Matching efficiency and business cycle fluctuations*

Francesco Furlanetto†

Norges Bank

Nicolas Groshenny‡

Reserve Bank of New Zealand and CAMA

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Abstract

A large decline in matching efficiency has been documented during the Great Recession in the US. We study the macroeconomic consequences of fluctuations in the matching efficiency in a simple New Keynesian model. We show that the transmission mechanism depends crucially on the form of the hiring cost function and on the presence of nominal rigidities. The same features are also crucial to determine the slope of the Beveridge curve conditional on shocks to the matching efficiency.

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†Address: Norges Bank, Bankplassen 2, PB 1179 Sentrum, 0107 Oslo, Norway. E-mail: francesco.furlanetto@norges-bank.no. Telephone number: +47 22316128.

‡Address: Economics Department, Reserve Bank of New Zealand, 2 The Terrace, PO Box 2498, Wellington, New Zealand. *email address:* nicolas.groshenny@rbnz.govt.nz.

1 Introduction

Between 2009-Q3 and 2010-Q4 the US labor market has been characterized by an increase in the vacancy rate by 20 per cent whereas the unemployment rate has not decreased at all. This fact can simply reflect insufficient aggregate demand and be part of the painful adjustment to a large negative shock like the recent Great Recession or it can be due to an outward shift in the Beveridge curve caused by structural factors. In particular, some policy-makers have related the absence of a decrease in unemployment to a less efficient matching process in the labor market (cf. Bernanke, 2010, Kocherlakota, 2010, Evans, 2010 among others for an overview on the debate). This view has received some support from recent empirical work by Barnichon and Figura (2011b) who find that a large decline in matching efficiency added 1.5 percentage points to the unemployment rate during the Great Recession.

Fluctuations in matching efficiency can be interpreted as variations in the degree of search and matching frictions in the labor market and reflect all the hiring behavior that cannot be explained by the stocks of unemployment and vacancies. Unemployment, vacancies, matching efficiency and hiring behavior are usually related through the aggregate matching function, one of the building blocks of models with search and matching frictions in the labor market (Blanchard and Diamond, 1989 and Petrongolo and Pissarides, 2001). When matching efficiency is very low, for given stocks of unemployment and vacancies, few new matches will be created. The opposite is true when matching efficiency is high. Barnichon and Figura (2011a) have estimated the aggregate matching function for the US over the period 1976-2010 by using data on the job finding rate and the labor market tightness. The regression residual, that represents fluctuations in matching efficiency, is relatively stable over time with the exception of the recent Great Recession when the matching efficiency is at historically low levels.[1]

Several factors could explain a lower degree of matching efficiency: skill mismatch (cf. Sahin, Song, Topa and Violante, 2011 and Herz and van Rens, 2011), geographical mismatch, possibly exacerbated by house-locking effects (cf. Nenov, 2011), reduction in search intensity by workers because of extended unemployment benefits (cf. Kuang and Valletta, 2010), reduction in firm recruiting intensity (cf. Davis, Faberman and Haltiwanger, 2010), shifts in the composition of the unemployment pool due, for example, to a larger share of long-term unemployment or to a larger share of permanent layoffs (cf. Barnichon and Figura, 2011a).

Importantly, in the framework of the aggregate matching function, matching efficiency has the same interpretation of the Solow residual in the context of the production function. Therefore, shocks to the matching efficiency play the same role of technology shocks in the production function and can be interpreted as structural shocks in modern business cycle models. However, while the literature has devoted a substantial effort to study the properties of technology shocks, little is known on the effects of shocks to the matching efficiency. This paper aims at filling this gap by providing a careful analysis of the transmission mechanism for shocks to the matching efficiency in the context of a very simple New Keynesian model with search and matching frictions in the labor market.[2]

Two contributions emerge from our analysis. First, the propagation of shocks to the matching efficiency depends crucially on the form of the hiring cost function.

[1] A substantial decline in matching efficiency during the Great Recession is documented also by Barlevy (2011), Borowczyk-Martins, Jolivet and Postel-Vinay (2011), Elsby, Hobijn and Sahin (2010) and Veracierto (2011). Notice that the large decline in matching effciency is a feature specific to the Great Recession. According to Barnichon and Figura (2011a), in fact, matching efficiency has increased in previous post-war recessions, and not decreased. Countercyclical matching efficiency is consistent with the theory developed by Michaillat (2010) that search frictions matter little in recessions.

[2] The use of search and matching frictions in business cycle models was pionereed by Merz (1995) and Andolfatto (1996) in the Real Business Cycle literature. More recently, the same labor market frictions have been studied in the New Keynesian model by Christiano, Trabandt and Walentin (2011), Gertler and Trigari (2008), Gertler, Sala and Trigari (2008), Groshenny (2009 and 2010), Krause and Lubik (2007), Krause, Lubik and López Salido (2008), Ravenna and Walsh (2008), Sveen and Weinke (2008 and 2009), Trigari (2006 and 2009) and Walsh (2005) among many others.

When we consider post-match hiring costs, in the form of training costs as in Gertler and Trigari (2008), we show analytically that the shock does not even propagate and unemployment is invariant to fluctuations in matching efficiency. Given that in the data post-match hiring costs happens to be the main component of total hiring costs (cf. Pissarides, 2009, Silva and Toledo, 2009 and Yashiv, 2000), our analysis seems to indicate a rather limited role for shocks to the matching efficiency in explaining business cycle fluctuations. When we consider pre-match hiring costs, in the form of linear costs of posting a vacancy as in Pissarides (2000), the shock propagates and unemployment declines in response to a positive impulse. However, the importance of these shocks is limited by the fact that they imply a large positive correlation between unemployment and vacancies whereas it is well known that this correlation is strongly negative in the data. Therefore, shocks to the matching efficiency cannot be a main driver of unemployment fluctuations although they can be seen as shifters of the Beveridge curve.

The second contribution of this paper is to show that when matching efficiency shocks propagate, i.e. under pre-match hiring costs, the presence of nominal rigidities is crucial for the transmission mechanism. In fact, the response of vacancies can be positive or negative depending on the presence of nominal rigidities in the model. The sign of the vacancy response is important because it is crucial to determine the slope of the Beveridge curve conditional on matching efficiency shocks. We show that when nominal rigidities are present, as in our baseline model, vacancies decrease and the conditional Beveridge curve has a positive slope. When prices are flexible, instead, vacancies increase and the conditional correlation between unemployment and vacancies declines substantially and can even become negative when the shock has limited persistence. Interestingly, nominal rigidities are also a feature that determine the sign of the employment/hours worked response to a technology shock (cf. Basu, Fernald and Kimball, 2006, Christiano, Eichenbaum and Vigfusson, 2003, Galí, 1999 and McGrattan, 2005 among many others).[3] Importantly, we show

[3]See also Francis and Ramey, 2005 for an alternative mechanism based on real rigidities (habit

analytically that the features that induce a negative response of employment/hours worked to a positive technology shock, imply also a negative response of vacancies to a positive matching efficiency shock.

Shocks to the matching efficiency were already present in the seminal paper by Andolfatto (1996) that introduced search and matching frictions in the standard RBC model. Since then, these shocks have been considered also in Arsenau and Chugh (2007), Krause, Lubik and Lopez-Salido (2008), Lubik (2009), Cheremukhin and Restrepo-Echevarria (2011), Justiniano and Michelacci (2011) and Mileva (2011). However, none of these papers relate the shock to the matching efficiency to the form of the hiring cost function, nor to the degree of nominal rigidities and to the slope of the Beveridge curve. Importantly, our theoretical analysis of the transmission mechanism can in part reconcile very different results on the importance of matching efficiency shocks that explain 92% of unemployment fluctuations in Lubik (2009), 37% in Krause, Lubik and Lopéz-Salido (2008) and only 11% in Michelacci and López-Salido (2011).

Our paper is also related to the literature on the importance of reallocation shocks for business cycle fluctuations initiated by Lilien (1982). Shocks to the matching efficiency, in fact, can be considered as reallocation shocks, at least as long as they capture some form of mismatch (in skills, in geography or in other dimensions), as argued in Andolfatto (1996) and Pissarides (2011). Abraham and Katz (1986) suggested that reallocation shocks play a limited role in explaining aggregate fluctuations because they imply a positive correlation between unemployment and vacancies (unlike aggregate demand shocks). However, that argument was not based on a general equilibrium analysis. Here, we confirm the statement by Abraham and Katz (1986) in the context of our New Keynesian model but we show that the slope of the conditional Beveridge curve can become negative when prices are flexible and the shock has low persistence.

persistence and capital adjustment costs) that can deliver a negative response of hours even in a Real Business Cycle (RBC) model.

The paper proceeds as follows: Section 2 describes briefly the model, section 3 presents our results, section 4 relates our results to the literature and section 5 concludes and offers an outline of our ongoing research.

2 The model

The model economy consists of a representative household, a continuum of wholesale goods-producing firms, a continuum of monopolistically competitive retail firms, and monetary and fiscal authorities which set monetary and fiscal policy respectively. The model is deliberately simple. We ignore features like capital accumulation, real rigidities (like habit persistence and investment adjustment costs) and wage rigidities. We include all these features in a companion paper (Furlanetto and Groshenny, 2011) where we estimate a more empirically plausible version of our model to study the evolution of the natural rate of unemployment. Based on the results from our companion paper, we can safely concentrate only on the features that are critical for the transmission of matching efficiency shocks and ignore the unnecessary complications. Our model is very similar to Kurozumi and Van Zandweghe (2010) in the version with pre-match hiring costs and is a simplified version of Gertler, Sala and Trigari (2008) in the version with post-mtch hiring costs.

The representative household There is a continuum of identical households of mass one. Each household is a large family, made up of a continuum of individuals of measure one. Family members are either working or searching for a job.[4] Following Merz (1995), we assume that family members pool their income before allowing the head of the family to optimally choose per capita consumption.

The representative family enters each period $t = 0, 1, 2, ...,$ with B_{t-1} bonds. At the beginning of each period, bonds mature, providing B_{t-1} units of money. The representative family uses some of this money to purchase B_t new bonds at nominal

[4] The model abstracts from the labour force participation decision.

cost B_t/R_t, where R_t denotes the gross nominal interest rate between period t and $t+1$.

Each period, N_t family members are employed. Each employee works a fixed amount of hours and earns the nominal wage W_t. The remaining $(1-N_t)$ family members are unemployed and each receives nominal unemployment benefits b, financed through lump-sum nominal taxes T_t. Unemployment benefits b are proportional to the steady-state nominal wage: $b = \tau W$. During period t, the representative household receives total nominal factor payments $W_t N_t + (1-N_t)\, b$ as well as profits D_t. The family purchases retail goods for consumption purposes.

The family's period t budget constraint is given by

$$P_t C_t + \frac{B_t}{R_t} \leq B_{t-1} + W_t N_t + (1-N_t)\, P_t b - T_t + D_t. \tag{1}$$

where C_t represents a Dixit-Stiglitz aggregator of retail goods and P_t is the corresponding price index.

The family's lifetime utility is described by

$$E_t \sum_{s=0}^{\infty} \beta^s \ln C_{t+s} \tag{2}$$

where $0 < \beta < 1$.

The representative intermediate goods-producing firm Each intermediate goods-producing firm $i \in [0,1]$ enters in period t with a stock of $N_{t-1}(i)$ employees. Before production starts, $\rho N_{t-1}(i)$ old jobs are destroyed. The job destruction rate ρ is constant. The workers who have lost their job start searching immediately and can possibly still be hired in period t (cf. Ravenna and Walsh, 2008). Employment at firm i evolves according to $N_t(i) = (1-\rho)\, N_{t-1}(i) + M_t(i)$ where the flow of new hires $M_t(i)$ is given by $M_t(i) = Q_t V_t(i)$. $V_t(i)$ denotes vacancies posted by firm i in period t and Q_t is the aggregate probability of filling a vacancy defined as $Q_t = \frac{M_t}{V_t}$. Similarly, the job finding rate (F_t) is defined as $F_t = \frac{M_t}{V_t}$

$M_t = \int_0^1 M_t(i)\,di$ and $V_t = \int_0^1 V_t(i)\,di$ denote aggregate matches and vacancies respectively. Aggregate employment $N_t = \int_0^1 N_t(i)\,di$ evolves according to

$$N_t = (1-\rho)\,N_{t-1} + M_t. \tag{3}$$

The matching process is described by an aggregate constant-returns-to-scale Cobb Douglas matching function

$$M_t = L_t S_t^{\sigma} V_t^{1-\sigma}, \tag{4}$$

where S_t denotes the pool of job seekers in period t

$$S_t = 1 - (1-\rho)\,N_{t-1}. \tag{5}$$

and L_t is a time-varying scale parameter that captures the efficiency of the matching technology. It evolves exogenously following the autoregressive process

$$\ln L_t = \rho_L \ln L_{t-1} + \varepsilon_{Lt}, \tag{6}$$

where ρ_L measures the persistence of the shock and ε_{Lt} is $i.i.d.N\left(0, \sigma_L^2\right)$.

Aggregate unemployment is defined by $U_t \equiv 1 - N_t$.

Newly hired workers become immediately productive. Hence, the firm can adjust its output instantaneously through variations in the workforce. However, firms face hiring costs, measured in terms of the finished good $\left(H_t^k(i)\right)$ where k is an index to distinguish the two kinds of hiring costs that we consider.

The first specification is a post-match hiring cost $\left(H_t^{post}(i)\right)$ in which total hiring costs are given by

$$H_t^{post}(i) = \frac{\phi_N}{2}\left[\frac{Q_t V_t(i)}{N_t(i)}\right]^2 N_t(i). \tag{7}$$

The parameter ϕ_N governs the magnitude of the post-match hiring cost. This kind of adjustment cost was used by Gertler and Trigari (2008) because it makes possible the derivation of the wage equation with staggered contracts and helps the model fit the persistence and the volatility of unemployment and vacancies that we observe in the data (Pissarides, 2009). Since then, this feature has become standard in the empirical literature (cf. Christiano, Trabandt and Walentin, 2011, Gertler, Sala and Trigari, 2007, Groshenny, 2009 and 2011, Sala, Söderstöm and Trigari, 2008). The post-match hiring cost can be interpreted as a training cost: it reflects the cost of integrating new employees into the employment pool.

The second specification that we consider is the hiring cost that is commonly used in the literature on search and matching frictions (Pissarides, 2000). Following the classification in Pissarides (2009), it is a pre-match hiring cost ($H_t^{pre}(i)$) and it represents the cost of posting a vacancy. We use a standard linear specification that reads as follows

$$H_t^{pre}(i) = \phi_N V_t(i)$$

The parameter ϕ_V governs the magnitude of the pre-match hiring cost.

Each period, firm i uses $N_t(i)$ homogeneous employees to produce $Y_t(i)$ units of intermediate good i according to the constant-returns-to-scale technology described by

$$Y_t(i) = A_t N_t(i). \tag{8}$$

A_t is an aggregate labor-augmenting technology shock that follows the exogenous stationary stochastic process

$$\ln(A_t) = (1-\rho_A)\ln(A) + \rho_A \ln(A_{t-1}) + \varepsilon_{At}, \tag{9}$$

where ε_{At} is $i.i.d.N\left(0,\sigma_A^2\right)$.

Each wholesale goods-producing firm $i \in [0,1]$ chooses employment and vacancies to maximize profits and sells its output $Y_t\left(i\right)$ in a perfectly competitive market at a relative price $Z_t(i)$. The firm maximizes

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s+1}}{\Lambda_{t+s}} \left(Z_{t+s}(i) Y_{t+s}\left(i\right) - \frac{W_{t+s}\left(i\right)}{P_{t+s}} N_{t+s}(i) - H_{t+s}^k(i) \right)$$

Wage setting $W_t\left(i\right)$ is determined through bilateral Nash bargaining,

$$W_t\left(i\right) = \arg\max \left[\Delta_t\left(i\right)^{\eta} J_t\left(i\right)^{1-\eta} \right]. \tag{10}$$

where $0 < \eta < 1$ represents the worker's bargaining power. The worker's surplus, expressed in terms of final consumption goods, is given by

$$\Delta_t\left(i\right) = \frac{W_t\left(i\right)}{P_t} - \frac{b}{P_t} + \beta E_t \left[\left(1-\rho\right)\left(1-F_{t+1}\right) \right] \left(\frac{\Lambda_{t+1}}{\Lambda_t} \right) \Delta_{t+1}\left(i\right). \tag{11}$$

The firm's surplus in real terms is given by

$$J_t\left(i\right) = Z_t\left(i\right) A_t - \frac{W_t\left(i\right)}{P_t} + \frac{\partial H_t^k(i)}{\partial N_t(i)} + \beta\left(1-\rho\right) E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} J_{t+1}\left(i\right) \right]. \tag{12}$$

Retail firms There is a continuum of retail goods-producing firms indexed by $j \in [0,1]$ that transform the wholesale good (bought at price Z_t, which is common across wholesale goods-producing firms) into a final good $Y_t^f\left(j\right)$ that is sold in a monopolistically competitive market at price $P_t\left(j\right)$. Demand for good j is given by $Y_t^f(j) = C_t(j) = (P_t(j)/P_t)^{-\theta} C_t$ where θ represents the elasticity of substititution across final goods. Firms choose their price subject to a Calvo (1983) scheme in which every period a fraction α is not allowed to re-optimize whereas the remaining fraction $1-\alpha$ chooses its price by maximizing the following discounted sum

$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \frac{\Lambda_{t+s}}{\Lambda_t} \left(\frac{P_t(j)}{P_{t+s}} - Z_{t+s} \right) Y_{t+s}^f (i)$$

Monetary and fiscal authorities The central bank adjusts the short-term nominal gross interest rate R_t by following a Taylor-type rule

$$\ln\left(\frac{R_t}{R}\right) = \rho_r \ln\left(\frac{R_{t-1}}{R}\right) + (1-\rho_r)\left[\rho_\pi \ln(\Pi_t) + \rho_y \ln(Y_t/Y_{t-1})\right], \tag{13}$$

where $\Pi_t = P_t/P_{t-1}$. The degree of interest-rate smoothing ρ_r and the reaction coefficients ρ_π and ρ_y are all positive.

The government budget constraint is of the form

$$(1-N_t)\, b = \left(\frac{B_t}{R_t} - B_{t-1}\right) + T_t, \tag{14}$$

3 Results

Table 1 reports the calibration of the model parameters for the US economy.

Table 1: Calibrated parameters

Discount rate	β	0.99
Elasticity of substitution between goods	θ	11
Interest rate smoothing	ρ_r	0.8
Response to inflation in the Taylor rule	ρ_π	1.5
Response to output growth in the Taylor rule	ρ_y	0.5
Calvo coefficient for price rigidity	α	0.75
Probability to fill a vacancy within a quarter	Q	0.7000
Separation rate	ρ	0.1
Unemployment rate	U	0.06
Unemployment benefits	τ	0.4
Pre-match hiring cost parameter	ϕ_V	1.0061
Post-match hiring cost parameter	ϕ_N	1.0144
Matching shock persistence	ρ_L	0.7
Technology shock persistence	ρ_A	0.7

The first set of parameters is standard in the monetary policy literature. The discount factor is set at $\beta = 0.99$, the elasticity of substitution final goods at $\theta = 11$ implying a steady-state markup of 10 percent. The parameters in the monetary policy rule are $\rho_r = 0.8$, $\rho_\pi = 1.5$, $\rho_y = 0.5$. The average degree of price duration is 4 quarters, corresponding to $\alpha = 0.75$.

The second set of parameter values is standard in the literature on search and matching in the labor market. The vacancy-filling rate Q is set equal to 0.70, the degree of exogenous separation is $\rho = 0.1$, the steady-state value of the unemployment rate is $U = 0.06$. The parameter τ that governs the value of unemployment benefits is set equal to 0.4 whereas the elasticity in the matching function is $\sigma = 0.5$. The two remaining parameters, the one that governs the size of hiring costs (ϕ_V or ϕ_N) and the degree of bargaining power of workers η, are linked by steady state conditions. Given the lack of convincing empirical evidence on the value of η, we set ϕ_V (or ϕ_N) such that hiring costs are equal to one percent of steady state output which implies that η is set around 0.9. This choice avoids indeterminacy issues that are widespread in this kind of models, as shown by Krause and Lubik (2010) and Kurozumi and Van Zandweghe (2010). Finally, the degree of persistence for the shock processes is set at 0.7.

The log-linear first order conditions that do not depend on the form of the hiring cost function are listed in Table 2:

Table 2: equilibrium equations

Euler equation	$c_t = E_t c_{t+1} - (r_t - E_t \pi_{t+1})$	(T 1)
production function	$y_t = a_t + n_t$	(T 2)
law of motion for employment	$n_t = (1-\rho)\, n_{t-1} + \rho(q_t + v_t)$	(T 3)
Definition of unemployment	$u_t = -\left(\frac{N}{U}\right) n_t$	(T 4)
Probability of filling a vacancy	$q_t = l_t - \sigma\left(v_t + \left(\frac{(1-\rho)N}{S}\right) n_{t-1}\right)$	(T 5)
Job finding rate	$f_t = l_t + (1-\sigma)\left(v_t + \left(\frac{(1-\rho)N}{S}\right) n_{t-1}\right)$	(T 6)
Definition of the hiring rate	$x_t = q_t + v_t - n_t$	(T 7)
New Keynesian Phillips curve	$\pi_t = \beta E_t \pi_{t+1} + \kappa z_t$	(T 8)
Monetary policy rule	$r_t = \rho_r r_{t-1} + (1-\rho_r)\left(\rho_\pi \pi_t + \rho_y (y_t - y_{t-1})\right)$	(T 9)
Matching efficiency shock	$l_t = \rho_L l_{t-1} + \epsilon_{L,t}$	(T 10)
Technology shock	$a_t = \rho_A a_{t-1} + \epsilon_{A,t}$	(T 11)

We define x_t as the hiring rate, the ratio between new matches and employment.

3.1 Matching efficiency shocks and post-match hiring costs

In this section we look at the transmission mechanism for the shock to the matching efficiency when the hiring cost is in the form of a training cost, as in Gertler and Trigari (2008).

In table 3 we report the three loglinearized first order conditions that depend on the form of the hiring cost function (the job creation condition, the wage equation and the market clearing condition):

Table 3: additional equations for the model with post-match hiring cost

$$x_t = -\left(\frac{W}{\phi_N \rho(1-2\rho)P}\right) rw_t + \left(\frac{Z}{\phi_N \rho(1-2\rho)}\right)(z_t + a_t) - \frac{\beta(1-\rho)}{(1-2\rho)}\left(i_t - E_t\pi_{t+1} + x_{t+1}\right) \qquad \text{(T 12)}$$

$$rw_t = \left(\frac{\eta Z P}{W}\right)(z_t + a_t) + \left(\frac{\eta 2\phi_N \rho^2 P}{W}\right) x_t - \left(\frac{\eta\beta(1-\rho)\phi_N F\rho P}{W}\right)\left(i_t - E_t\pi_{t+1} + E_t x_{t+1} - E_t f_{t+1}\right) \qquad \text{(T 13)}$$

$$(1-)\, y_t = \left(1 - \frac{\phi_N \rho^2}{2}\right) c_t + \phi_N \rho^2 x_t + \frac{\phi_N \rho^2}{2} n_t \qquad \text{(T 14)}$$

Impulse responses in figure 1 show that only vacancies and the probability of filling a vacancy react to the shock. A positive shock to the matching efficiency makes it easier to fill a vacancy because the job market is more efficient (q_t increases) but firms react by posting less vacancies (v_t decreases). Importantly, with post-match hiring costs the response of the two variables is of the same magnitude. This implies that employment does not react (see T.3) and, in turn, unemployment and output are also invariant to the shock (see T.4 and T.2). All the variables unrelated to the matching process are invariant to the matching efficiency shock or, in other words, the shock does not propagate.

This neutrality result hinges on the form of the hiring costs function. In a model with post-match hiring decision the choice variable for firms is the hiring rate (x_t). Vacancy positing, which is now costless, is determined residually from the matching function equation, once the decision on hiring has been made. This point can be

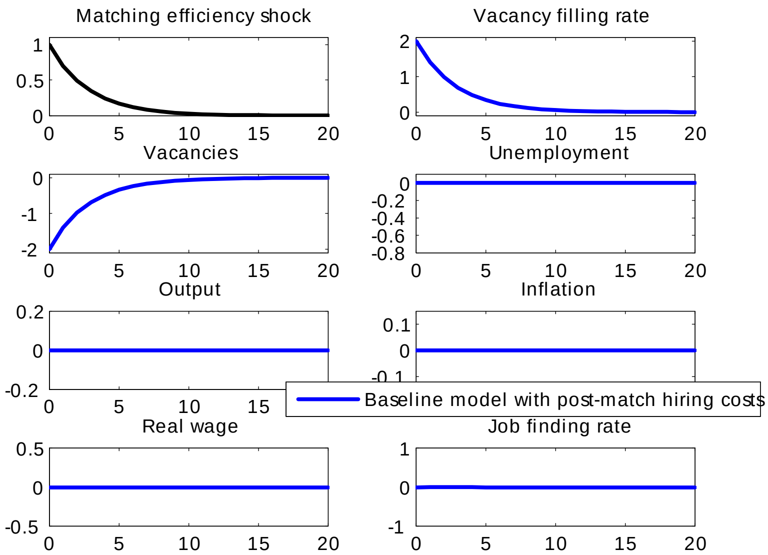

Figure 1: Impulse-responses in the model with pre-match hiring cost

seen analytically by using the list of equilibrium conditions in table 2 and 3. By substituting T7 into T3, we obtain

$$n_t = n_{t-1} + \frac{\rho}{1-\rho} x_t \tag{15}$$

and by substituting T.5, T.6 and T.7 into T.13, we have

$$rw_t = \left(\frac{\eta Z P}{W}\right)(z_t + a_t) + \left(\frac{\eta 2 \phi_N \rho^2 P}{W}\right) x_t - \left(\frac{\eta \beta (1-\rho) \phi_N F \rho P}{W}\right)\left(r_t - E_t \pi_{t+1} - E_t n_{t+1} - \frac{(1-\rho)\,\ldots}{1-(1-\rho\ldots}\right. \tag{16}$$

In the system of 9 equilibrium conditions (from T1 to T4, T8, T9, T12, T14, 15,16) with 9 endogenous variables, q_t, f_t and v_t never appear. Therefore, that block of equations is not affected by how the matching function is specified. More specifically, unemployment dynamics are invariant to shocks to the matching efficiency and to different values of the elasticity in the matching function (σ). q_t, s_t and v_t are determined residually by T5, T6 and T7.[5]

To sum-up, our model predicts that the larger is the importance of post-match hiring costs in total hiring costs, the lower is the propagation of shocks to the matching efficiency. Importantly, Silva and Toledo (2009) and Yashiv (2000) have looked at the importance of post-match hiring costs in the data. Both studies find that post-match hiring costs are substantial, accounting for at least 70 percent of total hiring costs. The same result is confirmed in an estimated New Keynesian model for Sweden by Christiano, Trabandt and Walentin (2011). Therefore, according to our analysis, given that the post-match component is dominant in the data, we should expect a very limited role for shocks to the matching efficiency in explaining business cycle fluctuations.

3.2 Matching efficiency shocks and pre-match hiring costs

In this section we look at the transmission mechanism for the shock to the matching efficiency when the hiring cost is in the form of a linear cost of posting a vacancy, as it is standard in the literature on search and matching frictions in the labor market (Pissarides, 2000).

In table 4 we report the three loglinearized first order conditions that depend on the form of the hiring cost function:

[5] This point was brought to our attention by Larry Christiano in a private conversation few years ago. The same concept is expressed in a note written by Thjis Van Rens (2008), who also refers to a conversation with Larry Christiano. The note is available at http://www.crei.cat/~vanrens/notes_comments/Gertler_Trigari_comment.pdf. At that time the point was relevant to understand why unemployment volatility was higher in the model by Gertler and Trigari (2008) rather than in standard search and matching models and there was no discussion on shocks to the matching efficiency.

Table 4: additional equations for the model with pre-match hiring cost

$q_t = \left(\frac{WQ}{P\phi_V}\right) rw_t - \left(\frac{ZQ}{\phi_V}\right)(z_t + a_t) + \beta(1-\rho)(r_t - E_t\pi_{t+1} + E_t q_{t+1})$	(T 15)
$rw_t = \left(\frac{\eta ZP}{W}\right)(z_t + a_t) - \left(\frac{\eta\beta(1-\rho)\phi_V FP}{WQ}\right)(r_t - E_t\pi_{t+1} + E_t q_{t+1} - E_t f_{t+1})$	(T 16)
$y_t = \left(1 - \frac{\phi_V V}{N}\right) c_t + \frac{\phi_V V}{N} v_t$	(T 17)

In figure 2 we plot impulse responses to a matching efficiency shock and we see that it propagates, unlike in the model with post-match hiring costs. A positive shock implies that the labor market is more efficient at matching workers and firms and, in fact, the probability of filling a vacancy and the probability of finding a job both increase. This expands the production possibilities in the economy, unemployment decreases and output increases.

We can understand why the shocks propagates under pre-match hiring costs by looking at the job creation conditions T.12 and T.15. In a model with pre-match hiring costs, the average cost of hiring a workers includes a component that depends on the expected duration of a vacancy, that itself depends on labor market tightness, which is taken as given by the firm. In a model with post-match hiring costs, instead, the average cost of hiring a worker does not depend on labor market tightness but only on the hiring rate which is a firm-specific variable. In a model with pre-match hiring cost search frictions imply a congestion externality in the job creation condition whereas in a model with post-match hiring costs search frictions are not active and the model is equivalent to a model with quadratic employment adjustment costs.

Importantly, even though it is easier to fill in a vacancy, firms react by posting less vacancies, as in the model with post-match hiring costs. This fact reminds to us the debate on the response of employment/hours worked to a positive technology shock in the standard New Keynesian model. The analogy is justified by the fact that a matching efficiency shock can also be seen as a technology shock in the production of new hires. Galí (1999) and Galí and Rabanal (2005) have linked the sign of the

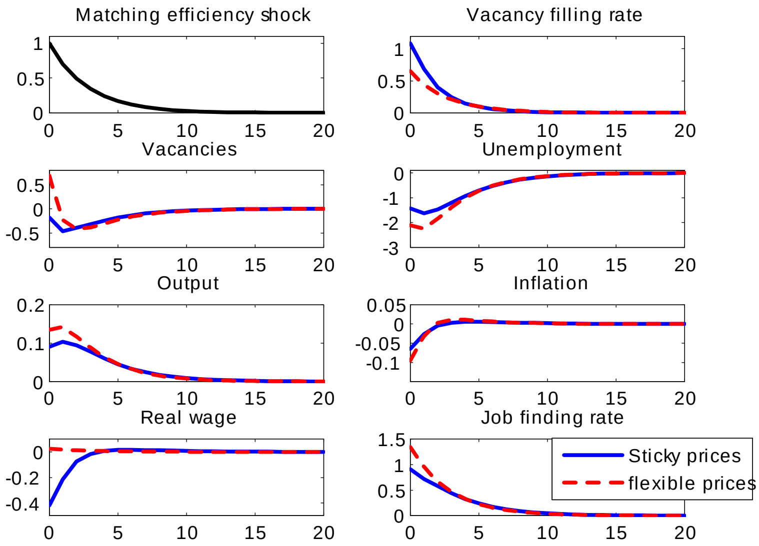

Figure 2: Impulse-responses in the model with pre-match hiring cost

employment/hours worked response to the presence of nominal rigidities and inertia in monetary policy. Interestingly, the same is true for the response of vacancies to a matching efficiency shock. The dotted line in figure 2 represents impulse responses in our model when prices are flexible: the response of vacancies is positive, as it is the one of employment when we simulate our model in response to a positive technology shock (see figure 3).

The relationship between the sign of the vacancy response and the degree of nominal rigidity can be shown also analytically in an extreme (but still interesting) case, following closely Galí (1999). For the sake of the argument, we consider the case of exogenous monetary policy (instead of an interest rate rule) and fixed prices (instead of sticky prices) and we postulate the following equation for money demand in log-linear terms

$$m_t - p_t = y_t$$

The assumptions of exogenous money and fixed prices imply that output is fixed

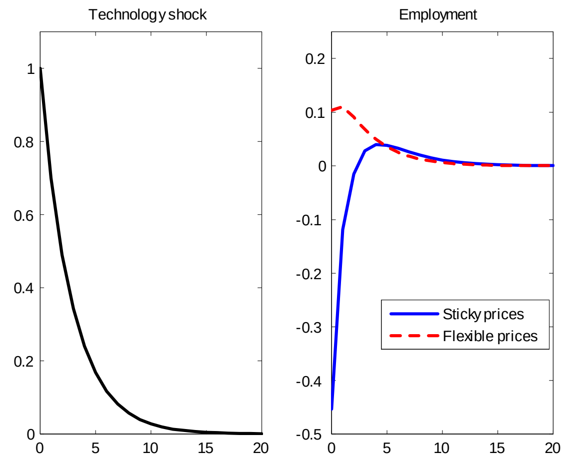

Figure 3: Impulse-responses to a technology shock in the model with pre-match hiring cost

in the period. Given fixed output and exogenous technology, employment is also fixed (see T.2). Then, from (T.3) there will be no job creation in response to the shock. Finally, the response of vacancies to matching efficiency shocks can be derived by using the matching function. Being new hires fixed in the period and searchers a predetermined variable, the following is true:

$$\ln \zeta_t = -(1-\sigma)v_t$$

According to our calibration ($\sigma = 0.5$), a one percent increase in the matching efficiency will be accompanied by a 0.5 percent decline in vacancies. Therefore, under the extreme case of exogenous money and fixed prices the vacancy response will be always negative. This is true also in our model although the decline in vacancies is of course lower, given that monetary policy is endogenous and prices are not fixed. Nevertheless, the larger is the degree of price rigidity (and the more inertial is monetary policy), the more negative is the vacancy response (as the more negative is the effect of a positive technology shock on the labor input).[6]

[6] Notice that the negative response of vacancies would be larger in models with additional nom-

Although a quantitative evaluation of the importance of matching efficiency shocks is not the objective of this paper, impulse responses in figure 2, and in particular the sign of the vacancy response, can give some insights on the relevance of this shock. In fact, unemployment and vacancies move in the same direction and they are almost perfectly positively correlated. Instead, it is well known that in the data unemployment and vacancies are strongly negatively correlated. This simple observation brings us to the conclusion that shocks to the matching efficiency cannot be an important source of aggregate fluctuations in a New Keynesian model with pre-match hiring costs, although they can be seen as shifters of the Beveridge curve. Interestingly, Galí (1999) used the same argument to downplay the importance of technology shocks in New Keynesian models.

Therefore, the argument based on the sign of the Beveridge curve reinforces even further the argument based on the importance of post-match hiring costs that we used in the previous section to downplay the importance of shocks to the matching efficiency in a New Keynesian model of the business cycle.

4 Our results in perspective

Our results from the previous section can be related to the literature on the importance of reallocation shocks that was initiated by Lilien (1982). Sectoral shifts in demand can have consequences in the aggregate macroeconomic variables if resources are not instantaneously mobile across sectors. The shock to the matching efficiency can be seen as a reallocation shock: if matches creation is easier within sector than across sectors, as it seems plausible, reallocation shocks will affect aggregate matching efficiency.

Lilien (1982) emphasize the importance of reallocation shocks that could ex-

inal (sticky wages) and real rigidities (habit persistence). However, since all the results presented in this paper would be reinforced in a more complicated model (with sticky wages, habit persistence and capital accumulation, cf. Furlanetto and Groshenny, 2011), we decided to consider the simplest set-up to make our point more transparent.

plain up to 50% of unemployment fluctuations in the postwar period. The empirical regularity underlying that result was a positive correlation between the dispersion of employment growth rates across sectors and the unemployment rate. However, Abraham and Katz (1986) show that this positive correlation is consistent not only with reallocation shocks but also with aggregate demand shocks under general conditions. Moreover, according to Abraham and Katz (1986), data on unemployment and vacancies are more useful to disentangle the importance of reallocation shocks. In fact, they argue that reallocation shocks should deliver a positive correlation between unemployment and vacancies as reallocation shocks can be seen as shifters of the Beveridge curve along a positively sloped job creation line.[7] Instead, aggregate demand shocks produce an inverse relationship between unemployment and vacancies, as it is observed in the unconditional data (summarized by a negatively sloped Beveridge curve). Therefore, according to Abraham and Katz (1986), data on unemployment and vacancies suggest the primacy of aggregate shocks, rather than reallocation shocks. That argument has been used as an identifying assumption in VARs (vector autoregressions) to reevaluate the importance of reallocation shocks. Blanchard and Diamond (1989) conclude that reallocation shocks play a minor role in unemployment fluctuations at least at business cycle frequencies.[8]

This paper contributes to the literature on the relationship between reallocation shocks and the slope of the Beveridge curve by highlighting the different role of pre-match and post-match hiring costs and by using a fully specified general equilibrium model rather than a partial equilibrium model as in the previous literature. On the one hand, the distinction between pre-match and post-match hiring costs is crucial: post-match hiring costs generate a vertical conditional Beveridge curve (given that unemployment is invariant to the shock) whereas pre-match hiring costs imply that unemployment and vacancies move in the same direction delivering a positively sloped conditional Beveridge curve. On the other hand, the general equilibrium

[7] The statement makes reference to a partial equilibrium model of the labor market with search and matching frictions, see Jackman, Layard and Pissarides (1989).

[8] For a useful review of empirical results in this literature, see Gallipoli and Pelloni (2008).

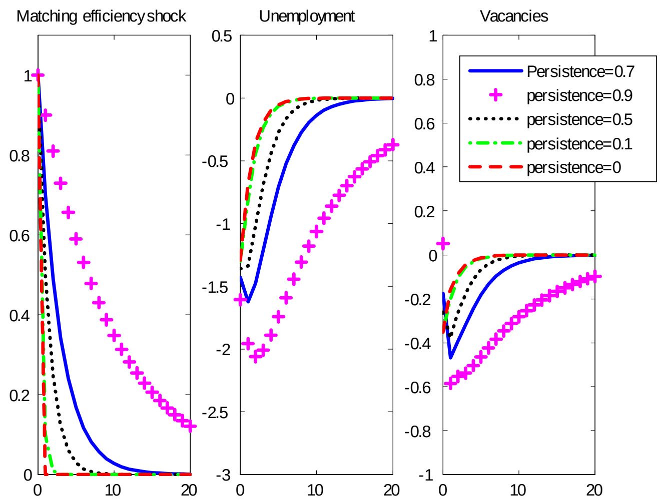

Figure 4: Impulse-responses in the model with pre-match hiring cost for different degrees of shock persistence

aspect becomes important when we investigate further the model with pre-match hiring costs. Our baseline model with sticky prices is fully consistent with the argument in Abraham and Katz (1986): conditional on matching shocks unemployment and vacancies are almost perfectly correlated and, importantly, the correlation does not depend on the autocorrelation in the shock process (see Figure 4 and Table 5) .

Table 5: corr(U_t, V_t) with pre-match hiring costs and sticky prices

$\rho_\zeta = 0.9$	0.95
$\rho_\zeta = 0.7$	0.97
$\rho_\zeta = 0.5$	0.99
$\rho_\zeta = 0.1$	1
$\rho_\zeta = 0$	1

However, this result is not as general as the previous literature has taken as given. In fact, it relies on the presence of nominal rigidities. From figure 5 and table 6, we see that a RBC version of our model ($\alpha = 0$) the correlation between on unemployment and vacancies depends on degree of autocorrelation in the shock process. When the shock process is very persistent we confirm the finding by Abraham and

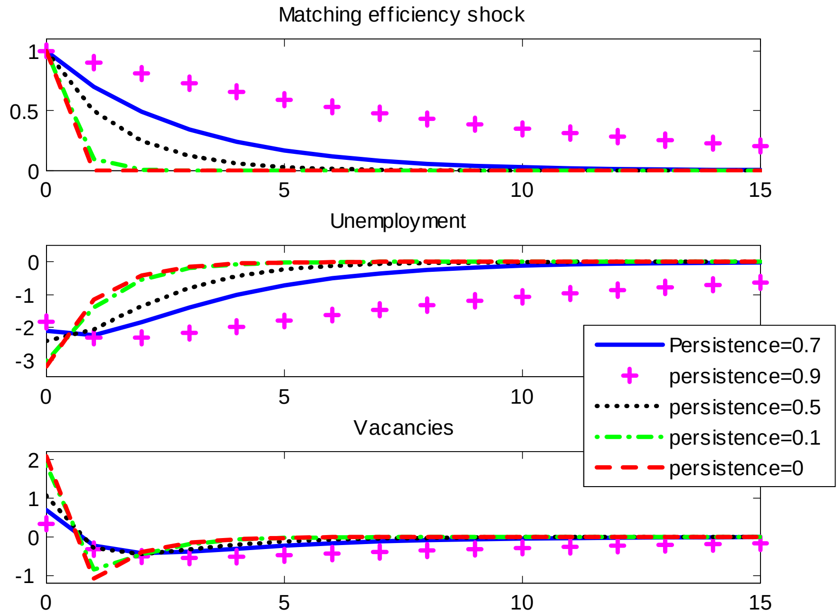

Figure 5: Impulse-responses in the model with pre-match hiring cost and flexible prices for different degrees of shock persistence

Katz (1986) also in an RBC set-up and the matching shock can be seen as a shifter of the Beveridge curve. But for lower degrees of persistence the correlation between unemployment and vacancies declines and becomes negative for values of ρ_m lower than 0.6. When the shock is iid the conditional correlation between unemployment and vacancies is -0.64, meaning that the conditional Beveridge curve has a negative slope, as in aggregate data. This point was raised also by Hosios (1994) but in a partial equilibrium model where the reallocation shock was modeled as a shock to the relative price dispersion across firms.[9] In his model, as in the flexible price version of our model with pre-match hiring costs, data on unemployment and vacancies are not conclusive to disentangle aggregate shocks and reallocation shocks. As far as we know, this is the first paper that show this point when the reallocation shock is given by a shock to the matching efficiency.

[9]Hosios (1994) considers also a second kind of reallocation shock, a shock to the job separation rate. That shock generate always a positively sloped Beveridge curve in his model. This is the case also in our model (results are available upon request).

Table 6: corr(U_t, V_t) with pre-match hiring costs and flexible prices

$\rho_\zeta = 0.9$	0.85
$\rho_\zeta = 0.7$	0.23
$\rho_\zeta = 0.5$	-0.23
$\rho_\zeta = 0.1$	-0.59
$\rho_\zeta = 0$	-0.64

We believe that our result has two implications. First, most of the literature on reallocation shocks is based on real business cycle models. We show that the assumption of flexible prices is not innocuous and that the interpretation of reallocation shocks as shifters of the Beveridge curve is robust only in a model with sticky prices. Second, this paper provides additional evidence that the presence of nominal rigidities crucially changes the transmission mechanism of shocks. This has been shown already for technology shocks (Galí, 1999), financial and different kind of investment shocks (Christiano, Motto and Rostagno, 2011, Del Negro, Eggertsson, Ferrero and Kyotaki, 2011, Furlanetto and Seneca, 2010 and 2011), fiscal shocks (Monacelli and Perotti, 2010 and Christiano, Eichenbaum and Rebelo, 2011). Here we show that this is relevant also for shocks to the matching efficiency.

Finally, our paper contributes to the literature on DSGE models with unemployment. Three recent papers by Lubik, 2009, Krause, Lubik and Lopez-Salido, 2008, and Justiniano and Michelacci, 2011 include shocks to the matching efficiency in estimated business cycle models for the US, although none of them focuses on the transmission mechanism for the shock to the matching efficiency. Importantly, the three papers reach very different conclusions on the role of matching efficiency shocks. Lubik (2009) finds that they explain 92% of unemployment and 38% of vacancy fluctuations in a RBC model very similar to our baseline model. Justiniano and Michelacci (2011) also estimate a RBC model for the US and for several other countries. However, in contrast to Lubik (2009), they find that matching efficiency shocks explain only 11% of unemployment and 3 percent of vacancy fluctuations in the US.[10] Our model can, at least in part,[11] reconcile these results: in Lubik (2009) hiring costs are only pre-match whereas in Justiniano and Michelacci (2011) there is also a post-match component. According to our analysis the larger is the weight of the post-match component, the lower should be the importance of matching shocks, in keeping with results in Lubik (2009) and Justiniano and Michelacci (2011). Finally, Krause, Lubik and López-Salido (2008) estimate a sticky price version of the model in Lubik (2009) where prices are flexible. They find that matching shocks explain 37 per cent of unemployment and only 1 per cent of vacancy fluctuations. According to our analysis the model with sticky prices imply a positively sloped conditional Beveridge curve whereas this is not always the case in a model with flexible prices (it depends on the persistence of the shock, that is not reported in Lubik, 2009). Therefore, our results can rationalize a more important role for matching shocks in RBC models.

5 Conclusion

Our analysis of the transmission mechanism for shocks to the matching efficiency emphasize the importance of the form of the hiring cost function and of the presence of nominal rigidities. When hiring costs are only post-match the shock does not propagate and matching efficiency shocks are irrelevant for business cycle fluctuations. When hiring costs are pre-match the shock propagates but generates a positively sloped Beveridge curve, in contrast to the unconditional empirical evidence, but in keeping with Abraham and Katz (1986), at least insofar prices are sticky and the shock is persistent.

More generally, our analysis shows that empirical models of the business cycle with unemployment should consider pre-match and post-match hiring costs in an

[10] Similar numbers are found for Germany, Norway and Sweden whereas there is evidence of a somewhat more important role for the shock in France and in the UK.

[11] The two models are similar but not identical. These differences can also influence the propagation of matching efficiency shocks.

integrated framework. This is the way we follow in a companion paper where we estimate a more empirically plausible model to see whether the natural rate of unemployment has increased during the Great Recession.

A further avenue for future research is to consider some of the determinants of matching efficiency in isolation. For example, the length of the unemployment benefit duration and the search effort of workers and firms can be modeled explicitly in simple extensions of the standard model. This exercise can be seen as a way to purify the Solow residual of the matching function, as it has been done for the production function. In that sense the role of endogenous search effort can play the same role of endogenous capital utilization in the production function. We leave these extensions for future research.

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List of common equilibrium conditions:

$$\Lambda_t = (C_t)^{-1}$$

$$\frac{\Lambda_t}{R_t} = \beta E_t \left(\frac{\Lambda_{t+1}}{\Pi_{t+1}}\right)$$

$$Y_t = A_t N_t$$

$$N_t = (1-\rho)\, N_{t-1} + Q_t V_t$$

$$U_t = 1 - N_t$$

$$S_t = 1 - (1-\rho)\, N_{t-1}$$

$$Q_t = L_t \left(\frac{V_t}{S_t}\right)^{-\sigma}$$

$$F_t = L_t \left(\frac{V_t}{S_t}\right)^{1-\sigma}$$

$$P_t(j) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \Lambda_{t+s} P_{t+s}^{\theta} C_{t+s} Z_{t+s}}{E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \Lambda_{t+s} P_{t+s}^{\theta-1} C_{t+s}}$$

Conditions specific to the model with post-match hiring costs

$$Y_t = C_t + \frac{\phi_N}{2} \left[\frac{Q_t V_t}{N_t}\right]^2 N_t$$

$$\frac{W_t}{P_t} = \eta \left[Z_t A_t + \phi_N X_t^2 + \beta (1 - \rho) \phi_N E_t \frac{\Lambda_{t+1}}{\Lambda_t} F_{t+1} X_{t+1}\right] + (1 - \eta) b$$

$$\phi_N X_t = Z_t A_t - \frac{W_t}{P_t} + \phi_N X_t^2 + \beta (1 - \rho) E_t \frac{\Lambda_{t+1}}{\Lambda_t} \phi_N X_{t+1}$$

Conditions specific to the model with pre-match hiring costs

$$Y_t = C_t + \phi_V V_t$$

$$\frac{W_t}{P_t} = \eta \left[Z_t A_t + \beta (1 - \rho) E_t \frac{\Lambda_{t+1}}{\Lambda_t} F_{t+1} \frac{\phi_V}{Q_{t+1}}\right] + (1 - \eta) b$$

$$\frac{\phi_V}{Q_t} = Z_t A_t - \frac{W_t}{P_t} + \beta\left(1-\rho\right) E_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\phi_V}{Q_{t+1}}$$

Steady state equations: common conditions

$$N = 1 - U$$

$$Y = N$$

$$S = 1 - \left(1-\rho\right) N$$

$$V = \frac{\rho N}{Q}$$

$$Z = \frac{\theta - 1}{\theta}$$

$$R = \frac{1}{\beta}$$

$$L = Q \left(\frac{V}{S}\right)^{\sigma}$$

$$F = L \left(\frac{V}{S}\right)^{1-\sigma}$$

Steady state equations: conditions specific to the model with post-match hiring costs

$$\frac{W}{P} = Z - \phi_N \rho \left(1-\rho\right)\left(1-\beta\right)$$

$$\phi_N = \frac{Z\left(1 - \frac{\eta}{1-\tau(1-\eta)}\right)}{\rho\left(1-\rho\right)\left(1-\beta\right) + \left(\frac{\eta}{1-\tau(1-\eta)}\right)\left(\rho^2 + \beta\left(1-\rho\right)F\rho\right)}$$

(check that...)

$$C = Y - \frac{\phi_N}{2}\rho^2 N$$

Steady state equations: conditions specific to the model with pre-match hiring costs

$$\frac{W}{P} = Z - \frac{\phi_V}{Q}\left(1 - \beta\left(1-\rho\right)\right)$$

$$\phi_V = \frac{QZ\left(1 - \tau\left(1 - \eta\right) - \eta\right)}{\left(1 - \beta\left(1 - \rho\right)\right)\left(1 - \tau\left(1 - \eta\right)\right) + \beta\left(1 - \rho\right)F\eta}$$

$$C = Y - \phi_V V$$