A DYNAMIC MODEL OF UNIVERSITY SELECTION

Ivan Anić, Vladimir Božin, Branko Urošević
Michael Spence – Job Market signaling

• Spence, A. M. (1973): “Job market signaling” Quart. J. Econ., 87
• Nobel prize 2001
• Negative correlation between the cost of education and worker productivity
• Two types of workers, according to productivity, “high” and “low”
Asymmetric information

• Employer does not know the productivity level of a worker, and has to infer it indirectly, while workers do know their productivity level
• Education level serves as a signaling message from workers to employers
Job Market signaling is an active area of research

- Cyclicality over time: Dellas and Sakellaris (2003)
- Generalizations: Jovanovic (1979), Farber and Gibbons (1996)
The Model

• Two types of universities (schools/programs) - “hard” and “easy”
• Two types of students, “high” and “low” ability ($n_h$ and $n_l$ - same every year)
• Dropout rates
• Wages set based on past experience with alumni
University policies set dropout rates

\[ p_{h1} > p_{l1} \quad p_{h2} > p_{l2} \]

\[ p_{h1} < p_{h2} \quad p_{l1} < p_{l2} \]
Employers set salaries

\[ S_1(t) = \frac{N_{h1}(t-1)p_{h1}}{N_{h1}(t-1)p_{h1} + N_{l1}(t-1)p_{l1}} S_h + \frac{N_{l1}(t-1)p_{l1}}{N_{h1}(t-1)p_{h1} + N_{l1}(t-1)p_{l1}} S_l \]

\[ S_2(t) = \frac{N_{h2}(t-1)p_{h2}}{N_{h2}(t-1)p_{h2} + N_{l2}(t-1)p_{l2}} S_h + \frac{N_{l2}(t-1)p_{l2}}{N_{h2}(t-1)p_{h2} + N_{l2}(t-1)p_{l2}} S_l \]

\[ N_{h1}(t) = \sum_{\tau=1}^{t} n_{h1}(\tau), \]

analogously for \( N_{h2}(t) \), \( N_{l1}(t) \) and \( N_{l2}(t) \)
At any moment $t$ there are 4 „states“:

$C_1$ – all prospective students choose harder university
$C_2$ – all prospective students choose easier university
$C_3$ – high ability students choose harder, and low ability easier university
$C_4$ – high ability students choose easier, and low ability harder university

$$Q(t) = \frac{S_2(t)}{S_1(t)}$$

One generation is taken as unit of time
Theorem 1.

(i) If the system at time $t$ is in states $C_1$ or $C_2$ then $Q(t + 1)$ is between $Q(t)$ and $\bar{Q}$

(ii) If the system at time $t$ is in state $C_3$ then $Q(t + 1)$ is between $Q(t)$ and $Q_{\text{min}}$

(iii) If the system at time $t$ is in state $C_4$ then $Q(t + 1)$ is between $Q(t)$ and $Q_{\text{max}}$

\[ Q_{\text{min}} = \frac{S_l}{S_h} \quad \quad Q_{\text{max}} = \frac{S_h}{S_l} \]

\[ \bar{S}_1 = \frac{n_h p_{h1}}{n_h p_{h1} + n_i p_{l1}} S_h + \frac{n_i p_{l1}}{n_h p_{h1} + n_i p_{l1}} S_l. \]

\[ \bar{S}_2 = \frac{n_h p_{h2}}{n_h p_{h2} + n_i p_{l2}} S_h + \frac{n_i p_{l2}}{n_h p_{h2} + n_i p_{l2}} S_l. \]

\[ \bar{Q} = \frac{\bar{S}_2}{\bar{S}_1} \]
<table>
<thead>
<tr>
<th>Student choice</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditions</td>
<td>$Q(t) &lt; R_h$</td>
<td>$Q(t) &gt; R_h$</td>
<td>$Q(t) &lt; R_h$</td>
<td>$Q(t) &gt; R_h$</td>
</tr>
<tr>
<td></td>
<td>$Q(t) &lt; R_l$</td>
<td>$Q(t) &gt; R_l$</td>
<td>$Q(t) &gt; R_l$</td>
<td>$Q(t) &lt; R_l$</td>
</tr>
</tbody>
</table>

$$R_h = \frac{p_{h1}}{p_{h2}}$$

$$R_l = \frac{p_{l1}}{p_{l2}}$$
State sequences for $D_1 < D_2$

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<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>$C_2 \rightarrow C_2 \rightarrow \cdots$</td>
</tr>
<tr>
<td>2</td>
<td>$C_1 \rightarrow C_1 \rightarrow \cdots$</td>
</tr>
<tr>
<td>3</td>
<td>$C_1 \rightarrow \cdots \rightarrow C_1 \rightarrow C_2 \rightarrow C_2 \rightarrow \cdots$</td>
</tr>
<tr>
<td>4</td>
<td>$C_1 \rightarrow \cdots \rightarrow C_1 \rightarrow C_4 \rightarrow \cdots \rightarrow C_4 \rightarrow C_2 \rightarrow C_2 \rightarrow \cdots$</td>
</tr>
<tr>
<td>5</td>
<td>$C_4 \rightarrow \cdots \rightarrow C_4 \rightarrow C_2 \rightarrow C_2 \rightarrow \cdots$</td>
</tr>
</tbody>
</table>

\[
D_1 = \frac{p_{h1}}{p_{l1}} \quad \text{and} \quad D_2 = \frac{p_{h2}}{p_{l2}}
\]
State sequences for
\(D_1 > D_2\)

<table>
<thead>
<tr>
<th></th>
<th>Dynamics</th>
<th>Diagrams</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>(C_1 \rightarrow C_1 \rightarrow \cdots)</td>
<td>2,4,6</td>
</tr>
<tr>
<td>2</td>
<td>(C_2 \rightarrow C_2 \rightarrow \cdots)</td>
<td>1,2,3,4,5,6</td>
</tr>
<tr>
<td>3</td>
<td>(C_1 \rightarrow \cdots \rightarrow C_1 \rightarrow C_2 \rightarrow C_2 \rightarrow \cdots)</td>
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<tr>
<td>4</td>
<td>(C_2 \rightarrow \cdots \rightarrow C_2 \rightarrow C_1 \rightarrow C_1 \rightarrow \cdots)</td>
<td>4,6</td>
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<tr>
<td>5</td>
<td>(C_1 \rightarrow \cdots \rightarrow C_1 \rightarrow C_1 \rightarrow C_3 \rightarrow \cdots C_3 \rightarrow C_1 \rightarrow C_1 \rightarrow \cdots)</td>
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<tr>
<td>6</td>
<td>(C_2 \rightarrow \cdots \rightarrow C_2 \rightarrow C_3 \rightarrow C_3 \rightarrow \cdots C_3 \rightarrow C_1 \rightarrow C_1 \rightarrow \cdots)</td>
<td>4,6</td>
</tr>
<tr>
<td>7</td>
<td>(C_3 \rightarrow C_3 \rightarrow \cdots)</td>
<td>1,5</td>
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<tr>
<td>8</td>
<td>(C_1 \rightarrow \cdots \rightarrow C_1 \rightarrow C_1 \rightarrow C_3 \rightarrow C_1 \rightarrow \cdots)</td>
<td>5</td>
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<td>9</td>
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<td>10</td>
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<td>2,4,6</td>
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<tr>
<td>11</td>
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<tr>
<td>12</td>
<td>(D \rightarrow D \rightarrow \cdots)</td>
<td>2,4</td>
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<tr>
<td>13</td>
<td>(C_3 \rightarrow \cdots \rightarrow C_3 \rightarrow D \rightarrow D \rightarrow \cdots)</td>
<td>2,4</td>
</tr>
<tr>
<td>14</td>
<td>(D \rightarrow \cdots \rightarrow D \rightarrow C_1 \rightarrow C_1 \rightarrow \cdots)</td>
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<tr>
<td>15</td>
<td>(C_3 \rightarrow \cdots \rightarrow C_3 \rightarrow D \rightarrow \cdots \rightarrow D \rightarrow C_1 \rightarrow C_1 \rightarrow \cdots)</td>
<td>2,4</td>
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<tr>
<td>16</td>
<td>(C_2 \rightarrow \cdots \rightarrow C_2 \rightarrow D \rightarrow \cdots \rightarrow D \rightarrow C_1 \rightarrow C_1 \rightarrow \cdots)</td>
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<tr>
<td>17</td>
<td>(C_2 \rightarrow \cdots \rightarrow C_2 \rightarrow D \rightarrow D \rightarrow \cdots)</td>
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<tr>
<td>18</td>
<td>(C_2 \rightarrow \cdots \rightarrow C_2 \rightarrow C_3 \rightarrow \cdots \rightarrow C_3 \rightarrow D \rightarrow \cdots \rightarrow D \rightarrow C_1 \rightarrow C_1 \rightarrow \cdots)</td>
<td>4</td>
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<tr>
<td>19</td>
<td>(C_2 \rightarrow \cdots \rightarrow C_2 \rightarrow C_3 \rightarrow \cdots \rightarrow C_3 \rightarrow D \rightarrow D \rightarrow \cdots)</td>
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</tbody>
</table>

Here \(D\) is a sequence \(C_1 \rightarrow \cdots \rightarrow C_1 \rightarrow C_3 \rightarrow \cdots \rightarrow C_3\).
Theorem 4. After sufficient number of generations, the following long run equilibria can occur:

• All students enroll in the more challenging university

• All students enroll in the less challenging university

• High productivity students enroll in the more challenging university while low productivity students enroll in the less challenging university

• High productivity students enroll in the more challenging university while low productivity students alternate enrollment in two universities indefinitely.
Expected salaries ratio over time.
Robustness analysis

- Dumping gives more weight to more recent graduates

\[ N_{h1}(t) = \sum_{\tau=1}^{t} q^{t-\tau} n_{h1}(\tau) \]

- Noise shifts student threshold randomly

\[ n_{l1}(t) = n_l \Pr(X_l(t) \leq 1), \quad X_l(t) \sim N\left(Q(t)/R_l, \sigma^2\right) \]

\[ n_{h1}(t) = n_h \Pr(X_h(t) \leq 1), \quad X_h(t) \sim N\left(Q(t)/R_h, \sigma^2\right) \]
Expected salaries ratio

Time

Low
Conclusions

• University policies – set distinction levels
• Employer prefers the separating equilibrium
• Incentives to increase distinction level of the more challenging university
Further research

• Optimal university policies (policies change over time – game theory)
• Time to reach equilibrium
• More types of students
• More universities/programs
• Analysis of extended models
• Studies that increase productivity
• Empirical research