Commitment Through Renegotiation-Proof Contracts under Asymmetric Information

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The Question

- Can a player change the outcome of a game with third-party contracts?
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- Prevent entry with a financial contract?
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- Commit to a target inflation rate or budget deficit?
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- Prevent entry with a financial contract?
- Commit to a target inflation rate or budget deficit?
- We analyze this question in dynamic games with asymmetric information
- Contracts can be

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Example: Entry Game

\[ z - w > x - y > 0 \]
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Unique PBE: 2 plays \( AA \) and 1 enters
Example: Entry Game

- $z - w > x - y > 0$
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- Can 2 deter entry?
Example: Entry Game

- $z - w > x - y > 0$
- Unique PBE: 2 plays $AA$ and 1 enters
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- Can it be supported with non-renegotiable contracts?
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- Unique PBE: 2 plays $AA$ and 1 enters
- Can 2 deter entry?
- Can it be supported with non-renegotiable contracts?
- How about renegotiable contracts?
Contracts in Strategic Settings

- Contracts with third parties matter in strategic interactions
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  - Schelling (1960)
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  - Optimal contracts for central bankers
Contracts in Strategic Settings

- Two possible forms
Contracts in Strategic Settings

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  1. Delegation games: Agent plays
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  2. Games with side contracts: Original player plays
Contracts in Strategic Settings

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Observable and Non-renegotiable Contracts

- Folk theorems
Observable and Non-renegotiable Contracts

- Folk theorems
Entry Game: Observable and Non-renegotiable Contracts

◮ $FF$ deters entry
Entry Game: Observable and Non-renegotiable Contracts

- $FF$ deters entry
- A contract that supports $FF$:

$$f(F) = \delta, \quad f(A) = \delta + (z - w)$$
Unobservable and Non-renegotiable Contracts

Katz (1991)

- NE outcomes of game with contracts = NE outcomes of original game
Katz (1991)

- NE outcomes of game with contracts $= \text{NE outcomes of original game}$
- In extensive form games:
  
  SE outcomes of game with contracts $\subseteq \text{NE outcomes of original game}$
Unobservable and Non-renegotiable Contracts

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Koçkesen and Ok (2004) and Koçkesen (2007)

- In extensive form games:
  \[
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  \]
Entry Game: Unobservable and Non-renegotiable Contracts

\[(O, F F')\] is a (Bayesian) Nash equilibrium
Entry Game: Unobservable and Non-renegotiable Contracts

\[(O, FF')\) is a (Bayesian) Nash equilibrium

- It can be supported with unobservable contracts
(\(O, FF'\)) is a (Bayesian) Nash equilibrium

- It can be supported with unobservable contracts
- Can use the same contract

\[
f(F) = \delta, \quad f(A) = \delta + (z - w)
\]
Entry Game: Renegotiable Contracts

Can we support $FF$ with renegotiable contracts?
Entry Game: Renegotiable Contracts

◮ Can we support $F F F$ with renegotiable contracts?
◮ Not if renegotiation is frictionless
Renegotiable Contracts

- Some form of friction in renegotiation process is necessary
Renegotiable Contracts

- Some form of friction in renegotiation process is necessary
- Previous literature
Renegotiable Contracts

- Some form of friction in renegotiation process is necessary
- Previous literature
  - Asymmetric information
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  - Exogenous asymmetric information between player 2 and third-party
  - Similar to Dewatripont (1988) but arbitrary games
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Previous literature

- Asymmetric information
  - Dewatripont (1988): Entry game
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This paper

- Exogenous asymmetric information between player 2 and third-party
- Similar to Dewatripont (1988) but arbitrary games
  - Also we look at unobservable contracts and let informed player initiate renegotiation
Model

- Original game $G = (A_1, A_2, \Theta, p, u_1, u_2)$
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- **Game with Third-Party Contracts** \( \Gamma(G) \)
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- **Game with Third-Party Contracts** \( \Gamma(G) \)
  - Contracts \( f : A_1 \times A_2 \to \mathbb{R} \)
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- **Game with Third-Party Contracts** $\Gamma(G)$
  - Contracts $f : A_1 \times A_2 \rightarrow \mathbb{R}$
  - Payoff functions

\[
\begin{align*}
    v_1 (f, a_1, a_2, \theta) &= u_1 (a_1, a_2, \theta) \\
    v_2 (f, a_1, a_2, \theta) &= u_2 (a_1, a_2, \theta) - f (a_1, a_2) \\
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- Note:
Model

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- **Note:**
  - Third-party can only observe $(a_1, a_2)$ (not $\theta$)
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    \]

  - **Note:**
    - Third-party can only observe \( (a_1, a_2) \) (not \( \theta \))
    - Only player 2 can write contracts
Non-renegotiable Contracts

Stage I. Player 2 offers a contract \( f : A_1 \times A_2 \to \mathbb{R} \)
Non-renegotiable Contracts

Stage I. Player 2 offers a contract $f : A_1 \times A_2 \rightarrow \mathbb{R}$

Stage II. Third party accepts or rejects $f$
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Stage III. Nature chooses $\theta$
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Stage III. Nature chooses $\theta$
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Stage V. Player 2 observes \((\theta, a_1)\)
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Stage VI. Player 2 chooses $a_2$
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Stage VI. Player 2 chooses $a_2$

- If player 1 observes $f$ before choosing $a_1$ → Observable contracts
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Stage V. Player 2 observes $(\theta, a_1)$

Stage VI. Player 2 chooses $a_2$ or a new contract $g$
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Stage VI. Player 2 chooses $a_2$ or a new contract $g$
  - $a_2 \rightarrow$ game ends
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- $g \rightarrow$
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Stage VI. Player 2 chooses $a_2$ or a new contract $g$

- $a_2 \rightarrow$ game ends
- $g \rightarrow$

Stage VII(i). Third party (without observing $\theta$) accepts or rejects $g$
Renegotiable Contracts

Stage I. Player 2 offers a contract $f : A_1 \times A_2 \rightarrow \mathbb{R}$

Stage II. Third party accepts or rejects $f$

Stage III. Nature chooses $\theta$

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Stage V. Player 2 observes $(\theta, a_1)$

Stage VI. Player 2 chooses $a_2$ or a new contract $g$

- $a_2 \rightarrow$ game ends
- $g \rightarrow$

  Stage VII(i). Third party (without observing $\theta$) accepts or rejects $g$

Stage VII(ii). Player 2 chooses $a_2$
Renegotiable Contracts

Stage I. Player 2 offers a contract $f : A_1 \times A_2 \rightarrow \mathbb{R}$

Stage II. Third party accepts or rejects $f$

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Stage V. Player 2 observes $(\theta, a_1)$

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  - $a_2 \rightarrow$ game ends
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$\checkmark$ $a_2 \rightarrow$ game ends

$\checkmark$ $g \rightarrow$

Stage VII(i). Third party (without observing $\theta$) accepts or rejects $g$

Stage VII(ii). Player 2 chooses $a_2$

$\checkmark$ If player 1 observes $f$ before choosing $a_1 \rightarrow$ Observable contracts
Extensions

- Arbitrary extensive form games with incomplete information
Extensions

- Arbitrary extensive form games with incomplete information
- Interested Third-Party
Extensions

- Arbitrary extensive form games with incomplete information
- Interested Third-Party
- Strong renegotiation-proofness
Incentive Compatibility

$\theta$ non-contractible $\rightarrow$ need more structure
Incentive Compatibility

\( \theta \) non-contractible \( \rightarrow \) need more structure

Strictly Increasing Differences

\( u_2 \) has strictly increasing differences in \( (\succsim_\theta, \succsim_2) \):

\[ \theta \succ_\theta \theta', a_2 \succ_2 a_2' \Rightarrow \]

\[ u_2(a_1, a_2, \theta) - u_2(a_1, a_2, \theta') > u_2(a_1, a_2', \theta) - u_2(a_1, a_2', \theta') \]
Incentive Compatibility

**Increasing Strategies**

\[ b_2 : A_1 \times \Theta \rightarrow A_2 \text{ is increasing in } (\succsim_\theta, \succsim_2) \text{ if for all } a_1 \]

\[ \theta \succsim_\theta \theta' \Rightarrow b_2(a_1, \theta) \succsim_2 b_2(a_1, \theta') \]

\( B_2^+ \): Set of all increasing \( b_2 \).
Incentive Compatibility

Increasing Strategies

\( b_2 : A_1 \times \Theta \rightarrow A_2 \) is increasing in \((\succsim_\theta, \succsim_2)\) if for all \(a_1\)

\[ \theta \succsim_\theta \theta' \Rightarrow b_2(a_1, \theta) \succsim_2 b_2(a_1, \theta') \]

\( B_2^+ \): Set of all increasing \(b_2\).

Incentive Compatibility

\( u_2 \) has strictly increasing differences \(\Rightarrow\)

incentive compatibility \(\Leftrightarrow\) \( b_2 \) increasing
Entry Game: IC Strategies

- $c_h \succ c_l$ and $A \succ F$
Entry Game: IC Strategies

- $c_h \succ c_l$ and $A \succ F$
- Increasing differences: $z - w > x - y$
Entry Game: IC Strategies

- $c_h \succ c_l$ and $A \succ F$
- Increasing differences: $z - w > x - y$
- Incentive compatible strategies: $FF, FA, AA$
Renegotiation-Proofness

Definition (Renegotiation-Proof Equilibria)

A PBE of $\Gamma_R(G)$ is renegotiation-proof if the equilibrium contract is not renegotiated after any $a_1$ and $\theta$. 
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A PBE of $\Gamma_R(G)$ is renegotiation-proof if the equilibrium contract is not renegotiated after any $a_1$ and $\theta$.

Definition (Renegotiation-Proofness)
We say that $(f, b_2^*)$ is renegotiation-proof if for all $a_1$ and $\theta$ for which there exists an incentive compatible $(g, b_2)$ such that

$$u_2(a_1, b_2(a_1, \theta), \theta) - g(a_1, b_2(a_1, \theta)) > u_2(a_1, b_2^*(a_1, \theta), \theta) - f(b_2^*(a_1, \theta))$$

there exists a $\theta'$ such that

$$f(a_1, b_2^*(a_1, \theta')) \geq g(a_1, b_2(a_1, \theta'))$$
Renegotiation-Proofness

Renegotiation-Proof Strategies

A strategy $b_2$ is renegotiation-proof if there exists a contract $f$ such that $(f, b_2)$ is IC and RP

$B^R_2$: Set of all RP strategies
Renegotiation-Proof Contracts

Theorem 1

\((f, b_2^*)\) is RP iff for any \(a_1, i, \) and \(b_2 \in \mathcal{B}(a_1, i, b_2^*)\) there exists \(k\):

\[
\begin{align*}
 u_2(a_1, b_2(a_1, \theta^i), \theta^i) &- u_2(a_1, b_2^*(a_1, \theta^i), \theta^i) \\
 &+ \sum_{j=k}^{i-1} \vec{U}_2(a_1, b_2)_{2j-1} \leq f_k - f_i
\end{align*}
\]

or there exists \(l\):

\[
\begin{align*}
 u_2(a_1, b_2(a_1, \theta^i), \theta^i) &- u_2(a_1, b_2^*(a_1, \theta^i), \theta^i) \\
 &+ \sum_{j=i+1}^{l} \vec{U}_2(a_1, b_2)_{2(j-1)} \leq f_l - f_i
\end{align*}
\]

Proof
Renegotiation-Proof Strategies

- Necessary and sufficient conditions for renegotiation-proof strategies
Renegotiation-Proof Strategies

- Necessary and sufficient conditions for renegotiation-proof strategies
- Characterization when there are only two types
Entry Game: RP Strategies

Why is $FF$ not RP?
Entry Game: RP Strategies

- **FF** is not RP
  - Why is FF not RP?
  - $c_h$ must play A
Entry Game: RP Strategies

- **FF** is not RP
  - Why is **FF** not RP?
  - $c_h$ must play **A**
- RP strategies: **FA, AA**
  - Why is **FA** RP?
Entry Game: RP Strategies

- **FF** is not RP
  - $c_h$ must play $A$
- RP strategies: $FA$, $AA$

- RP strategy: Allow best response for the type who benefits most
Entry Game: RP Strategies

- **$FF$ is not RP** (Why is $FF$ not RP?)
  - $c_h$ must play $A$

- RP strategies: $FA, AA$ (Why is $FA$ RP?)

- RP strategy: Allow best response for the type who benefits most
  - High cost benefits most from accommodating
Entry Game: RP Strategies

- **FF** is not RP
  - \( c_h \) must play **A**
- RP strategies: **FA**, **AA**

RP strategy: Allow best response for the type who benefits most
- High cost benefits most from accommodating
- High unemployment benefits most from higher inflation or higher budget deficits
Entry Game: RP Strategies

- **FF** is not RP
  - Why is **FF** not RP?
  - $c_h$ must play **A**

- RP strategies: **FA, AA**
  - Why is **FA** RP?

- RP strategy: Allow best response for the type who benefits most
  - High cost benefits most from accommodating
  - High unemployment benefits most from higher inflation or higher budget deficits

- Credibility requires tolerance for the worst case scenarios
Observable and Non-renegotiable Contracts

Stackelberg Payoffs

\[
\bar{U}_2^B = \max_{b_2 \in B_2^+} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2) \\
\bar{U}_2^W = \max_{b_2 \in B_2^+} \min_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)
\]
Observable and Non-renegotiable Contracts

Stackelberg Payoffs

\[
\bar{U}_2^B = \max_{b_2 \in B_2^+} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)
\]

\[
\bar{U}_2^W = \max_{b_2 \in B_2^+} \min_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)
\]

Proposition 1

\( \bar{U}_2^B - \delta \) can be supported with non-renegotiable contracts.
Observable and Non-renegotiable Contracts

Stackelberg Payoffs

\[
\bar{U}_2^B = \max_{b_2 \in B_2^+} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)
\]

\[
\bar{U}_2^W = \max_{b_2 \in B_2^+} \min_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)
\]

Proposition 1
\(\bar{U}_2^B - \delta\) can be supported with non-renegotiable contracts.

Proposition 2
\(\bar{U}_2^W - \delta\) is the smallest payoff that can be supported with non-renegotiable contracts.
Entry Game: Observable Non-renegotiable Contracts

- Unique outcome that can be supported is no-entry
Unique outcome that can be supported is no-entry
Supported with strategy $FF$
Entry Game: Observable Non-renegotiable Contracts

- Unique outcome that can be supported is no-entry
- Supported with strategy $FF$
- A contract that supports $FF$

$$f(F) = \delta, \quad f(A) = \delta + (z - w)$$
Observable and Renegotiable Contracts

Stackelberg Payoffs

$$\bar{U}_2^{BR} = \max_{b_2 \in B_2^R} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$$

$$\bar{U}_2^{WR} = \max_{b_2 \in B_2^R} \min_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)$$
Observable and Renegotiable Contracts

Stackelberg Payoffs

\[
\bar{U}_{2}^{BR} = \max_{b_2 \in B_2^R} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)
\]

\[
\bar{U}_{2}^{WR} = \max_{b_2 \in B_2^R} \min_{b_1 \in BR_1(b_2)} U_2(b_1, b_2)
\]

Proposition 3

\(\bar{U}_{2}^{BR} - \delta \) can be supported with renegotiation-proof contracts.
Observable and Renegotiable Contracts

Stackelberg Payoffs

\[ \bar{U}^{BR}_2 = \max_{b_2 \in B^R_2} \max_{b_1 \in BR_1(b_2)} U_2(b_1, b_2) \]

\[ \bar{U}^{WR}_2 = \max_{b_2 \in B^R_2} \min_{b_1 \in BR_1(b_2)} U_2(b_1, b_2) \]

Proposition 3
\[ \bar{U}^{BR}_2 - \delta \] can be supported with renegotiation-proof contracts.

Proposition 4
\[ \bar{U}^{WR}_2 - \delta \] is the smallest payoff that can be supported with renegotiation-proof contracts.
Entry Game: Observable and RP Contracts

- RP strategies: $FA, AA$
Entry Game: Observable and RP Contracts

- RP strategies: $FA, AA$
- RP contract that supports $FA$:
  \[ f(F) = \delta, \quad f(A) = \delta + (x - y) \]
Entry Game: Observable and RP Contracts

- RP strategies: $FA, AA$
- RP contract that supports $FA$:
  \[ f(F) = \delta, \quad f(A) = \delta + (x - y) \]
- Pl. 1’s best response

\[
br_1(AA) = E
\]
\[
br_1(FA) = \begin{cases} 
O, & p(c_l) > 2/3 \\
E, & p(c_l) < 2/3 
\end{cases}
\]
Entry Game: Observable and RP Contracts

- **RP strategies**: $FA, AA$
- **RP contract that supports $FA$**: $f(F) = \delta, f(A) = \delta + (x - y)$
- **Pl. 1’s best response**

\[
br_1(AA) = E \\
\begin{align*}
br_1(FA) &= \begin{cases} O, & p(c_l) > 2/3 \\ E, & p(c_l) < 2/3 \end{cases} \end{align*}
\]

- If $p(c_l) > 2/3$ unique outcome that can be supported is no-entry
Entry Game: Observable and RP Contracts

- RP strategies: $FA, AA$
  - RP contract that supports $FA$:
    $$f(F) = \delta, \quad f(A) = \delta + (x - y)$$
- Pl. 1’s best response
  $$br_1(AA) = E$$
  $$br_1(FA) = \begin{cases} O, & p(c_l) > 2/3 \\ E, & p(c_l) < 2/3 \end{cases}$$

- If $p(c_l) > 2/3$ unique outcome that can be supported is no-entry
  - Using strategy $FA$
Entry Game: Observable and RP Contracts

- RP strategies: $FA, AA$
  - RP contract that supports $FA$:
    $$f(F) = \delta, \quad f(A) = \delta + (x - y)$$
- Pl. 1’s best response
  $$br_1(AA) = E$$
  $$br_1(FA) = \begin{cases} O, & p(c_l) > 2/3 \\ E, & p(c_l) < 2/3 \end{cases}$$

- If $p(c_l) > 2/3$ unique outcome that can be supported is no-entry
  - Using strategy $FA$
- If $p(c_l) < 2/3$ unique outcome that can be supported is entry and accommodate
Unobservable and Non-renegotiable Contracts

Proposition 5

\((b_1^*, b_2^*)\) can be supported iff

1. \((b_1^*, b_2^*)\) is a Bayesian Nash equilibrium of \(G\)
2. \(b_2^*\) is increasing
Unobservable and Non-renegotiable Contracts

Individually rational payoff of player 1:

\[ U_1^+ = \max_{a_1} \min_{b_2 \in B_2^+} U_1(a_1, b_2) \]
Unobservable and Non-renegotiable Contracts

Individually rational payoff of player 1:

$$U_1^+ = \max_{a_1} \min_{b_2 \in B_2^+} U_1(a_1, b_2)$$

Corollary 1

Outcome $\left(a_1^*, a_2^*\right)$ can be supported iff

1. $a_2^*(\theta) \in BR_2(a_1^*, \theta)$ for all $\theta$ and
2. $U_1(a_1^*, a_2^*) \geq U_1^+$
Proposition 6

\((b_1^*, b_2^*)\) can be supported iff

1. \((b_1^*, b_2^*)\) is a Bayesian Nash equilibrium of \(G\)
2. \(b_2^*\) is increasing and renegotiation-proof
Individually rational payoff of player 1:

$$U_1^R = \max_{a_1 \in A_1} \min_{b_2 \in B_2^R} U_1(a_1, b_2)$$
Individually rational payoff of player 1:

$$U_1^R = \max_{a_1 \in A_1} \min_{b_2 \in B_2^R} U_1(a_1, b_2)$$

**Corollary 2**

*Outcome \((a_1^*, a_2^*)\) can be supported iff*

1. \(a_2^*(\theta) \in BR_2(a_1^*, \theta)\) for all \(\theta\) and
2. \(U_1(a_1^*, a_2^*) \geq U_1^R\)
Entry Game: Unobservable Contracts

In addition to no-entry, entry and accommodate also supported
Entry Game: Unobservable Contracts

In addition to no-entry, entry and accommodate also supported

Unobservable contracts expand the set of equilibrium outcomes
Conclusions

- Observable contracts as commitment devices
Conclusions

- Observable contracts as commitment devices
  - non-renegotiable $\rightarrow$ Stackelberg payoff
Conclusions

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  - renegotiation-proofness weakens their power but does not erase it
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- Observable contracts as commitment devices
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Conclusions

- Observable contracts as commitment devices
  - non-renegotiable $\rightarrow$ Stackelberg payoff
  - renegotiation-proofness weakens their power but does not erase it
- Unobservable contracts expand the set of equilibrium outcomes
- Credibility requires tolerance for some scenarios
Entry Game

- Why is $FF$ not RP?
Why is $FF$ not RP?

Consider

$$g(F) = g(A) = f(F) + \frac{x - y}{2}$$
Entry Game

- Why is $FF$ not RP?
- Consider

\[ g(F) = g(A) = f(F) + \frac{x - y}{2} \]

- Under $g$ optimal strategy is $AA$
Entry Game

- Why is $FF$ not RP?
- Consider

$$g(F) = g(A) = f(F) + \frac{x - y}{2}$$

- Under $g$ optimal strategy is $AA$
- Type $c_l$ is better off

$$x - g(A) > y - f(F)$$
Entry Game

- Why is $FF$ not RP?
- Consider

\[ g(F) = g(A) = f(F) + \frac{x - y}{2} \]

- Under $g$ optimal strategy is $AA$
- Type $c_l$ is better off

\[ x - g(A) > y - f(F) \]

- Third-party is better off too

\[ g(A) > f(F) \]
Entry Game

- Why is $FF$ not RP?
- Consider

$$g(F) = g(A) = f(F) + \frac{x - y}{2}$$

- Under $g$ optimal strategy is $AA$
- Type $c_l$ is better off

$$x - g(A) > y - f(F)$$

- Third-party is better off too

$$g(A) > f(F)$$
Entry Game

- Why is $FF$ not RP?
- Consider

$$g(F) = g(A) = f(F) + \frac{x - y}{2}$$

- Under $g$ optimal strategy is $AA$
- Type $c_l$ is better off

$$x - g(A) > y - f(F)$$

- Third-party is better off too

$$g(A) > f(F)$$
Entry Game

- Why is $FA$ RP?
Entry Game

- Why is $F_A$ RP?
- Incentive compatibility

\[ x - y \leq f(A) - f(F) \leq z - w \]  (1)
Entry Game

- Why is $FA$ RP?
- Incentive compatibility

$$x - y \leq f(A) - f(F) \leq z - w$$  \hspace{1cm} (1)

- Only $c_l$ is not best responding. Can he renegotiate to $AA$?
Entry Game

- Why is \( FA \) RP?
- Incentive compatibility

\[
x - y \leq f(A) - f(F) \leq z - w
\]  

(1)

- Only \( c_l \) is not best responding. Can he renegotiate to \( AA \)?
- Need a \( g \) that gives incentives to play \( AA \) and

\[
x - g(A) > y - f(F)
\]  

(2)

\[
g(A) > f(F)
\]  

(3)

\[
g(A) > f(A)
\]  

(4)
Entry Game

- Why is $FA$ RP?
- Incentive compatibility

$$x - y \leq f(A) - f(F) \leq z - w \quad (1)$$

- Only $c_l$ is not best responding. Can he renegotiate to $AA$?
- Need a $g$ that gives incentives to play $AA$ and

$$x - g(A) > y - f(F) \quad (2)$$

$$g(A) > f(F) \quad (3)$$

$$g(A) > f(A) \quad (4)$$

- (2) and (4) imply

$$f(A) < g(A) < f(F) + (x - y)$$

contradicting (1)
Entry Game

- Why is $FA$ RP?
- Incentive compatibility

\[ x - y \leq f(A) - f(F) \leq z - w \]  \hspace{1cm} (1)

- Only $c_l$ is not best responding. Can he renegotiate to $AA$?
- Need a $g$ that gives incentives to play $AA$ and

\[ x - g(A) > y - f(F) \]  \hspace{1cm} (2)
\[ g(A) > f(F) \]  \hspace{1cm} (3)
\[ g(A) > f(A) \]  \hspace{1cm} (4)

- (2) and (4) imply

\[ f(A) < g(A) < f(F) + (x - y) \]

contradicting (1)
Entry Game

- Why is $FA$ RP?
- Incentive compatibility

\[
x - y \leq f(A) - f(F) \leq z - w \tag{1}
\]

- Only $c_l$ is not best responding. Can he renegotiate to $AA$?
- Need a $g$ that gives incentives to play $AA$ and

\[
x - g(A) > y - f(F) \tag{2}
\]

\[
g(A) > f(F) \tag{3}
\]

\[
g(A) > f(A) \tag{4}
\]

- (2) and (4) imply

\[
f(A) < g(A) < f(F) + (x - y)
\]

contradicting (1)
Proof of Theorem 1

\[ \theta^n \preceq \cdots \preceq \theta^1 \]
Proof of Theorem 1

1. $\theta^n \preccurlyeq \ldots \preccurlyeq \theta^1$
2. $f \in \mathbb{R}^n$, $f_j = f(a_1, b_2(a_1, \theta^j))$
Proof of Theorem 1

\[
\begin{align*}
\theta^n &\succ \cdots \succ \theta^1 \\
f &\in \mathbb{R}^n, \quad f_j = f(a_1, b_2(a_1, \theta^j)) \\
\text{Increasing differences } \Rightarrow \text{ IC equivalent to } \\
&f_j - f_{j+1} \leq u_2(a_1, b_2(a_1, \theta^j), \theta^j) - u_2(a_1, b_2(a_1, \theta^{j+1}), \theta^j) \\
&-f_{j-1} + f_j \leq u_2(a_1, b_2(a_1, \theta^j), \theta^j) - u_2(a_1, b_2(a_1, \theta^{j-1}), \theta^j)
\end{align*}
\]
Proof of Theorem 1

- $\theta^n \succeq \ldots \succeq \theta^1$
- $f \in \mathbb{R}^n$, $f_j = f(a_1, b_2(a_1, \theta^j))$
- Increasing differences $\Rightarrow$ IC equivalent to

\[
\begin{align*}
    f_j - f_{j+1} &\leq u_2(a_1, b_2(a_1, \theta^j), \theta^j) - u_2(a_1, b_2(a_1, \theta^{j+1}), \theta^j) \\
    -f_{j-1} + f_j &\leq u_2(a_1, b_2(a_1, \theta^j), \theta^j) - u_2(a_1, b_2(a_1, \theta^{j-1}), \theta^j)
\end{align*}
\]

- Can write these as $Df \leq \vec{U}_2(b_2)$
Proof of Theorem 1

\((f, b_2^*)\) not RP iff there exist \(a_1, i, \) and IC \((g, b_2)\):

1. \(u_2(a_1, b_2(a_1, \theta^i), \theta^i) - g_i > u_2(a_1, b_2^*(a_1, \theta_i), \theta_i) - f_i\)

2. \(g_j > f_j\) for all \(j\)
Proof of Theorem 1

\((f, b_2^*)\) not RP iff there exist \(a_1, i,\) and IC \((g, b_2)\):

1. \(u_2(a_1, b_2(a_1, \theta^i), \theta^i) - g_i > u_2(a_1, b_2^*(a_1, \theta_i), \theta_i) - f_i\)
2. \(g_j > f_j\) for all \(j\)

\((f, b_2^*)\) not RP iff there exist \(a_1, i, b_2, \varepsilon\):

1. \(D(f + \varepsilon) \leq \vec{U}_2(b_2)\)
2. \(\varepsilon_i < u_2(a_1, b_2(a_1, \theta^i), \theta^i) - u_2(a_1, b_2^*(a_1, \theta^i), \theta^i)\)
3. \(\varepsilon \gg 0\)
Proof of Theorem 1

\((f, b^*_2)\) not RP iff there exist \(a_1, i,\) and IC \((g, b_2)\):
\begin{enumerate}
\item \(u_2(a_1, b_2(a_1, \theta^i), \theta^i) - g_i > u_2(a_1, b^*_2(a_1, \theta_i), \theta_i) - f_i\)
\item \(g_j > f_j\) for all \(j\)
\end{enumerate}

\((f, b^*_2)\) not RP iff there exist \(a_1, i, b_2, \varepsilon\):
\begin{enumerate}
\item \(D(f + \varepsilon) \leq \vec{U}_2(b_2)\)
\item \(\varepsilon_i < u_2(a_1, b_2(a_1, \theta^i), \theta^i) - u_2(a_1, b^*_2(a_1, \theta^i), \theta^i)\)
\item \(\varepsilon \gg 0\)
\end{enumerate}

\((f, b^*_2)\) not RP iff \([Ax \gg 0, Cx \geq 0\) has a solution \(x\)]
Proof of Theorem 1

\((f, b_2^*)\) not RP iff there exist \(a_1, i,\) and IC \((g, b_2)\):

1. \(u_2(a_1, b_2(a_1, \theta^i), \theta^i) - g_i > u_2(a_1, b_2^*(a_1, \theta^i), \theta^i) - f_i\)
2. \(g_j > f_j\) for all \(j\)

\((f, b_2^*)\) not RP iff there exist \(a_1, i, b_2, \varepsilon:\)

1. \(D(f + \varepsilon) \leq \vec{U}_2(b_2)\)
2. \(\varepsilon_i < u_2(a_1, b_2(a_1, \theta^i), \theta^i) - u_2(a_1, b_2^*(a_1, \theta^i), \theta^i)\)
3. \(\varepsilon \gg 0\)

\((f, b_2^*)\) not RP iff \([Ax \gg 0, Cx \geq 0\) has a solution \(x)\]

RP iff \([A'y_1 + C'y_2 = 0, y_1 > 0, y_2 \geq 0\) has a solution \(y_1, y_2)\]
Proof of Theorem 1

\((f, b_2^*)\) not RP iff there exist \(a_1\), \(i\), and IC \((g, b_2)\):

1. \(u_2(a_1, b_2(a_1, \theta^i), \theta^i) - g_i > u_2(a_1, b_2^*(a_1, \theta_i), \theta_i) - f_i\)
2. \(g_j > f_j\) for all \(j\)

\((f, b_2^*)\) not RP iff there exist \(a_1\), \(i\), \(b_2\), \(\varepsilon\):

1. \(D(f + \varepsilon) \leq \bar{U}_2(b_2)\)
2. \(\varepsilon_i < u_2(a_1, b_2(a_1, \theta^i), \theta^i) - u_2(a_1, b_2^*(a_1, \theta^i), \theta^i)\)
3. \(\varepsilon \gg 0\)

\((f, b_2^*)\) not RP iff \([Ax \gg 0, Cx \geq 0\) has a solution \(x\)]

RP iff \([A'y_1 + C'y_2 = 0, y_1 > 0, y_2 \geq 0\) has a solution \(y_1, y_2\)]