The Role of Budget Constraints in the Theory of Rational Expectations, A Framework for Modelling and Discussing Financial Stability

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Abstract

This paper extends the work on modelling with two particular objectives: first the paper revisits the rational expectations argument to propose a new method to bring models closer to reality; and second to provide a definition and a general framework for the analyses of financial stability issues in the framework of monetary policy. The paper observes that current literature has not taken full advantage of Blanchard and Kahn (B&K) solution of rational expectation models. In a practical way the paper proposes to substitute current resource identities in the model with cointegration relationships, as a way to introduce more flexibility in the model without affecting the priorities or the total number of restrictions which are necessary to solve the model in the rational expectations setup. This is done by augmenting the DSGE with the power of VECM (cointegration). The main assumption here is that agents are rational but this is true in the long run, as they experience intentional or accidental irrationalities and/or information loss in the short run. Despite allowing more flexibility, this proposal yields a more realistic and logical approach solution for the hybrid model or structural shocks proposed by other authors.

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Key words: Rational Expectations, Financial Stability, DSGE, Budget Constraint, Cointegration.

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Introduction

Modelling of macroeconomic behaviour has been one of the defining trends in the central banks until recently. Models are traditionally used for the benefit of monetary policy; however they have recently been adopted to cover issues of financial stability. These models range from simple single equations to the most sophisticated Dynamic Stochastic General Equilibrium (DSGE) models. The evolution within the same model categories and the “revolution” from one class of models to another has been mainly driven by the need for a better and more natural representation of the real world events. Despite such advancement, recently used models have suffered a setback, so academics and practitioners are back at the drawing board to fix the problem. The objective as always is to build better, more representative models or improve and enhance the current ones. Among other traditional models, currently the efforts are focused on the introduction of financial stability issues in the macroeconomic thinking. This paper takes a shot at this problem, after defining financial stability in terms of the broader economy.

Among others, two distinct works on macroeconomic modelling have played a very important role in development of the theory of macroeconomics and its practical application to policy design. The first approach is the work of Sims (1980) Vector Autoregressive Model (VAR) approach that tries to explain the interconnection between variables with the VAR system, which so to say, lets the data to explain the economic story and reveal relationship between variables. This could also include as a subcategory the structural and cointegrated VAR approach or the unit root econometrics.

The second approach is the DSGE modelling or the rational expectations approach, which starts form micro theoretic fundamentals to develop a model that describes economy at its largest macro-scale. One important characteristic of both these approaches is that they are build upon structural mathematic models in the form of systems of linear equations (or equations which can be reduced to the linear form, starting from a nonlinear pattern), and both rely on Gaussian methods to find a unique solution. In this respect they both need a set of restrictions which will allow for recursive substitution in the system, in order to provide a single solution to this system.[2] In principal, the problem converts into identifying and imposing a set of restrictions in the state matrix of the system, (in other words: starting from an unconstrained system, authors introduce the minimum number of constraints in the

[2] In the case of VAR the solution was introduced by Sims and in the case of DSGE the conditions for a single solution were described by Blanchard and Kahn.

system) that will yield a single identifiable solution, making this optimization problem a constrained problem.

Despite these advances, each approach has encountered different obstacles to fully and realistically describe the economy. First, the VAR approach required that the researcher decides the structure of the economic relationship including the map and variables order of shock transmissions to the economy. This solution means that the researcher is imposing the structure of economic relationship, which in fact is like setting up a model assuming that we know the theoretic fundamentals without actually formalizing it. Another significant problem with VAR models is that these models are backward-looking and are subject to Lucas Critique.[3]

On the other hand the DSGE approach is considered a way too stylized to replicate real economic fluctuations and in the meantime very limited in the ability to generate shocks to economy. While it is all based on the micro theoretic fundaments, it is not able to explain the life as we know or at least as we observe it. In principal, this was (and continues to be) such a problem that was impossible to bring the model to data. Last but not least, this critique is extended to include the fact that DSGE failed to predict the last financial crisis. This is due to the fact that such models did not address directly or indirectly any issues that deal with financial stability.

Ireland (2004) brings together both these approaches into a single hybrid system, which solves the above-mentioned inability of DSGE models to replicate the observed economic fluctuations in the data. Basically Ireland (2004) proposes to incorporate the VAR structure of errors (meaning adding a vector of errors, which brings more flexibility in the model and enables it to match the observed economic data) in the DSGE model. Eventually, the solution is to introduce a structure of autoregressive shocks into the model, similar to what we observe in the real life. This solution has two benefits: first it brings the models to the data by regarding every single difference between the model and the data as an estimation error, and second each error is treated simultaneously as a potential shock to economy. While practical benefits for model builders and policy makers are enormous because it practically brought DSGE models from the laboratory to the real life, this modification is motivated by a convenient assumption that economic data collection and calculation is

[3] Lucas Critique highlights the fact that parameters that result from empirical inferences, which are later used in model building, might be subject to change in response to policy changes, or policy interventions. Using models with varying parameters would yield to unpredictable results. Therefore only models with stable parameters must be used to evaluate the impact of policy changes.

inevitably incorporating errors of measurements, rather than founded in the economic reasoning.

This approach, however, did not solve several fundamental problems faced by DSGE models[4], the singularity issue being the most important. Due to their general design assumptions, all DSGE models have only one technological shock that drives the economy.

In response to this problem, another similar solution from a very different approach was to introduce structural shocks in the DSGE models. But this solution is arbitrary and subject to critique as well. Despite the critique, this was great, because in general it brought models to the data and in fact opened the way for a more realistic use and interpretation of results in the policy decision making. Therefore, DSGE was a very promising tool for policy design and implementation.

Yet, despite these adjustments, empirical investigation reveals that the data behaves differently from what DSGE models would assume in the setup process, and predict in policy evaluation exercises. The work by Juselius and Frenchi (2007) is one very fine example which empirically shows that: “the story that the data wants to tell, is in fact very different from the RC theory” (pp. 33). In fact, they find that most of the underlying assumptions on which the DSGE model is built are rejected by the data[5]. The differences between theoretic (read DSGE) and empirical models in which data story gets the precedence (read VAR) “suggest that conclusions based on strong economic priors and many untested assumptions might say more about the faith of the researcher than the economic reality”

However, the story of Juselius and Franchi (2007) is in principal not different from the story of Sims (1980). As in Sims identification scheme, they also need to impose several restrictions, in order to find the unique solution for their model. They do so by introducing a set of cointegration restrictions (long-run theoretic relationships), which in fact is like imposing the structure to the model. After this the data does it all, and we do not understand why the economy behaves such or that way. The data do tell the story without “structural” content, meaning that we do not have a fundamental economic understanding of the observed shocks. Moreover, Lucas critique remains relevant, as it is demonstrated by the instability of parameters for two different sub-periods of the entire sample, respectively,

[4] In addition to singularity problem and poor fit, modelling challenges; incorporation of financial stability issues such as financial frictions, currency risk premia; datasets; the role of fiscal policy; invalid cross equation restrictions; measurement errors and identification.
[5] A brief description of these assumptions and their implications in the context of the economic behaviour is discussed latter in the paper.

before and after 1979.[6] To the authors "it seems obvious that the major difference (between two periods), is to be found in the degree of globalization, worldwide capital deregulation and increasing international competitiveness".[7]

These difficulties have not stopped policy makers to build and use such models for policy purposes. DSGE models are widely used in the process of policy-making in the most respected central banks in the worlds. Tovar (2009) provides a brief review of the increasing role that DSGE models are playing in the policy-making process, the nature of their limitations, and the problems and challenges faced by central banks in the use of such models. The main focus of such work is to predict the effects of policy actions undertaken by monetary authorities.

This was true until 2008 when the models failed "miserably" to predict the financial crisis.[8] Since then, the DSGE framework has experienced a setback. The Nobel laureate Robert Solow (2010), with his statement: "I do not think that the currently popular DSGE models pass the smell test" (pp.2), probably makes one of the biggest opponents of the DSGE model. His remarks with regard to: rationality, the total exclusion of conflicts of interests, incompatible expectations, and deceptions, are well appointed observations and add to the long list of critique that challenges DSGE model and the arguments against its imprudent use in policy making.

New efforts are now devoted to address some of these problems with the objective to improve and/or build alternative models. They focus on identifying the correct shocks and/or financial frictions and estimating better the values of parameters in the models using VAR-s, as in Del Negro and SchorFheide (2007) or Hall (2009). In both cases, authors try to get better and more realistic input from the data, in order to incorporate financial stability issues and bring models closer to the real world. Sims (2008) finds that there is still potential for improvement in this particular area.

Well this paper makes the point that there is still potential in the DSGE, if we were to take full advantage of several overlooked aspects of current solutions method, rational and conflicting expectations. We do this by combining the DSGE and cointegrated VAR, in a way that goes beyond the "hypothesis evaluation" used by Juselius and Franchi (2007). This paper discusses issues in the following way. Section 2 describes the DSGE setup and its solution. Section 3 discusses models and crisis; Section 4 discusses the modelling of rational expectations and discusses our proposal that transforms the existing model with a

[6] Based on recursive tests authors find support to the existence of a structural break around 1979.
[7] Johansen and Franchi (2007), pp.32
[8] Despite the set back, DSGE models are still used in the policy making and forecasting process.

new trick without violating conditions for the solution of rational expectation models; Section 5 proposes a framework for definition of financial stability and its interaction with monetary policy; and Section 6 concludes.

2. The general DSGE setup: the intuition behind the solution.

The standard DSGE model portrays the economy in the framework of the real business cycle model of Kydland and Prescott (1982); later representations of the model have incorporated prices and are known as the New Keynesian models. Currently, the efforts are focused on introducing the financial stability issues and rethinking the incorporation of the financial sector. In the simplest of such models one single agent, called the representative one, tries to chose the combination of consumption and labour to maximize his utility function, *(1)* typically represented by a constant elasticity of substitution, subject to constant returns to scale production function *(2)*,

$$E_t \sum_{i=1}^{\infty} \beta^t (C_{t+i}; H_{t+i}) \qquad (1)$$

$$Y_t = A_t K_t^{\alpha} (\eta^t H_t)^{1-\alpha} \qquad (2)$$

The model is completed by a set of identities, which describe capital formation *(3)*, and budget constraint or aggregate demand *(4)*

$$K_t = I_t + (1-\delta) K_{t-1} \qquad (3)$$

$$Y_t = C_t + I_t \qquad (4)$$

Of all the elements of the model only the total factor productivity is assumed to follow a stochastic first order autoregressive model *(5)*. The error $\varepsilon_{A,t}$ is the only source of stochastic movement “unpredictability” in the model.

$$A_t = \rho_A A_{t-1} + \varepsilon_{A,t} \qquad (5)$$

The model is a collection of dynamic equations that result from the optimization procedure of the above problem. First order conditions of the Lagrangian of the dynamic system are comprised by the utility function, production function, resource constraint and laws of motion *(3) & (5)*. The model is organized as a system of linear first difference equations *(6)* below. The solution to this dynamic system is at the same time the solution to the agents’ utility maximizing problem in time.

$$
\begin{cases}
y_t = a_t k_t^{\theta} h_t^{1-\theta} \\
\ln(a_t) = (1-\rho)\ln(a) + \rho \ln(a_{t-1}) + \varepsilon_t \\
y_t = c_t + i_t \\
\eta k_t = (1-\delta) k_t + i_t \\
\gamma c_t h_t = (1-\theta) y_t \\
\frac{\eta}{c_t} = \beta E\{(\frac{1}{c_{t+1}})[\theta(\frac{y_{t+1}}{k_{t+1}}) + 1 - \delta]\}
\end{cases}
\qquad (6)
$$

The lower case variables represent the original model after it is first normalized by the gross rate of labour augmenting technological process η. Since the later is considered to grow with time (bearing a time trend), this procedure is required to make the model stationary.

The next step requires log-liberalizing the dynamic relationships of (6) around the steady state of the model (which yields also the solution for the model), with the log linearization, due to the fact that the model is non linear. After the log-linearization, the model (6) transforms into the following system of linear difference equations:

$$
\begin{cases}
y_t = a_t + \theta k_t \; (1-\theta)\, h_t \\
a_t = \rho a_{t-1} + \varepsilon_t \\
(\eta/\beta - 1 + \delta) y_t = [(\eta/\beta - 1 + \delta) - \theta(\eta - 1 + \delta)] c_t + \theta(\eta - 1 + \delta) i_t] \\
y_t = c_t + h_t \\
\eta k_{t+1} = (1-\delta) k_t + (\eta - 1 + \delta)\, i_t \\
-\eta/\beta\, c_t = E(c_{t+1}) + (\eta/\beta - 1 + \delta) E_t y_{t+1} - (\eta/\beta - 1 + \delta) k_{t+1}
\end{cases}
\qquad (7)
$$

The interpretation of the final solution of the model yields that at the steady state all the variables grow simultaneously at a constant rate equivalent to the labour augmenting technological progress. In the mean time, the steady state is only function of the deep structural parameters $(\alpha, \beta, \delta, \theta, \rho, \eta)$ of the model, which are invariant of policy shifts. One important observation in the model above only the level of capital and the level of technology are known at the beginning of each period (meaning are carried over from previous period), the rest of the variables are determined within each period. These two groups of variables are called predetermined and non-predetermined variables, respectively.

“Rational expectations” is one of the fundamental assumptions of the modern macroeconomic modelling, and a fundamental requirement for this model. It was Blanchard and Kahn (1981) who provided the mathematical condition that yielded the conditions for the solution of rational expectations models. Mathematically speaking, rationality means that the matrix of coefficients of the system of equations that describes the model *(7)* above has as many linearly independent vectors as there are predetermined variables in the model. In other words, the rationality requires that the number of stable (less than one) unit roots in the system (in the matrix of coefficients) is equal to the number of predetermined variables. This condition, according to Blanchard and Kahn (1981) (referred as B&K) is necessary to yield a single and uniquely identified solution for the model, which falls on the steady state or along a steady state path.

This would mean that there are two independent (predetermined variables) in the model as there are two different processes that drive the model. In general, our simple model must have a deterministic trend (the labour augmented technological process) that usually is embodied in the behaviour of capital and a stochastic trend that emerges from the random shock in the productivity function.

If this model were to represent the economy reasonably well, then the observed behaviour of economic variables in the real word must be similar to the predictions of the model. Moreover, this can easily be tested. The first such test failed as researchers tried to bring the models to the data. Due to rigidity of the theoretic model, it failed to fit and replicate the data generating process, as the data are subject to various and frequent shocks that the model could not accommodate because of the singularity issue. As it is specified above, Ireland (2004) provided the solution by augmenting the model with a series of random stochastic shocks, assuming that they would follow an AR (1) process in the short run. While the argument for such addition is justified by measurement errors rather than found in the principles of economic theory, it allows the model to fit the data. This solution is in fact a shortcut that permits the model to fit the data but does not solve the problem fundamentally.

The theoretic setup presented above and the assumptions, on which it is based, imply that the time series of economic variables that are represented in the model must satisfy a set of specific individual or simultaneous restrictions. These restrictions emerge from the rational expectation solution of the theoretic model; from the structural economic relationship and exogeneity assumptions that are observed in the system *(7)*; and from the statistical properties of the parameters and variables of the model. These restrictions are formalized by Juselius and Franchi (2007) and tested by the same authors using data for US economy with the use of Cointegrated VAR. While the Juselius and Franchi (2007) authors fully discuss all the set of restrictions that derive from the theoretic setup, we would only refer to the set of exogeneity and stationary assumptions, as they are of the primary interest from the point of view of rational expectations. According to Juselius and Franchi (2007), the rational expectation solution of B&K implies the following restrictions:

- Exogeneity assumptions require that the evolution of $a_t \,\&\, k_t$ is the driving force to the system and is weekly exogenous. This assumption relates to the fact that both capital and technology are predetermined in the context of rational expectations.

- Stationary assumptions require:
  a. that $y_t, c_t \; and \; k_t$ are trend stationary, with identical linear growth rates, driven by the labour augmented technological progress. This would further imply that any linear combinations of $y_t \,\&\, c_t \; and \; y_t \,\&\, k_t$ are as well stationary.
  b. that h_t is stationary. The stationary of h_t resulting from the fact that since the linear combination of output and consumption is stationary than the linear combination of output, consumption and labour is stationary only if labour is stationary.

In principle, the above assumptions are used by Juselius and Franchi (2007) to work out the identification scheme that represents the correct number of stationary and non stationary relationships in the CVAR model for the US economy.

In the findings, Juselius and Franchi (2007) conclude that these assumptions are not supported by the data. They find that linear combinations in *(a)* above are not stationary; labour was found to be nonstationary as well. The weak exogeinity of capital is strongly rejected by the data, meaning that a cumulated shock to capital is not one of the driving forces in the model; it is rather the shocks to consumption that has been the driving force of the model. This is completely at odds with the theory prediction.

We believe that the story told by the data can be easily accommodated in the theoretic model naturally. For this, we would like to discus the intuition behind the rational expectations, from the point of view of latest financial crisis. One main problem with a model like this is the restriction of the shock to only productivity shock, a shortcoming that is the Achilles heel of the model. We would start the argument with an intuitive description of the rational expectations.

3. Why do models fail to predict crisis: intuitive description of rational expectations

"Rational expectations" is one of the fundamental assumptions of the modern macroeconomic modelling. Models assume that agents make rational choices, but the question remains: Are agents really rational? It seems that the answer to this question might depend more on the moment rather than on our understanding of the agent's behaviour. Ten years ago the answer seemed to have been yes. Economists were so sure about it that even invented a name to define the collective set of economic behaviour that was not explained by the traditional common knowledge and understanding, calling it "the new economy". In the light of the recent unpleasant experiences of full blown crisis, an easy tentative answer for the rationality can be: agents simply aren't rational. Solow (2010) articulates "Clearly they (agents) do not always behave in this rational way, and systematic deviations are worth studying". After all, anything that ends up in a crisis has to be irrational, and this imposes a strong problem for the DSGE models. A model that is designed with a rational agent in mind will not yield reliable and predictable results in a real world full of irrational ones. So it is important to discuss this fundamental institution of modern macro-modelling in the context of real world and in the models.

Beyond the rigorous explanation given in the theoretic setup (which we will discuss later), the standard economic text book will define rationality a set of choices or decisions (made in time) under the assumption that the agent has all the information about past and future events and uses all this information in the best possible way to maximize its utility; all this is based on a set of constant fundamental parameters that describe his preferences. For this agent to be irrational it means that he either does have only a subset of the entire information set and does his best to optimize or that he has the entire information set but is not able to process the information correctly, or both conditions are satisfied simultaneously.

In the context of current crisis, it must have taken a large number of irrational decisions from all agents that created the environment (the bubble) for crisis. Then all of sudden some of the agents become rational and did in fact stop supplying additional resources to support irrational choices of the irrational agents, causing the later to default on their financial obligations. What can cause that "on" and "off" rationality behaviour in some agents? Based on the above explanation, this could either result from the extension of the agents set of information to the true set of information or because the agents start to process this information. As long as crisis happens, the agent has to do either the one or the other or both of them simultaneously. *The agent can not remain irrational, even in case he dies he learns the notion of rationality in the last period. With or without his intention he will become rational.*

This intuitive description must fall within the framework of the B&K solution. In order to process the information along the same way as B&K describes in their paper, a model builder will have to build a system of *m* first difference equations where $m \in R$ represents the number of the variables in the system out of which *r* are known in period *t* (the set of predetermined variables in the economy), and the remaining $u = m - r$ are not predetermined variables.

In this $\Re^m$ space there is only one single point (the steady state) and a single line that goes through this point (the steady state path) that can guaranty stability. The condition, defined by Blanchard and Kahn is necessary to put our agent on the steady state or along a steady state path. As long as the agent is on the steady state or on the steady state path, there is no other possibility but to stay there or converge at the steady state equilibrium of the model. Therefore, once there, the rational agent has no way to end up in crisis. In other words, identifying the steady state path and placing the representative agent there the author excludes the possibility of a crisis. Simply in such model, a crisis can not emerge without a foreign disturbance. This foreign disturbance is referred to as a shock. Once the shock is observed, our agent will be taken away from the steady state or steady state path, eventually leading him toward starvation or eating his economy. However, we do not have a mechanism inside the model that explains why, how and when this agent will pull the trigger to start the crisis. Yet, real life data reveal that agents make consistent errors every period. Could it also relate to the fact that agents follow a consistent goal in the long run, but different goals in the short run? Moreover, in spite of these consistent errors they do not usually and frequently end up in crisis. Why exactly is that?

The definition of rationality described above explains how irrational agents get themselves in trouble. In order to do so they would need to jump out of the steady state path. Therefore, in order to introduce instability in the model, the model builders would have to implement a structural shock in the model that can literally create this jump.[9]

[9] This is a mandatory practical solution that provides the necessary mechanisms that allow models to fit the data and explain the crisis.

Basically this is the direction in which several authors are going; currently the trend is to introduce financial instability in the model. Hall (2009) is a good example of this work. However, as in the case of Ireland (2004), these shocks which are usually called financial frictions, are not founded on theoretic arguments as the reasons behind their emergence and size are not well understood. Moreover, to prevent the crisis they have to come back either to the same steady state path or to a new one depending in the nature of the shock. DSGE model and its rational expectation solution do not have a mechanism to explain these patterns of behaviour.

4. Solution of rational expectations models: a discussion of the B&K conditions in practical terms?

It is essential, however, to highlight an important observation, that the formal B&K conditions with regard to the number r of independent vectors is similar to the condition that a VAR process has r cointegrated relationships. In the cointegrated VAR literature, this is the same as saying that the underlying VAR process has as many cointegration relationships as there are predetermined variables in the system, and the rest of the columns $u = m - r$ (the number of predetermined variables) is represented as a linear combination of the first r columns. In other words, the requirement proposed for DSGE solution is not different from a VAR system constrained to have as many cointegration relationships as there are predetermined variables in the system. In addition, it requires that the corresponding coefficients of the loading matrix for the predetermined variables are 0 (meaning that the predetermined variables are weakly exogenous).

We believe that there are two particular elements of the DSGE and in general macro modelling, which have impaired the ability of the models to perform better or enable them to become more realistic or more stretchable, so that they can fit the observed data and economic trends. First is the number and second the nature of constraints that are required to find a unique solution for the system of linear difference that describes the behaviour of the economic variables in the model.

In a standard Gaussian decomposition for the solution of systems of equations the standard procedure takes a system of equations, describing the behaviour of a set of n variables, in the $\Re^n$ space, and projects it in the $\Re^m$ space by means of assumption that the first u variables (where $u = n - m$ with $1 \leq u < n$) are either representation of deterministic constants in $\Re^N$ or a linear combination of any two or more well defined elements in the solution space in $\Re^M$. From here the modeller applies the B&K procedure to solve the model and find the unique solution Z in the form of an *(1 x m)* vector in $\Re^m$, given the condition that first u relationships is given by Ω (in the form of a *1 x u vector)* the set of known constants or linear combination of known constants (in other words identities). Therefore, the solution in $\Re^N$ is given by a X an (1 x n) vector as follows

$$X = Z \cup \Omega \tag{8}$$

Let's discuss a bit the nature of restrictions starting with the notion of rationality in the real life and the way it is portrayed and applied in the DSGE models. As described above, rational models try to optimize given a rational agent, implying that some of the future variables are known at time *t–1*, the rest of the variables are expressed as linear combination of the predetermined yielding solution of the problem at time *t* (now). The solution is guaranteed by the fact that markets clear in period *t* (now) as implied by the budget constraint, which is represented in the form of an identity where income y_t equals the sum of consumption c_t and investment i_t (with the assumption that savings equal investment). This restriction in the form of an identity is consistent with the requirement that prices adjust immediately, as it requires that all markets clear. However, such adjustment is not observed in reality.

This is recognized by the modellers who have proposed the introduction of different market frictions and/or imperfections in the form of sticky prices and wages, intermediate goods market, asymmetries in information etc, in the modelling in order to make models behave as they would in the real world. However, again these are arbitrary decisions forcefully implemented in models. The modelling work again is based on the assumption that markets clear at each discrete moment in time. We, however, observe that it is possible that markets do clear in the long run, but must not do so in the short run.

The intuition is as follows: if output equals the sum of consumption government expenditures and private investments, then finding the value of the last three automatically gives the value of output. If the model builder can figure n-m such identities, then he can reduce the computational burden by reducing the number of estimated coefficients in the model. This is of course an efficient solution but it comes at a huge cost. It requires that the agent is being rational at each period, given that the market clears each period.

However based on our personal experiences we can't help asking the following questions: Is aggregate demand or resource constraint an identity? It surely is in the national accounts from the accounting point of view. By definition, GDP measures everything that is being produced and sold in the markets. However, total production must not be equal each period with what is sold in the markets. This is very similar to measurement errors mentioned by Ireland 2004. It is easily logical to assume that a farmer could misjudge market supply and produces few extra bushels of wheat that he could not deliver in the market; or how for some reasons his tractor broke down at the very end of the period and part of the production is not delivered in the market. These differences between production and market supply will not be accounted as part of GDP and therefore the choice of capital labour and leisure and raw materials used in the production process, given current technology, would not be consistent with the calculated GDP. The same is true, if someone consumes part of the output that is produced by personal resources (consuming a fraction of its own production). This will not show either in the consumption or the GDP statistics, yet given used resources and production function, the figures will not add up to what the model calls rational choice. One can think of many more reasons (exogenous factors, like weather conditions, accidents, etc.) for which the consumption of labour and capital will not be consistent with the reported or forecasted output. Under such circumstances our agent will be pushed away from its equilibrium by random forces outside the system (since they operate in the R^{n+1} with the extra dimension representing for example the weather).

Second even if it is an “identity”, could the errors in budget constraint become intentional? There are several situations under which the agents can intentionally (endogenous factors) generate similar situations as the farmer above. In the real life we have the agent that makes an investment that is a half build factory which was an investment but is not part of the production process yet. In this respect output Y_t and investment I_t are not bound in a uniquely identified relationship as generated by the production function. In a different situation, a consumption smoothing agent might borrow few dollars from the future and consume more than what is possible by at time t, given current resources. Therefore consumption will increase faster than the increase in output and investment which will grow with the same trend as before. In all above examples the level of savings will be different from the level of investment and therefore, an error will emerge in the budget constraint. Our rational agent will intentionally jump away from the steady state or the steady state path. The only way to stay away from crisis our agent must converge to a steady equilibrium (regardless whether is the old equilibrium or a new one) or the corresponding steady state path. Taking another page from the Cointegrated VAR literature, the occurrence of this convergence hints to the existence of an error correcting mechanism that prevents our agent from total catastrophe. In other words, despite moving away from its equilibrium bearing “location”, our agent does not face a crisis (otherwise we could say he is financially stable) because there exists a strong enough ***error correction mechanism*** that brings the agent back to the equilibrium bearing "location".[10]

Usually models have accommodated this consumption smoothing behaviour via introduction of debt and the additional restriction that corrects its outstanding stock and debt service to zero as time $t \rightarrow \infty$. The shocks that are generated in the extra dimensional space $\Re^x$ where $x \in R$ such that $x > n$ and $\Re^n \cap \Re^x = \Re^n$ or $\Re^n \subset \Re^x$, can be approximated by the autoregressive VAR errors, which are used by Ireland's measurement errors. The AR structure is essential for the stability condition of the system.

However, we can introduce these shocks to our one sector, one representative agent economy in the same way that we would introduce a current account deficit, assuming that due to consumption smoothing or external shocks our agent will experience an error (a current account deficit or surplus) in his budget constraint. The sustainability condition of this deficit requires that at the end of time the current account position must be equal to zero.[11] Therefore, assuming that budgetary position is sustainable is equivalent to saying that the budget constraint $y_t = c_t + i_t$ is an identity in the long run, but can experience sustainable deficits/surpluses in the short run. These sustainable deficits/surpluses are not different from an error (shock) ε_t with $E(\varepsilon_t) = 0$ or in the extreme case of permanent sustainable level of debt $E_\infty(\varepsilon_t) =$ to whatever sustainable level. Trehan and Walsh (1981) and Taylor (2002) respectively prove that this is indeed the case.

Intuitively, this would mean that occasionally the agent can increase/decrease c_t or i_t deviating from the steady state trajectory or the steady state path at a given moment t in time with a "***strong commitment"*** that in the next v periods (exactly at time $t+v$) where $1 \leq v \leq \infty$, the agent will sacrifice consumption (slow down consumption) to increase investment in order to return to its equilibrium growth rate implied by the steady state or the steady state path exactly at time $t+v$. Mathematically, the fact that the shocks ε_t are stationary around "*0*" or whatever sustainable non zero level, makes y_t, c_t and i_t cointegrated with restricted long run coefficients equal to 1, and the "strong commitment" an error correction mechanism with v deciding the speed of returning to the long run equilibrium. This procedure of unit root testing as described by Trehan and Walsh (1981)

[10] "Equilibrium Bearing Location" as in the set of all points that represent the steady state path in the phase diagram of the corresponding model.

[11] Following Taylor (2002) the zero condition can become a sustainable level of debt depending on the constant growth rate of the economy as it reaches its steady state equilibrium.

and Taylor (2002) is a standard procedure in testing the sustainability of current account or other similar constrained problems.

In this respect “budgetary deficits” (the discrepancy between savings and investments) and “budgetary surpluses can emerge and disappear during “short” periods of time v in response to a jump from the steady state path in favour of consumption or investment, respectively. In the light of the above discussion, the choice of the modeller of a “budget constraint identity” and the reduced system that results from this assumption is not only more restrictive than the original choice of the representative agent preventing the model to fit the data, but can also yield to incorrect inferences assuming that B&Q are satisfied, while in fact they might be violated, yielding incorrect conclusion on the stable solution.

4.1. New Contribution

Now, if we were to assume that the first u relationship does not represent identities and therefore Ω in the equation *(8)* above does not represent a set of deterministic constants but a set of random variables such that any $\varpi \in \Omega$ is known to be distributed in time as $\mathrm{N}(\mu;\delta^2)$ which would be the case if any of the first *S* relationships were in fact bound in a cointegration relationship with one or more elements of $z \in Z$ rather than an identity, we can still use the same logic above to linearly project our original $\Re^N$ space into the $\Re^M$ space without losing information. In this specific case the rank of the matrix that describes the system in $\Re^n$ is *r* assuming that the rank of the $\Re^m$ is *r*.

From here it would be easy to abstract our $\Re^m$ solution to the identical $\Re^n$ space by augmenting the matrix of the $\Re^m$ space with an additional row and column that will incorporate the cointegration relationship while substituting *Z* for *X* preserving the identical solution to the $\Re^n$ space. So in practical terms any DSGE in a $\Re^n$ space can be solved by first projecting it linearly in the $\Re^m$ space and find the general unique solution of the system in $\Re^m$. Later we can abstract it in $\Re^n$ by augmenting *Z* with Ω. This would not affect the B&K solution as long as the number of stable roots in $\Re^n$ and $\Re^n$ remains the same.

If one were to adopt the definition of identity rather than the cointegration relationship, the context of the B&K condition (or solution) requires only “long run rationality”, rather than “single period rationality”. This formulation makes a considerable difference with the current state of solution since it allows our representative agent to “disregard” some information or “stop” making rational choices in the short run (read each period) without compromising the overall solution of the system in the context of long run rationality. In other words, as long as the B&K conditions are satisfied and the budget constraint is a cointegration relationship, which would be to say that income, consumption and investment share the same stochastic trend, any external shock or irrational choice or the pursuit of a second objective (any deviation from this trend of the variables above) in the short run will be corrected by the ECM back to its equilibrium conditions in the long run. Therefore, for any irrationality that emerges in the system, B&K makes sure that the system corrects it appropriately. Bottom line agents can become irrational in the short run but rational in the long run. This looks closer to the real word. In addition, the very existence of such error correcting mechanism guaranties that whenever the agent will move away any equilibrium or equilibrium bearing position, it (ECM) will start the correction action to bring him back to the sustainable and unique equilibrium. In this respect the existence of ECM will make our agent financially stable, in other words, in the presence of ECM he can not face a crisis in spite of endogenous or exogenous shocks in the system. It is the existence of cointegration relationship among the non predetermined variables that guaranties the presence of the ECM, and hence, the presence of the error correcting mechanism, which will prevent the agent from total catastrophe.

On the other hand, by adopting the definition of identity, one will forego the opportunity to identify possible situations that will violate the very B&K condition in the current rational expectations models. By its virtue as an identity the budget constraint excludes the possibility of the emergence of an additional stochastic trend (e.g. in consumption) in the system and therefore the presence of an additional stable unit root in the matrix. If a separate stochastic or time trend develops in consumption, the DSGE models would not be able to incorporate the new trend, but will however incorporate the resulting shocks as measurement errors. Therefore assuming that, the B&K condition, which is satisfied for the reduced system is simultaneously satisfied for the original system as well, might yield incorrect conclusions regarding the existence of a unique and stable solution for the system.

4.2. Can data say anything about this?

It is reasonable to assume that if the above arguments have any merits, then these problems must be evident in the data. At this point it is very interesting to discuss what Juselius and Franchi (2007) have observed in section 6 of their paper, as they allow the data to speak freely.

The first conclusion that emerges from their empiric investigation of the entire period 1960-2005, is that they observe strong evidence of a structural break in the data at around 1979. This is also supported by the fact that the assumed stable relationships, respectively consumption income and production, behave very differently in the first and second period.

Despite a reasonably well behaved consumption income framework during the first period, different from the prediction of the theoretic model that consumption income ratio must be stationary, the authors find that it exhibits pronounced persistence and needs to be combined with another variable to achieve stability.

The behaviour of consumption-income and capital-labour rations suggests that US investment was primary financed by domestic savings in the first period. There is evidence that during the second period US reliance on foreign savings has increased.

Their results show that there is much more dynamic in the system that what is predicted by the theoretic model with both capital and output equilibrium correcting to income consumption ratio and to savings ratio respectively, in the first period. These dynamics increase in the second period to include a significant adjustment of labour supply to consumption-income relation. In particular they find that shocks to consumption are one of the main driving forces in the system, which is totally at odds with the assumptions of the theoretic models that assign this role to total factor productivity. In addition, shocks seem to be more demand than supply driven, again at odds with theoretic predictions that assign a leading role to supply; while consumption must adjust, otherwise its explosive root will make the system unstable. In the first period the equilibrium correcting behaviour of capital to both savings rate and production function adds to the stability of the system, whereas the poor and inadequate adjustment toward equilibrium is a dominant characteristic of the second period.

Based on all this evidence Juselius and Franchi (2007) conclude that DSGE tells a “structural” story but with very little empirical content. To us it seems, however, that the existence of a cointegration relationship in the budget constraint might accommodate at least some of the observed inconsistencies.

5. Financial Stability, Monetary Policy and Their Interaction

The recent crisis took us all by surprise and did in fact leave a big puzzle behind. While today we have a good understanding of the phenomenon and events that lead to the crisis, it is evident that we do not have a clear understanding why such financial imbalances grew unnoticed by the authorities that are mandated to safeguard financial stability. Despite the existing early warning systems in place, it is clear that authorities and mainstream academics did not have a universal, updated, and practical framework and a suitable set of tools to identify the emergence of financial stability issues in the presence of financial innovation. Since the crisis, the central banks which are mandated with the objective of price stability are also required to define and explicitly or implicitly include financial stability in the set of objectives. Providing a definition and incorporating it in the framework of policy analysis and decision-making, is however proving very hard.

Well, the way in which the banking community and academics are dealing with the possible and suggested solutions to financial stability problem, shows how difficult it is to define it. Currently, we are calling for more regulations, meaning either impose new rules or tighten the existing ones. Either one or the other converges to some quantitative measures of financial sector indicators, capital, total liabilities and/or assets, etc., just to name a few. This way of thinking portrays financial stability more as a scale problem within the banking supervision area rather than a broader and deeper phenomenon that relates to agents' rational choices. However this does not address one of the fundamental problems of the crisis. Gordon (2009) observes: "Blinders' first finger points to the core of the initial problem, gullible consumers who signed up for mortgages that they did not understand and could not afford" (pp.5). This is an important aspect of financial stability that relates to the economic behaviour and incentives of the supposedly rational agents, but that is not getting much attention. It is sure a reminder that understanding and addressing the problems of financial sector is only part of the solution. The same is happening in modelling. None originates from the micro fundamentals that characterize the agent's behaviour. Is financial stability a quantitative element modelled as indicative percentage or ration of financial sector balance sheet or a qualitative element founded in the microeconomic behaviour? Answering this question correctly might be the first step toward the correct definition.

One way to think of this question is to approach the problem in a different way. Assume that we do not have a financial system in the picture, meaning in a model without financial system. Can financial instability arise in a model like this? If we were to define financial stability by the ability of the agent to stay in the steady state, or steady state path than in the presence of random exogenous and endogenous shocks to non predetermined variables, we would observe instability taking over and the agent would either starve to death or eat the entire economy. In other words, this behaviour would be unstable. Therefore, instability can emerge in the model without the presence of the financial sector. In the absence of a correcting mechanism every exogenous or endogenous shock would end up in crisis, in other words the agent would be financially unstable. Since the answer to the question above is yes, then definitively, the financial stability is not a scale problem within the area of banking or financial supervision, but a totally different much broader problem that originates at the micro behaviour.

This pursue of scale, ratios and frictions are probably one of the reasons why the highly stylized micro-based rational models were not useful in the detection of mounting problems in the financial stability or in the understanding of the mechanisms behind it. To this day, researchers are still struggling to identify shocks, understand them and justify their artificial integration in a reasonably acceptable way

What is of most importance it seems that we do not have a consensus on the role and incentives that monetary policy could have played in the current financial crisis. In fact we do not have a framework to study this relationship among the areas of monetary policy, banking supervision and financial stability. Financial frictions and other shocks that researchers are incorporating their models are not well understood and therefore not related to monetary policy.

Yet we know very well that monetary policy has a strong and deep effect on agent's economic decision-making and its economic incentives. As the interest rates go south, people are more prone to borrow and increase their consumption and investments individually or simultaneously will exceed current output. Short run budget constraints will be broken, but will still be sustainable as long as everybody is willing to correct these deficits in the future, in other words, as long as everybody has an inner commitment in the error correction mechanism to correct this short term excesses to the point where they are financially stable in the long run. This interaction becomes more obvious in the case when a financially stable agent has opened a financial gap with a strong commitment to close it in the next v periods. Under such circumstances, a monetary policy rate change will affect the outstanding debt and its service and therefore the size of the existing financial gap will also change. This change will affect the agent's ability to close it in the next v periods and will potentially jeopardize the financial stability situation of the agent. It is due to this fact that we propose to define and model financial stability with the existence of an error correcting mechanism in the agents' behaviour, which would imply that income, consumption and investment are bound together in a cointegration relationship in the long run. The suggested definition provides a mechanism to link financial stability with the general framework of monetary policy and its decision-making.

6. Conclusion

This paper combines two important and contemporary topics of economics and policy making, the issues that surround the financial stability and macroeconomic modelling. Both are discussed form the point of view of rational expectations. Rational expectations have played an important role in understanding and application of economics and economic policies in the real world. Despite its appeal the models that are built in this framework, are highly stylized to fit the data and accurately represent their generation process. In addition, current rational expectation models suffer from the absence of a reliable framework that deals with the problems of financial stability. Therefore, such models are being improved in several directions.

The literature is developing in several directions to include financial frictions, structural errors, etc. However, one important feature is that in order to incorporate financial stability, all these models need the presence of financial market in the models in one way or the other. Different from this research, we have adopted the view that financial stability is an economic phenomenon that can arise in the absence of financial market in the model.

This paper discusses the problems of financial stability and modelling from the point of view of rational expectations. After analyzing the structure of DSGE model and the B&K condition for solution of rational expectation models, we propose to transform the conditions of budget constraint from an identity into a cointegrated relationship.

Taking these two conditions together we have the opportunity to relax the budget constraint moving from a discrete one to a continuous one, assuming that it has to hold in the long run rather than each period. This will permit representative agent to endogenously generate shocks to consumption, and/or investment, and accommodate external shocks in the short run, without compromising the stable equilibrium in the long run. This is achieved by the presence of the error correction mechanism in the model.

The advantage of such proposed solution in this paper toward the one presented by other authors is that here we try to develop a method which can introduce endogenous shocks in the model innovation to the hybrid model that can bring model to the data based on reasonable economic theory and not on convenient measurement errors and unexplained structural shocks that are implemented in the models without much content. The proposed solution is able to accommodate several “odd” trends that are observed in the data by Juselius and Franchi (2007)

While this process is more realistic and provides a reasonable and formal way to introduce errors in the DSGE not just as measurement errors, it represents an opportunity to naturally incorporate (y, c, i) shocks into the model. This will allow models to fit the observed trends in the data unmatched by theoretic models. Assuming a cointegration relationship, the model builder can attach the observed trend to any of the variables allowing the other two to adjust, simply by shifting the position of the “cointegration bearing row” to the state matrix of the model, so that it corresponds to the desired variable in the vector of explanatory variables.

However, the main element of this proposed solution relates to the fact that the cointegration structure provides a framework for the definition of Financial Stability, relating it to the rational behaviour of the representative agent. Most importantly, it outlines the principles that show how monetary policy interacts with and influences of financial stability.

The bottom line is that different from the current practice, the original solution given by B&Q allows irrationalities in the short run without affecting rationality assumption in the long run. Therefore, the proposal here is to substitute this budget constraint with a long run relationship in the form of identity, meaning that the budget constraint is satisfied in the long run as an identity not necessarily every period. For all the period during which the intra-temporal constraint is not satisfied, the error correcting mechanism will bring it back to its own equilibrium. This means that each time when the agent makes one or few irrational choices in the short run, there exists an ECM, which brings him back to the equilibrium steady state path, making him financially stable.

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