse happens when screening intensity is high. That is, overall, there is an inverse-U relationship between screening intensity and manipulation frequency. By assumption in our model, manipulation is unobservable in real time. Ex post, however, prosecutions or regulatory actions for fraudulent reporting might be a useful indicator of the frequency with which such manipulation occurs. In the spirit of treating λ as a proxy for α, one might imagine examining how the incidence of manipulation changes after the passage of the Dodd-Frank Act, which made CRAs more accountable for their credit ratings (Dimitrov, Palia, and Tang, 2015).

Finally, we highlight one policy implication of the model. An important goal of regulatory policy is to minimize rating errors. Our model highlights that the quality of the rating process depends both on the behavior of the CRA and the behavior of the issuer. The model implies that policies which increase the cost of manipulation to the issuer (such as, e.g., stricter disclosure requirements) can both directly increase the accuracy of ratings and also have the side-effect of improving screening by the CRA. Conversely, if a security has a low manipulation cost and increasing this cost is not feasible, strengthening the penalty on the CRA for rating errors leads to a better rating process.

6 Conclusion

We argue that strategic disclosure by issuers is an important friction to consider in the ratings process. Our broad message is that the quality of credit ratings depends on both the quality of screening and the type and disclosure strategy of the issuer. In our model, information manipulation by a low-value issuer can to some extent undo the effect of better screening by a credit rating agency. The result is that better screening may simply lead to a greater cost for the CRA without a corresponding benefit in terms of more informative credit ratings.

When assessing periods during which the quality of ratings has been perceived to be low, it is important to remember that issuers are likely to know more about their own asset qualities than a CRA. Any policy design intended to improve the overall ratings process must include providing incentives to issuers to truthfully report the quality of their assets as an important component.

Appendix: Proofs

Proof of Lemma 1

Recall from equation (1) that the price of the high-rated security is $p(\alpha,\sigma) = \frac{\eta-(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)}$.
(i) Now, suppose the issuer knows that its project has low cash flow. Its payoff from making a high report is

$$\pi_\ell(g_h) = (1-\alpha)(1-\phi)p(\alpha,\sigma) - m. \tag{13}$$

Its payoff from making a low report is zero. Therefore, it is a strict best response to make a high report if $\pi_\ell(g_h) > 0$. Setting $\sigma = 1$ in the expression for the price, the condition $\pi_\ell(g_h) > 0$ reduces to $\frac{m}{1-\phi} < (1-\alpha)p(\alpha,1)$. Noting that $p(\alpha,1) = \frac{\eta-(1-\eta)(1-\alpha)}{\eta+(1-\eta)(1-\alpha)}$, this last inequality may be written as $\frac{m}{1-\phi} < (1-\alpha)\frac{\eta-(1-\eta)(1-\alpha)}{\eta+(1-\eta)(1-\alpha)} = (1-\alpha)p(\alpha,1)$. Finally, observe that when $\frac{m}{1-\phi} = (1-\alpha)p(\alpha,1)$, it remains a best response to make a high report.

Therefore, when $\frac{m}{1-\phi} \leq (1-\alpha)p(\alpha,1)$ and $\sigma^* = 1$, we have $(1-\alpha)(1-\phi)p(\alpha,\sigma^*) \geq m$, so the low-type issuer plays a best response by setting $\sigma^*(\alpha) = 1$.

(ii) Similarly, it is a best response to make a low report if $\pi_\ell(g_h) \leq 0$. Setting $\sigma = 0$ in the expression for the price, we obtain $p = 1$. Therefore, the condition $\pi_\ell(g_h) \leq 0$ reduces to $\frac{m}{1-\phi} \geq 1-\alpha$. That is, when $\frac{m}{1-\phi} \geq 1-\alpha$ and $\sigma^* = 0$, we have $(1-\alpha)(1-\phi)p(\alpha,\sigma^*) \leq m$, so that the low-type issuer plays a best response by setting $\sigma^*(\alpha) = 0$.

(iii) Finally, suppose that $\frac{m}{1-\phi} \in ((1-\alpha)p(\alpha,1), 1-\alpha)$. The low-type issuer is indifferent between reporting high and reporting low if $\frac{m}{1-\phi} = p(\alpha,\sigma)$. Observe that $p(\alpha,\sigma)$ is strictly decreasing in σ. Further, by assumption, $\frac{m}{1-\phi} > (1-\alpha)p(\alpha,1)$ and $\frac{m}{1-\phi} < p(\alpha,0) = 1-\alpha$. Therefore, there exists some $\hat{\sigma} \in (0,1)$ such that $\frac{m}{1-\phi} = p(\alpha,\hat{\sigma})$, so that in equilibrium the low-type issuer reports high with probability $\hat{\sigma}$ and low with probability $1-\hat{\sigma}$.

Set $p(\alpha,\hat{\sigma}) = \frac{m}{1-\phi}$ and solve for $\hat{\sigma}$. We obtain the expression for σ^* shown in equation (2). That is, here $\sigma^*(\alpha) = \hat{\sigma} = \frac{\eta}{1-\eta}\left(\frac{(1-\alpha)-\frac{m}{1-\phi}}{(1-\alpha)[(1-\alpha)+\frac{m}{1-\phi}]}\right)$. Finally, note that substituting in $\frac{m}{1-\phi} = 1-\alpha$ into this expression yields $\sigma^* = 0$, and substituting in $\frac{m}{1-\phi} = (1-\alpha)p(\alpha,1)(1-\alpha)\left(\frac{\eta-(1-\eta)(1-\alpha)}{\eta+(1-\eta)(1-\alpha)}\right)$ yields $\sigma^* = 1$. ■

Proof of Proposition 1

When $\sigma^* \in (0,1)$, from equation (2) in the statement of Lemma 1, it follows that

$$(1-\eta)\sigma^*(1-\alpha) = \eta \left(\frac{1-\alpha-\frac{m}{1-\phi}}{1-\alpha+\frac{m}{1-\phi}} \right). \tag{14}$$

Denote $y = \frac{1-\alpha-\frac{m}{1-\phi}}{1-\alpha+\frac{m}{1-\phi}}$. Then, we can write $p = \frac{\eta(1-y)}{\eta(1+y)} = \frac{1-y}{1+y}$.

Further, from the expression for p, it follows that $\frac{\partial p}{\partial \alpha} = \frac{2\eta(1-\eta)\sigma^*}{[\eta+(1-\eta)\sigma^*(1-\alpha)]^2} = \frac{1}{1-\alpha}\frac{2\eta^2 y}{\eta^2(1+y)^2} = \frac{1}{1-\alpha}\frac{2y}{(1+y)^2}$. Therefore, the inequality $p < (1-\alpha)\frac{\partial p}{\partial \alpha}$ reduces to

$$\frac{1-y}{1+y} < \frac{2y}{(1+y)^2}, \quad \text{or} \quad y^2 + 2y - 1 > 0.$$

Now, the equation $y^2+2y-1=0$ has two roots, $y = -1 \pm \sqrt{2}$. It follows that $y^2+2y-1>0$ if $y < -1-\sqrt{2}$ or $y > \sqrt{2}-1$. As $y \geq 0$ by definition, only the positive range is relevant for us. Now, using $y = \frac{1-\alpha-\frac{m}{1-\phi}}{1-\alpha+\frac{m}{1-\phi}}$, some straightforward algebra shows that $y > \sqrt{2}-1$ if and only if $\frac{m}{1-\phi} < (1-\alpha)(\sqrt{2}-1)$, or $\alpha < 1 - \frac{m}{1-\phi}(\sqrt{2}+1) = \bar{\alpha}\left(\frac{m}{1-\phi}\right)$.

That is, if $\alpha < \bar{\alpha}\left(\frac{m}{1-\phi}\right)$, we have $p < (1-\alpha)\frac{\partial p}{\partial \alpha}$, so that $\frac{d\sigma^*}{d\alpha} > 0$. Conversely, if $\alpha > \bar{\alpha}\left(\frac{m}{1-\phi}\right)$, then $p > (1-\alpha)\frac{\partial p}{\partial \alpha}$, so that $\frac{d\sigma^*}{d\alpha} < 0$. ∎

Proof of Proposition 2

(i) Observe that $(1-\alpha)p(\alpha,1) = (1-\alpha)\frac{\eta-(1-\eta)(1-\alpha)}{\eta+(1-\eta)(1-\alpha)}$. Define M_1 to be the value of this expression when $\alpha = 0$; that is, $M_1 = 2\eta - 1$, and define $M_2 = \sqrt{2}-1 = \frac{1}{\sqrt{2}+1}$. Observe that when $\eta < \frac{1}{\sqrt{2}}$, we have $M_1 < M_2$. Define $\bar{\eta} = \frac{1}{\sqrt{2}}$.

Suppose that $\eta \in \left(\frac{1}{2}, \bar{\eta}\right)$, and $\frac{m}{1-\phi} \in (M_1, M_2)$. At $\alpha = 0$, from equation (2) we have $\sigma^* = \frac{\eta}{1-\eta}\left(\frac{1-m/(1-\phi)}{1+m/(1-\phi)}\right)$. Clearly, $\sigma^* > 0$. Further, as $\frac{m}{1-\phi} > M_1$, it follows that $\sigma^* < 1$. Finally, observe that the condition $\frac{m}{1-\phi} < M_2$ implies that $\bar{\alpha}\left(\frac{m}{1-\phi}\right) = 1 - (\sqrt{2}+1)\frac{m}{1-\phi} > 0$. Therefore, when $\alpha = 0$, by Proposition 1, we have $\frac{d\sigma^*}{d\alpha} > 0$.

Now, the thresholds $(1-\alpha)p(\alpha,1)$ and $1-\alpha$ identified in Lemma 1 are continuous in α. Therefore, σ^* is continuous in α. It therefore follows that there exists an $\alpha_1 > 0$ such that, for all $\alpha \in [0, \alpha_1)$, we have both $\sigma^* \in (0,1)$ and $\frac{d\sigma^*}{d\alpha} > 0$.

(ii) Proposition 1 (ii) shows that when $\alpha > \bar{\alpha} = 1 - (\sqrt{2}+1)\frac{m}{1-\phi}$, we have $\frac{d\sigma^*}{d\alpha} < 0$. Further, recall that Lemma 1 (ii) implies that when $\alpha = 1 - \frac{m}{1-\phi}$, we have $\sigma^* = 0$. Therefore, again invoking continuity of σ^* in α, there must exist an $\alpha_2 < 1 - \frac{m}{1-\phi}$ such that, for all $\alpha \in \left(\alpha_2, 1 - \frac{m}{1-\phi}\right)$, we have $\sigma^* \in (0,1)$ and $\frac{d\sigma^*}{d\alpha} < 0$. The interval $\left(1 - (\sqrt{2}+1)\frac{m}{1-\phi}, 1 - \frac{m}{1-\phi}\right)$

is non-empty for all values of $\frac{m}{1-\phi} < 1$ and all values of η. Therefore, a screening intensity threshold α_2 with the desired properties exists for all values of η and $\frac{m}{1-\phi} < 1$. ■

Proof of Lemma 2

First, suppose that $\sigma^*(\alpha) = 1$. Then, $p = \frac{\eta-(1-\eta)(1-\alpha)}{\eta+(1-\eta)(1-\alpha)}$, and $q = (1-\eta)(1-\alpha)$. It is immediate that q is decreasing in α and that p is decreasing in q, so that p increases in α.

Next, suppose that $\sigma^* \in (0,1)$. We show in equation (14) in the proof of Proposition 1 that in this case

$$(1-\eta)\sigma^*(1-\alpha) = \eta\left(\frac{1-\alpha-\frac{m}{1-\phi}}{1-\alpha+\frac{m}{1-\phi}}\right). \tag{15}$$

The left-hand side has been defined as q, the probability of a ratings error. A direct derivation shows that $\frac{dq}{d\alpha} = \frac{-2\eta m}{\left(1-\alpha+\frac{m}{1-\phi}\right)^2} < 0$.

Now, $p = \frac{\eta-(1-\eta)\sigma^*(1-\alpha)}{\eta+(1-\eta)\sigma^*(1-\alpha)} = \frac{\eta-q}{\eta+q}$. We have $\frac{dp}{d\alpha} = \frac{dp}{dq} \times \frac{dq}{d\alpha}$. Here, $\frac{dp}{dq} = -\frac{2\eta}{(\eta+q)^2} < 0$. Further, as shown above, $\frac{dq}{d\alpha} < 0$. Therefore, $\frac{dp}{d\alpha} > 0$. ■

Proof of Lemma 3

Recall that $\Psi(\alpha) = \eta\phi-(1-\eta)(\phi+\lambda)\sigma(1-\alpha)-c(\alpha)$. At any α such that Ψ is differentiable, we have

$$\Psi'(\alpha) = (1-\eta)(\phi+\lambda)\left(\sigma-(1-\alpha)\frac{d\sigma}{d\alpha}\right) - c'(\alpha), \tag{16}$$

$$\Psi''(\alpha) = (1-\eta)(\phi+\lambda)(1+\alpha)\left((1+\alpha)\frac{d\sigma}{d\alpha} - (1-\alpha)\frac{d^2\sigma}{d\alpha^2}\right) - c''(\alpha). \tag{17}$$

Denote $y = \frac{m}{1-\phi}$. At $\alpha = 0$, we have $\sigma^*(0) = \min\left\{\frac{\eta}{1-\eta}\frac{1-y}{1+y}, 1\right\}$. Further, when $\sigma^* \in (0,1)$, we have $\frac{\partial\sigma^*}{\partial\alpha}\mid_{\alpha=0} = h'(0) = \frac{\eta}{1-\eta}\frac{1-y^2-2y}{(1+y)^2}$.

Now, suppose $\sigma^* \in (0,1)$. Also, suppose that $\alpha = 0$. As $c'(0) = 0$, the payoff Ψ is increasing at $\alpha = 0$ if $\sigma^* > (1-\alpha)\frac{\partial\sigma^*}{\partial\alpha}$. This last condition is equivalent to

$$\frac{\eta}{1-\eta}\frac{1-y}{1+y} > \frac{\eta}{1-\eta}\frac{1-y^2-2y}{(1+y)^2}, \text{ or,} \tag{18}$$

$$1-y^2 > 1-y^2-2y, \tag{19}$$

which always holds when $y > 0$. Therefore, at $\alpha = 0$, if $\sigma^* \in (0,1)$, we have $\sigma^* > (1-\alpha)\frac{\partial\sigma^*}{\partial\alpha}$, which implies that $\Psi'(0) > 0$. Therefore, it cannot be optimal for the CRA to set $\alpha = 0$.

Now, suppose that at $\alpha = 0$, we have $\sigma^* = 1$. There are two possibilities:
(a) a small increase in α has no effect on σ^*. Then, $\frac{\partial \sigma^*}{\partial \alpha} = 0$, so again we have $\Psi'(0) > 0$.
(b) any small increase in α leads to a reduction in σ^* below 1. Then, for all α strictly positive but sufficiently small, the arguments outlined above when $\sigma^* \in (0,1)$ go through, and $\Psi'(\alpha) > 0$. Taking the right derivative as $\alpha \to 0_+$, we have that the right derivative of Ψ at $\alpha = 0$ is strictly positive.

In both cases, $\Psi(\alpha)$ is strictly increasing at $\alpha = 0$. Now, observe that $\Psi(0) = \eta\phi - (1-\eta)(\phi+\lambda)\sigma^*(0) \geq \eta\phi - (1-\eta)(\phi+\lambda)$, because $\sigma^*(0) \leq 1$. Therefore, the condition $\frac{\lambda}{\phi} < \frac{2\eta-1}{1-\eta}$ directly implies that $\Psi(0) \geq 0$. As Ψ is strictly increasing at $\alpha = 0$, there must exist an $\alpha > 0$ at which the CRA earns a strictly positive profit.

Finally, to complete the proof, observe that substituting $\alpha = 1$ into equation (5) yields $\Psi(1) = \eta\phi - c(1)$. Hence, under Assumption 1 part (b), it follows that the CRA makes a loss at $\alpha = 1$. Therefore, the optimal α must be strictly between zero and one. ■

Proof of Proposition 3

(i) First, suppose that $\sigma^* = 1$. In this case, as shown in the text in equation (8), the CRA's optimal screening intensity satisfies the first-order condition $c'(\alpha) = (1-\eta)(\phi+\lambda)$. Convexity of $c(\cdot)$ ensures that the second-order condition is satisfied. Denote α_f as the value of α that solves $c'(\alpha) = (1-\eta)(\phi+\lambda)$. Now, from Lemma 1 (i), it follows that for all $\frac{m}{1-\phi} < (1-\alpha_f)p(\alpha_f, 1)$, it is a best response by the issuer to set $\sigma = 1$ if the CRA's screening intensity is α_f.

Define $\alpha^\dagger$ to be the value of α at which $c(\alpha) = \eta\phi$. From Assumption 1 (ii), it follows that $\alpha^\dagger < 1$. Further, it must be that $\alpha^* \leq \alpha^\dagger$.

Denote $\alpha_z = 1 - \frac{m}{1-\phi}$. If the CRA sets $\alpha = \alpha_z$, from Lemma 1 (ii), the issuer sets $\sigma^* = 0$. The CRA's optimal choice of α is now one out of α_f, α_z, or some α such that $\sigma^*(\alpha) \in (0,1)$.

From Assumption 1 (b), it follows that there exists some $M' > 0$ such that if $\frac{m}{1-\phi} \leq M'$, $c(1 - \frac{m}{1-\phi}) > \eta\phi$, and so it cannot be optimal to set $\alpha = \alpha_z$ when $\frac{m}{1-\phi} \leq M'$.

That leaves the choice of an α such that $\sigma^*(\alpha) \in (0,1)$. From equation (7) it follows that, if $\sigma^*(\alpha) \in (0,1)$, the derivative of the CRA's payoff function with respect to α is $\Psi'(\alpha) = \frac{2\eta(\phi+\lambda)\frac{m}{1-\phi}}{\left(1-\alpha+\frac{m}{1-\phi}\right)^2} - c'(\alpha)$. Denote $g(\alpha) = \Psi'(\alpha)$.

Recall that $\alpha^\dagger < 1$. Therefore, as $\frac{m}{1-\phi} \to 0$, the first term $\frac{2\eta(\phi+\lambda)\frac{m}{1-\phi}}{\left(1-\alpha+\frac{m}{1-\phi}\right)^2}$ converges to 0 for all $\alpha \in [0, \alpha^\dagger]$. Recall that $c'(\alpha_f) = (1-\eta)(\phi+\lambda) > 0$. Therefore, in the limit as $\frac{m}{1-\phi} \to 0$, we have $g(\alpha) < 0$ for all $\alpha \in [\alpha_f, \alpha^\dagger]$. It follows that there exists some $M'' > 0$ such that, when $\frac{m}{1-\phi} < M''$, we have $g(\alpha) < 0$ for all $\alpha \in [\alpha_f, \alpha^\dagger]$. As $g(\cdot)$ is the derivative of $\Psi(\cdot)$, it follows that when $\frac{m}{1-\phi} < M''$, $\Psi(\alpha_f) > \Psi(\alpha)$ for all $\alpha > \alpha_f$ whenever $\sigma^*(\alpha) \in (0,1)$.

Define $\underline{M} = \min\{M', M'', (1-\alpha_f)p(\alpha_f, 1)\}$, and consider any $\frac{m}{1-\phi} < \underline{M}$. Then, as $\frac{m}{1-\phi} < M'$, it follows that $\Psi(\alpha_f) > \Psi(\alpha_z)$. As $\frac{m}{1-\phi} < M''$, $\Psi(\alpha_f) > \Psi(\alpha)$ for all $\alpha > \alpha_f$ such that $\sigma^*(\alpha) \in (0,1)$. Finally, $\frac{m}{1-\phi} < (1-\alpha_f)p(\alpha_f, 1)$ ensures that it is optimal for the issuer to set $\sigma^* = 1$ when the screening intensity is α_f.

(ii) Suppose that $\sigma^* = 0$. Since $p(\alpha, 0) = 1$, it follows from Lemma 1 (ii) that $\alpha \geq \alpha_z = 1 - \frac{m}{1-\phi}$. Further, when $\alpha > \alpha_z$, we have $\Psi'(\alpha) = -c'(\alpha) < 0$, so it is immediate that $\Psi(\alpha_z) > \Psi(\alpha)$ for all $\alpha > \alpha_z$.

Now, consider the function $h(\alpha) = (1-\alpha)p(\alpha, 1)$. The function is strictly less than 1 at $\alpha = 0$ and $\alpha = 1$. At $\alpha \in (0,1)$, each component is strictly less than 1, so again $h(\alpha) < 1$. The function is continuous, so has a maximum for $\alpha \in [0,1]$. Denote the maximizer by $\hat{\alpha}$, so that the maximized value is $h(\hat{\alpha}) < 1$.

From Lemma 1 (iii), it follows that if $\frac{m}{1-\phi} > h(\hat{\alpha})$, then $\sigma^*(\alpha) < 1$ for all $\alpha \in [0,1]$, and so in particular for all $\alpha \in [0, \alpha_z]$. Hence, for $\frac{m}{1-\phi}$ in this range, the optimal screening intensity is either $\alpha^* = \alpha_z$ (in which case $\sigma^* = 0$) or $\alpha^* < \alpha_z$ such that $\sigma^* \in (0,1)$.

As in the proof of part (i), let $g(\alpha) = \frac{2\eta(\phi+\lambda)\frac{m}{1-\phi}}{\left(1-\alpha+\frac{m}{1-\phi}\right)^2} - c'(\alpha)$ denote the derivative of Ψ with respect to α when $\sigma^*(\alpha) \in (0,1)$. Observe that $g(0) > 0$, because $c'(0) = 0$. Now, if there exists a value of α such that $g(\alpha) = 0$, define $\alpha^\ddagger$ to be this value. Otherwise, define $\alpha^\ddagger = \hat{\alpha}$. Note that if $\frac{m}{1-\phi} > 1 - \alpha^\ddagger$, then $\alpha_z = 1 - \frac{m}{1-\phi} < \alpha^\ddagger$. Then, for $\frac{m}{1-\phi} \in (1-\alpha^\ddagger, 1]$, we have $g(\alpha) > 0$ for all $\alpha \in [0, \alpha_z]$. Therefore, $\Psi(\alpha_z) > \Psi(\alpha)$ for all $\alpha < \alpha_z$.

Define $\overline{M} = \max\{1-\alpha^\ddagger, 1-h(\hat{\alpha})\}$. Consider $\frac{m}{1-\phi} > \overline{M}$. As $\frac{m}{1-\phi} > 1 - h(\hat{\alpha})$, it follows that $\sigma^*(\alpha) < 1$ for all $\alpha \in [0, \alpha_z]$. As $\frac{m}{1-\phi} > 1 - \alpha^\ddagger$, it follows that $\Psi(\alpha_z) > \Psi(\alpha)$ for all $\alpha < \alpha_z$. Therefore, the optimal value of α is α_z, so that $\sigma^*(\alpha^*) = 0$. ■

Proof of Proposition 4

We provide a proof by contradiction.

As in the proof of Proposition 3, define $\alpha_f = c'^{-1}((1-\eta)(\phi+\lambda))$ and $\alpha_z = 1 - \frac{m}{1-\phi}$. From Proposition 3, when $\frac{m}{1-\phi}$ is low, the equilibrium has $\alpha^* = \alpha_f$ and $\sigma^*(\alpha^*) = 1$, and when $\frac{m}{1-\phi}$ is high, the equilibrium has $\alpha^* = \alpha_z$ and $\sigma^*(\alpha^*) = 0$.

Now, suppose that for some value of $\frac{\lambda}{\phi}$, there does not exist an interval (M_3, M_4) such that $\sigma^*(\alpha^*) \in (0,1)$ when $\frac{m}{1-\phi} \in (M_3, M_4)$. Then, there must exist a threshold $\hat{M}$ such that, as $\frac{m}{1-\phi}$ increases from $\hat{M} - \epsilon$ to $\hat{M} + \epsilon$, the optimal screening intensity changes from α_f to

α_z. Further, since α_f does not depend on $\frac{m}{1-\phi}$, it follows from Lemma 1 (i) that

$$\hat{M} \leq K_1 \equiv \max\left\{\frac{m}{1-\phi}\middle|\sigma^*(\alpha_f) = 1\right\} = (1-\alpha_f)p(\alpha_f, 1). \tag{20}$$

Now, it must be that the CRA's payoff function, $\Psi(\alpha)$, is continuous at $\frac{m}{1-\phi} = \hat{M}$. That is,

$$\hat{\Psi}_- \equiv \lim_{\frac{m}{1-\phi}\uparrow\hat{M}} \Psi(\alpha_f) = \lim_{\frac{m}{1-\phi}\downarrow\hat{M}} \Psi(\alpha_z) \equiv \hat{\Psi}_+. \tag{21}$$

Suppose not; instead, suppose that $\hat{\Psi}_- > \hat{\Psi}_+$. Note that $\sigma^*(\alpha_f)$ is continuous in $\frac{m}{1-\phi}$, so it must be that for some $\frac{m}{1-\phi}$ just greater than $\hat{M}$, the CRA can earn a higher payoff by choosing $\alpha^* = \alpha_f$ rather than $\alpha^* = \alpha_z$. An analogous argument holds if $\hat{\Psi}_- < \hat{\Psi}_+$.

Denote the value of α_z when $\frac{m}{1-\phi} = \hat{M}$ as $\hat{\alpha} = 1 - \hat{M}$. Note that $\sigma^*(\alpha_f) = 1$ and $\sigma^*(\hat{\alpha}) = 0$. Now, $\hat{\Psi}_- = \hat{\Psi}_+$ implies that

$$\eta\phi - (1-\eta)(1-\alpha_f)(\phi+\lambda) - c(\alpha_f) = \eta\phi - c(\hat{\alpha}). \tag{22}$$

Letting α' denote the unique solution to the equation,

$$c(\alpha') = (1-\eta)(1-\alpha_f)(\phi+\lambda) + c(\alpha_f). \tag{23}$$

The condition in equation (21) requires that $\alpha' = \bar{\alpha}$. Now, α' converges to α_f as $\phi+\lambda \to 0$. As $\alpha' = \hat{\alpha}$, this implies that, for any $\epsilon > 0$, there exists an $L > 0$ such that $|\hat{M} - (1-\alpha_f)| < \epsilon$, for all $\phi+\lambda < L$. However, from equation (20), it follows that $K_1 \leq (1-\alpha_f)p(\alpha_f, 1) < 1-\alpha_f$ as long as $\alpha_f < 1$. Therefore, it cannot be that $\hat{M} \leq K_1$, which is a contradiction.

Thus, when $\phi + \lambda < L$, the conjectured $\hat{M}$ cannot exist. In other words, there must exist an open interval (M_3, M_4) such that $\sigma^*(\alpha^*) \in (0,1)$ for all $\frac{m}{1-\phi} \in (M_3, M_4)$.

The CRA's optimal choice of screening intensity, α^*, must satisfy the first-order condition $\Psi'(\alpha^*) = 0$. As shown in equation (7), when $\sigma^* \in (0,1)$, this first-order condition reduces to $c'(\alpha^*) = \frac{2\eta(\phi+\lambda)\frac{m}{1-\phi}}{\left(1-\alpha^*+\frac{m}{1-\phi}\right)^2}$. ■

Proof of Proposition 5

We consider the change in α as each of η, m, and ϕ change.

First, consider a change in η. Denote $\frac{m}{1-\phi} = y$. As discussed in the proof of Proposition 4, the first-order condition for optimal screening intensity in a partial manipulation equilibrium is $c'(\alpha) = \frac{2\eta\,(\phi+\lambda)y}{(1-\alpha+y)^2}$. Write this as $(1-\alpha+y)^2 c'(\alpha) = 2\eta(\phi+\lambda)y$. Applying the implicit function theorem, we directly have

$$\frac{\partial \alpha^*}{\partial \eta} = \frac{2(\phi+\lambda)y}{(1-\alpha^*+y)[c''(\alpha^*)(1-\alpha^*+y) - 2c'(\alpha^*)]}. \tag{24}$$

Now, when $\sigma^*(\alpha) \in (0,1)$, the derivative of Ψ with respect to α is $\Psi'(\alpha) = \frac{2\eta(\phi+\lambda)y}{(1-\alpha+y)^2} - c'(\alpha)$. Therefore,

$$\Psi''(\alpha) = \frac{4\eta(\phi+\lambda)y}{(1-\alpha+y)^3} - c''(\alpha) = \frac{2c'(\alpha)}{1-\alpha+y} - c''(\alpha). \tag{25}$$

If α^* maximizes Ψ, it must be that $\Psi''(\alpha^*) < 0$, so that $c''(\alpha^*) > \frac{2c'(\alpha^*)}{1-\alpha^*+y}$, or $c''(\alpha^*)(1-\alpha^*+y) > 2c'(\alpha^*)$. Further, $\alpha^* < 1$ by Assumption 1 (b). Therefore, both the numerator and denominator of the RHS of equation (24) are strictly positive, so that $\frac{\partial \alpha^*}{\partial \eta} > 0$.

Next, consider a change in m. As before, denote $\frac{m}{1-\phi} = y$, and let $f(y) = \frac{y}{(1-\alpha+y)^2}$. Then, the first-order condition in equation (7) may be written as $c'(\alpha) = 2\eta(\phi+\lambda)f(y)$. Now,

$$f'(y) = \frac{(1-\alpha+y)^2\,(1) - y(2(1-\alpha+y)}{(1-\alpha+y)^4} = \frac{(1-\alpha+y)(1-\alpha-y)}{(1-\alpha+y)^4} \tag{26}$$

$$= \frac{(1-\alpha-y)}{(1-\alpha+y)^3}. \tag{27}$$

Now, in a partial manipulation equilibrium, it must be that $\frac{m}{1-\phi} < 1-\alpha$ (else the low-type issuer chooses $\sigma^* = 0$). Therefore, in such an equilibrium, $f'(y) > 0$. As $c'(\alpha)$ is strictly increasing, it is immediate that an increase in y results in an increase in α^*. Further, an increase in m is directly an increase in y. Therefore, $\frac{\partial \alpha^*}{\partial m} > 0$.

Finally, consider a change in ϕ. Again, write the first-order condition for optimal α when $\sigma^*(\alpha) \in (0,1)$ as $(1-\alpha+\frac{m}{1-\phi})^2 c'(\alpha) = 2\eta(\phi+\lambda)\frac{m}{1-\phi}$. Note that $y = \frac{m}{1-\phi}$, so $\frac{\partial y}{\partial \phi} = \frac{m}{(1-\phi)^2} = \frac{y}{1-\phi}$. Applying the implicit function theorem, we have

$$\frac{\partial \alpha^*}{\partial \phi} = \frac{2\eta y\,\frac{1+\lambda}{1-\phi} - 2c'(\alpha^*)(1-\alpha^*+y)\,\frac{y}{1-\phi}}{(1-\alpha^*+y)[c''(\alpha^*)(1-\alpha^*+y) - 2c'(\alpha^*)]}. \tag{28}$$

We have established above that the denominator of the RHS of equation (28) is positive.

Consider the numerator. Noting that $c'(\alpha^*) = \frac{2\eta(\phi+\lambda)y}{(1-\alpha^*+y)^2}$, we can write the numerator as

$$\frac{2\eta y}{1-\phi}\left(1+\lambda-\frac{2(\phi+\lambda)y}{1-\alpha^*+y}\right). \tag{29}$$

Consider the term inside the parentheses in expression (29). Because this is a partial manipulation equilibrium, $\alpha^* < 1-y$, or $1-\alpha^* > y$. Therefore, $\frac{2(\phi+\lambda)y}{1-\alpha^*+y} < \frac{2(\phi+\lambda)y}{2y} = \phi+\lambda$. Thus, $1+\lambda-\frac{2(\phi+\lambda)y}{1-\alpha^*+y} > 1+\lambda-(\phi+\lambda) = 1-\phi > 0$, where the last inequality follows as $\phi < 1$.

Therefore, the term in the parentheses in expression (29) is strictly positive. The numerator and denominator of the RHS of equation (28) are both thus positive, so that $\frac{\partial\alpha^*}{\partial\phi} > 0$.

■

Proof of Lemma 4

(i) Suppose the CRA has received a high signal. If it assigns a high rating, it obtains the fee ϕp with probability 1. Conditional on receiving a high signal, the firm has low cash flow with probability $\frac{(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)}$. The payoff of the CRA is therefore $\phi\hat{p} - \frac{(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)}\lambda$.

In equilibrium, the price of the high-rated security is $\hat{p} = \frac{\eta-(1-\eta)\sigma(1-\alpha\beta)}{\eta+(1-\eta)\sigma(1-\alpha\beta)}$. The lowest value this price attains in equilibrium occurs if $\beta = 0$ (i.e., the CRA assigns a high rating even when it has a low signal). Then, the price of the security is $\frac{\eta-(1-\eta)\sigma}{\eta+(1-\eta)\sigma}$. Therefore, it is a best response for the CRA to assign a high rating as long as

$$\phi\frac{\eta-(1-\eta)\sigma}{\eta+(1-\eta)\sigma} \geq \frac{(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)}\lambda, \text{ or,} \tag{30}$$

$$\frac{\lambda}{\phi} \leq \frac{\eta-(1-\eta)\sigma}{\eta+(1-\eta)\sigma}\left(\frac{\eta+(1-\eta)\sigma(1-\alpha)}{(1-\eta)\sigma(1-\alpha)}\right). \tag{31}$$

Now, the RHS of the last inequality is strictly decreasing in σ and strictly increasing in α. It therefore attains its lowest value when $\sigma = 1$ and $\alpha = 0$. Making these substitutions, this lowest value equals $\frac{2\eta-1}{1-\eta}$.

Therefore, a sufficient condition to ensure that the CRA issues a high rating on obtaining a high signal, regardless of the values of α and σ, is $\frac{\lambda}{\phi} \leq \frac{2\eta-1}{1-\eta}$.

(ii) Suppose the CRA has received a low signal. As mentioned in the text, its payoff is $\phi p - \lambda$. In equilibrium, the price of the security is $\hat{p}(\alpha,\sigma,\beta) = \frac{\eta-(1-\eta)\sigma(1-\alpha\beta)}{\eta+(1-\eta)\sigma(1-\alpha\beta)}$.

It is a strict best response for the CRA to assign a high rating if $\phi\hat{p}(\alpha,\sigma,0) > \lambda$ and to assign a low rating if $\phi\hat{p}(\alpha,\sigma,1) < \lambda$. Now, $\hat{p}(\alpha,\sigma,0) = \frac{\eta-(1-\eta)\sigma}{\eta+(1-\eta)\sigma}$ and $\hat{p}(\alpha,\sigma,1) = \frac{\eta-(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)}$. It is immediate that, if $\alpha > 0$, we have $\hat{p}(\alpha,\sigma,1) > \hat{p}(\alpha,\sigma,0)$. Therefore, in any equilibrium of the continuation game at stage 5, it must be that $\beta = 0$ if $\frac{\lambda}{\phi} < \frac{\eta-(1-\eta)\sigma}{\eta+(1-\eta)\sigma} = p(0,\sigma)$, and $\beta = 1$ if $\frac{\lambda}{\phi} > \frac{\eta-(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)} = p(\alpha,\sigma)$.

Finally, observe that $\hat{p}(\alpha,\sigma,\beta)$ is strictly increasing in β. Therefore, if $\sigma > 0, \alpha > 0$, and $\frac{\lambda}{\phi} \in (p(0,\sigma), p(\alpha,\sigma))$, there exists some $\hat{\beta}$ such that $\hat{p}(\alpha,\sigma,\hat{\beta}) = \frac{\lambda}{\phi}$. At this price, the CRA is indifferent between assigning a high rating and assigning a low rating, conditional on having obtained a low signal. Therefore, in equilibrium, the CRA assigns a high rating with probability $1-\hat{\beta}$ and a low rating with probability $\hat{\beta}$. Solving the equation $\hat{p}(\alpha,\sigma,\beta) = \frac{\lambda}{\phi}$ for β yields the value of $\hat{\beta}$ shown in equation (10). ■

Lemma 5. *Suppose that $\alpha > 0$ and $\frac{m}{1-\phi} \leq 1-\alpha$. Then, in the continuation game starting at time 3, there is a unique equilibrium as follows. The CRA assigns a high rating when it obtains a low signal. Further, if it obtains a low signal,*

(a) The CRA assigns a low rating (i.e., sets $\beta^* = 1$) in the following two cases:

(i) $\frac{\lambda}{\phi} \geq p(\alpha,1)$ and $\frac{m}{1-\phi} \leq (1-\alpha)p(\alpha,1)$. Here, $\sigma^* = 1$.

(ii) $\frac{\lambda}{\phi} \geq \frac{m}{(1-\alpha)(1-\phi)}$ and $\frac{m}{1-\phi} \in ((1-\alpha)p(\alpha,1), 1-\alpha)$. Here, $\sigma^* = \frac{1-\alpha-m/(1-\phi)}{(1-\alpha)[1-\alpha+m/(1-\phi)]} \in (0,1)$.

(b) The CRA assigns a high rating (i.e., sets $\beta^* = 0$) in the following two cases:

(i) $\frac{\lambda}{\phi} \leq 2\eta - 1$ and $\frac{m}{1-\phi} \leq 2\eta - 1$. Here, $\sigma^* = 1$.

(ii) $\frac{\lambda}{\phi} \leq \frac{m}{1-\phi}$ and $\frac{m}{1-\phi} \in (2\eta-1, 1-\alpha)$. Here, $\sigma^* = \frac{1-m/(1-\phi)}{1+m/(1-\phi)} \in (0,1)$.

(c) The CRA mixes between assigning a high rating and a low rating (i.e., chooses $\beta^* \in (0,1)$) in the following two cases:

(i) $\frac{\lambda}{\phi} \in (2\eta-1, p(\alpha,1))$ and $\frac{m}{1-\phi} < \frac{\eta(1-\lambda/\phi)\lambda/\phi}{(1-\eta)(1+\lambda/\phi)}$. Here, $\beta^* = \frac{\lambda/\phi-(2\eta-1)}{\alpha(1-\eta)(1+\lambda/\phi)}$ and $\sigma^* = 1$.

(ii) $\frac{\lambda}{\phi} \in \left(\frac{m}{1-\phi}, \frac{m}{(1-\alpha)(1-\phi)}\right)$ and $\frac{m}{1-\phi} \geq \frac{\eta(1-\lambda/\phi)\lambda/\phi}{(1-\eta)(1+\lambda/\phi)}$. Here, $\beta^* = \frac{\lambda/\phi - m/(1-\phi)}{\alpha\lambda/\phi}$ and $\sigma^* = \frac{\eta(1-\lambda/\phi)\lambda/\phi}{(1-\eta)(1+\lambda/\phi)m/(1-\phi)} \in (0,1)$.

Proof

Throughout the proof, we use the results in Lemma 4 and Lemma 1 to find a pair (σ^*, β^*) that satisfies the criteria for both the CRA and the low-type issuer to be playing best responses. Recall from equation (1) that $p(\alpha,\sigma) = \frac{\eta-(1-\eta)\sigma(1-\alpha)}{\eta+(1-\eta)\sigma(1-\alpha)}$.

(a) (i) Suppose that $\frac{\lambda}{\phi} \geq p(\alpha, 1)$. As $p(\alpha, \sigma) \geq p(\alpha, 1)$ for all σ, it is immediate from Lemma 4 part (ii) that the optimal rating strategy for the CRA is to set $\beta^* = 1$. Given that $\beta^* = 1$, it now follows from Lemma 1 part (i) that if $\frac{m}{1-\phi} \leq (1-\alpha)p(\alpha, 1)$, the best response of the low-type issuer is to choose $\sigma^* = 1$.
(ii) Suppose that $\frac{m}{1-\phi} \in ((1-\alpha)p(\alpha, 1), 1-\alpha)$. Then, it follows from Lemma 1 part (iii) that, if $\beta = 1$, it is a best response for the low-type issuer to choose some $\sigma^* \in (0, 1)$. Further, $\frac{\lambda}{\phi} \geq \frac{m}{(1-\alpha)(1-\phi)}$ implies that $\frac{\lambda}{\phi} > p(\alpha, 1)$, so that it is a best response for the CRA to set $\beta^* = 1$. The expression for σ^* is then the same as in equation (2).

(b) (i) Suppose $\sigma = 1$. Then, $p(0, \sigma) = 2\eta - 1$, so that if $\frac{\lambda}{\phi} \leq 2\eta - 1$, from Lemma 4 part (ii), it is optimal for the CRA to set $\beta^* = 0$. The payoff to a low-type that misreports its type as high is $(1-\alpha\beta)\hat{p} - \frac{m}{1-\phi}$. Extending the argument in Lemma 1 (ii), if $\frac{m}{1-\phi} \leq (1-\alpha\beta^*)(2\eta-1) = 2\eta-1$, it is indeed a best response for the low-type issuer to set $\sigma^* = 0$.
(ii) Extending the argument in Lemma 1 (iii), if $\frac{m}{1-\phi} \in ((1-\alpha\beta)(2\eta-1), 1-\alpha)$, it is optimal for the low-type issuer to choose a $\sigma^* \in (0, 1)$. Suppose that $\beta = 0$. Then, the relevant range of $\frac{m}{1-\phi}$ is $(2\eta - 1, 1-\alpha)$. Further, the price of the high-rated security is $\hat{p}(\alpha, \sigma, 0) = \frac{\eta - (1-\eta)\sigma}{\eta + (1-\eta)\sigma} = p(0, \sigma)$. Because $\sigma \in (0, 1)$ and $\beta = 0$, it must be that $\frac{m}{1-\phi} = p(0, \sigma)$. Now, the condition that $\frac{\lambda}{\phi} \leq \frac{m}{1-\phi}$ implies that $\frac{\lambda}{\phi} \leq p(0, \sigma)$, so that from Lemma 4 (ii), it is a best response for the CRA to set $\beta^* = 0$. Finally, setting $p(0, \sigma^*) = \frac{m}{1-\phi}$ and solving for σ^* yields the expression in the statement of the Lemma.

(c) (i) Suppose that $\sigma = 1$. We have $p(0, 1) = 2\eta - 1$, so that when $\frac{\lambda}{\phi} \in (2\eta - 1, p(\alpha, 1))$, from Lemma 4 (ii), the CRA should choose some $\beta^* \in (0, 1)$. Now, suppose $\beta \in (0, 1)$. Then, the price of the high rated security is equal to $\frac{\lambda}{\phi}$. Further, when $\sigma = 1$, we have $p = \frac{\eta - (1-\eta)(1-\alpha\beta)}{\eta + (1-\eta)(1-\alpha\beta)}$. Using $p = \frac{\lambda}{\phi}$, we therefore have $1 - \alpha\beta = \frac{\eta}{1-\eta} \frac{1-\lambda/\phi}{1+\lambda/\phi}$, so that $(1-\alpha\beta)p = \frac{\eta}{1-\eta} \frac{1-\lambda/\phi}{1+\lambda/\phi} \frac{\lambda}{\phi}$. The low-type issuer sets $\sigma^* = 1$ as long as $\frac{m}{1-\phi} \leq (1-\alpha\beta)p = \frac{\eta(\phi-\lambda)\lambda}{(1-\eta)(\phi+\lambda)\phi} = \frac{\eta(1-\lambda/\phi)\lambda/\phi}{(1-\eta)(1+\lambda/\phi)}$. Fixing $\sigma^* = 1$, solve the equation $\hat{p}(\alpha, 1, \beta^*) = \frac{\lambda}{\phi}$ for β^*, which yields the expression in the statement of the Lemma.
(ii) Suppose that $\beta \in (0, 1)$. The same argument as in part (c) (i) above implies that when $\frac{m}{1-\phi} \in \left(\frac{\eta(\phi-\lambda)\lambda}{(1-\eta)(\phi+\lambda)\phi}, 1-\alpha\right)$, it is optimal for the low-type issuer to choose some $\sigma^* \in (0, 1)$. As the low-type issuer is mixing, it must be that $(1-\alpha\beta)\hat{p} = \frac{m}{1-\phi}$, so that $\hat{p}(\alpha, \sigma, \beta) = \frac{1}{(1-\alpha\beta)} \frac{m}{1-\phi}$. Now, $p(0, \sigma) = \frac{m}{1-\phi}$, and $p(\alpha, \sigma) = \frac{1}{1-\alpha} \frac{m}{1-\phi}$. Therefore, $\frac{\lambda}{\phi} \in \left(\frac{m}{1-\phi}, \frac{m}{(1-\alpha)(1-\phi)}\right)$ implies that $\frac{\lambda}{\phi} \in (p(0, \sigma), p(\alpha, \sigma))$, so that from Lemma 4 (ii), it is optimal for the CRA to choose some $\beta^* \in (0, 1)$. This implies that $\hat{p}(\alpha, \sigma, \beta) = \frac{\lambda}{\phi}$. Divide this equation by

$\hat{p}(\alpha,\sigma,\beta) = \frac{1}{(1-\alpha\beta)}\frac{m}{1-\phi}$, and solve for β to obtain the expression for β^* in the expression of the Lemma. Finally, substituting the expression for β^* back into $\hat{p}(\alpha,\sigma,\beta) = \frac{\eta-(1-\eta)\sigma(1-\alpha\beta^*)}{\eta+(1-\eta)\sigma(1-\alpha\beta^*)}$ and using the equation $\hat{p}(\alpha,\sigma,\beta) = \frac{\lambda}{\phi}$ to solve for σ yields the expression for σ^*. ■

Proof of Proposition 6

(i) Suppose that $\alpha^* > 0$ and $\beta^* < 1$. Observe that σ^* depends only on $\alpha^*\beta^*$ rather than separately on α^* and β^*.

Now, $\hat{\Psi}(\alpha^*) = \eta\phi-(1-\eta)\sigma^*(1-\alpha^*\beta^*)-c(\alpha^*)$. Denote $\tilde{\Psi}(\alpha,\beta) = \eta\phi-(1-\eta)\sigma^*(1-\alpha\beta)$, where σ^* is the best response given α,β. Set $\hat{\beta} = \beta^*+\epsilon$ for some small $\epsilon \in (0, 1-\beta^*)$, and set $\hat{\alpha} = \frac{\alpha^*\beta^*}{\hat{\beta}} < \alpha^*$. As $c(\hat{\alpha}) < c(\alpha^*)$, and all other terms in σ^* is unaffected, it follows that $\tilde{\Psi}(\hat{\alpha},\hat{\beta}) > \hat{\Psi}(\alpha^*,\beta^*)$. Further, for a suitably small $\epsilon > 0$, it must be that $(\sigma^*,\hat{\beta})$ represent equilibrium strategies in the continuation game at stage 3, given $\hat{\alpha}$. We therefore have a direct contradiction to the assumption that α^* and β^* represent equilibrium values.

Therefore, in any equilibrium, if $\alpha^* > 0$, it must be that $\beta^* = 1$.

(ii) Suppose $\sigma^* = 0$. Then, the price of the asset is $\hat{p}(\alpha,0,\beta) = 1$. Therefore, for any $\frac{\lambda}{\phi} < 1$, it must be that $\frac{\lambda}{\phi} < \hat{p}$. Thus, the best response of the CRA is to set $\beta^* = 0$ if it receives a low signal (of course, obtaining a low signal represents an unreached information set in the game). However, if $\beta^* = 0$ and $\hat{p} = 1$, the payoff to the low-type issuer from deviating and reporting high is $(1-\alpha\beta^*)\hat{p}-\frac{m}{1-\phi} = 1-\frac{m}{1-\phi} > 0$, as long as $\frac{m}{1-\phi} < 1$. Therefore, the low-type issuer should deviate and play $\sigma = 1$ instead, breaking the conjectured equilibrium. ■

Proof of Proposition 7

First, suppose that $\frac{\lambda}{\phi} < 2\eta-1$. Now, the lowest possible value of p, obtained when $\sigma = 1$ and $\alpha\beta = 0$, is $p = 2\eta-1$. Therefore, $\frac{\lambda}{\phi} < p$. Now, when $\frac{\lambda}{\phi} < p$, regardless of what the value of α^* is, the CRA must set $\beta^* = 0$; i.e., it assigns a high rating regardless of the signal it received. It cannot be optimal to choose a positive α^* and then set $\beta^* = 0$; the CRA obtains a strictly higher payoff by simply setting $\alpha^* = 0$, because it saves on the direct cost of screening. Therefore, in equilibrium, $\alpha^* = 0$.

Next, suppose $\frac{\lambda}{\phi} = 2\eta-1$. For any value of α and β such that $\alpha\beta > 0$, it follows that $p > 2\eta-1 = \frac{\lambda}{\phi}$. As argued in the previous paragraph, it is therefore optimal to set $\beta^* = 0$, regardless of the value of α. Therefore, as in the previous paragraph, in equilibrium, it must be that $\alpha^* = 0$.

Now, consider $\frac{m}{1-\phi} \in [2\eta-1, 1]$ and $\frac{\lambda}{\phi} \leq \frac{m}{1-\phi}$. Suppose first that $\frac{m}{1-\phi} > (1-\alpha\beta)p$. Then, it follows that the best response of the issuer is to set $\sigma^* = 0$. But then it must be that $p = 1$, and further that $\alpha\beta = 1 - \frac{m}{1-\phi}$. When $\alpha\beta = 1 - \frac{m}{1-\phi}$ and $p = 1$, we have $(1 - \alpha\beta)p = \frac{m}{1-\phi}$, which contradicts the assumption $\frac{m}{1-\phi} > (1 - \alpha\beta)p$.

Therefore, it must be that $(1-\alpha\beta)p \geq \frac{m}{1-\phi}$. Now, if $\alpha\beta > 0$, it follows that $p > \frac{m}{1-\phi} \geq \frac{\lambda}{\phi}$. As $p > \frac{\lambda}{\phi}$, from the arguments in the first paragraph, it then follows that it must be that $\beta^* = 0$, which in turn implies that $\alpha^* = 0$.

Suppose instead that $\alpha\beta = 0$. There are two possibilities: We either directly have that $\alpha^* = 0$, or $\beta = 0$. In the latter case, the arguments of the first paragraph imply that $\alpha^* = 0$.

■

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